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HOLOMORPHIC L^p-FUNCTIONS ON COVERINGS OF STRONGLY PSEUDOCONVEX MANIFOLDS

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ABSTRACT. In this paper we show how to construct holomorphic L^p-functions on unbranched coverings of strongly pseudoconvex manifolds. Also, we prove some extension and approximation theorems for such functions.

1. INTRODUCTION

1.1. The present paper continues the study of holomorphic functions of slow growth on unbranched coverings of strongly pseudoconvex manifolds started in [2]–[4]. Our work was inspired by the seminal paper [7] of Gromov, Henkin and Shubin on holomorphic L^2-functions on coverings of pseudoconvex manifolds. This subject is of particular interest because of its possible applications to the Shafarevich conjecture on holomorphic convexity of universal coverings of complex projective manifolds. The results of this paper don't imply directly any new results in the area of the Shafarevich conjecture. However, one obtains a rich complex function theory on coverings of strongly pseudoconvex manifolds that together with some additional methods and ideas would lead to progress in studying this conjecture.

The main result of [4] deals with holomorphic L^2-functions on unbranched coverings of strongly pseudoconvex manifolds. In the present paper we use this result to construct holomorphic L^p-functions, $1 \le p \le \infty$, on such coverings. Also, we prove some extension and approximation theorems for these functions. In our proofs we exploit some ideas based on infinite-dimensional versions of Cartan's A and B theorems originally proved by Bungart [1] (see also [8] and references therein for some generalizations of results from complex function theory to the case of Banach-valued holomorphic functions).

1.2. To formulate our results we first recall some basic definitions.

Let $M \subset\subset N$ be a domain with smooth boundary bM in an n-dimensional complex manifold N; specifically,

$$M = \{z \in N \ : \ \rho(z) < 0\} \tag{1.1}$$

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where ρ is a real-valued function of class $C^2(\Omega)$ in a neighbourhood Ω of the compact set $\overline{M} := M \cup bM$ such that

$$d\rho(z) \neq 0 \quad \text{for all} \quad z \in bM. \tag{1.2}$$

Let $z_1, \dots, z_n$ be complex local coordinates in N near $z \in bM$. Then the tangent space T_zN at z is identified with $\mathbb{C}^n$. By $T_z^c(bM) \subset T_zN$ we denote the complex tangent space to bM at z, i.e.,

$$T_z^c(bM) = \{w = (w_1, \dots, w_n) \in T_z(N) \ : \ \sum_{j=1}^n \frac{\partial \rho}{\partial z_j}(z) w_j = 0\}. \tag{1.3}$$

The *Levi form* of ρ at $z \in bM$ is a hermitian form on $T_z^c(bM)$ defined in the local coordinates by the formula

$$L_z(w, \overline{w}) = \sum_{j,k=1}^n \frac{\partial^2 \rho}{\partial z_j \partial \overline{z}_k}(z) w_j \overline{w}_k. \tag{1.4}$$

The manifold M is called *strongly pseudoconvex* if $L_z(w, \overline{w}) > 0$ for all $z \in bM$ and all $w \neq 0$, $w \in T_z^c(bM)$. Equivalently, strongly pseudoconvex manifolds can be described as manifolds which locally, in a neighbourhood of any boundary point, can be presented as strictly convex domains in $\mathbb{C}^n$. It is also known (see [6], [9]) that any strongly pseudoconvex manifold admits a proper holomorphic map with connected fibres onto a normal Stein space.

1.3. Without loss of generality we may and will assume that $\pi_1(M) = \pi_1(N)$ for M as above and N strongly pseudoconvex as well. (Here $\pi_1(X)$ stands for the fundamental group of X.) Let $r : N' \to N$ be an unbranched covering of N. For $U \subset N$ we set $U' := r^{-1}(U)$.

Let $\psi : N' \to \mathbb{R}_+$ be such that $\log \psi$ is uniformly continuous with respect to the path metric induced by a Riemannian metric pulled back from N. By $\mathcal{H}_{p,\psi}(M')$, $1 \le p < \infty$, we denote the Banach space of holomorphic functions g on M' with norm

$$|g|_{p,\psi} := \sup_{x \in M} \left(\sum_{y \in r^{-1}(x)} |f(y)|^p \psi(y) \right)^{1/p}. \tag{1.5}$$

By $\mathcal{H}_{\infty,\psi}(M')$ we denote the Banach space of holomorphic functions g on M' with norm

$$|g|_{\infty,\psi} := \sup_{z \in M'} \{|g(z)|\psi(z)\}. \tag{1.6}$$

Example 1.1. Let d be the path metric on N' obtained by the pullback of a Riemannian metric defined on N. Fix a point $o \in M'$ and set

$$d_o(x) := d(o, x), \quad x \in N'.$$

It is easy to show by means of the triangle inequality that as the function ψ one can take, e.g., $(1 + d_o)^\alpha$ or $e^{\alpha d_o}$ with $\alpha \in \mathbb{R}$.

Remark 1.2. Let $dV_{M'}$ be the Riemannian volume form on the covering M' obtained by a Riemannian metric pulled back from N. Note that every $f \in \mathcal{H}_{p,\psi}(M')$,

$1 \le p < \infty$, also belongs to the Banach space $H^p_\psi(M')$ of holomorphic functions g on M' with norm

$$\left(\int_{z\in M'} |g(z)|^p \psi(z) dV_{M'}(z)\right)^{1/p}.$$

Moreover, one has a continuous embedding $\mathcal{H}_{p,\psi}(M') \hookrightarrow H^p_\psi(M')$.

Next, we introduce the Banach space $l_{p,\psi,x}(M')$, $x \in M$, $1 \le p < \infty$, of functions g on $r^{-1}(x)$ with norm

$$|g|_{p,\psi,x} := \left(\sum_{y\in r^{-1}(x)} |g(y)|^p \psi(y)\right)^{1/p}, \tag{1.7}$$

and the Banach space $l_{\infty,\psi,x}(M')$, $x \in M$, of functions g on $r^{-1}(x)$ with norm

$$|g|_{\infty,\psi,x} := \sup_{y\in r^{-1}(x)} \{|g(y)|\psi(y)\}. \tag{1.8}$$

Let $C_M \subset M$ be the union of all compact complex subvarieties of M of complex dimension ≥ 1. It is known that if M is strongly pseudoconvex, then C_M is a compact complex subvariety of M.

In the following, for Banach spaces E and F, $\mathcal{B}(E, F)$ denotes the space of all linear bounded operators $E \to F$ with norm $||\cdot||$.

Our main result is the following interpolation theorem.

Theorem 1.3. *Suppose that $\Omega \subset M \setminus C_M$ is an open Stein subset and $K \subset\subset \Omega$. Then for any $p \in [1, \infty]$ there exists a family $\{L_z \in \mathcal{B}(l_{p,\psi,z}(M'), \mathcal{H}_{p,\psi}(M'))\}_{z\in\Omega}$, holomorphic in z, such that*

$$(L_z h)(x) = h(x) \quad \text{for any} \quad h \in l_{p,\psi,z}(M') \quad \text{and} \quad x \in r^{-1}(z).$$

Moreover,

$$\sup_{z\in K} ||L_z|| < \infty.$$

A similar result for M being a bounded domain in a Stein manifold was proved in [5, Theorem 1.3].

1.4. To formulate applications of Theorem 1.3 we recall some definitions from [5].

Definition 1.4. Let $r : N' \to N$ be a covering and $X \subset N$ a complex submanifold of N. By $\mathcal{H}_{p,\psi}(X')$, $X' := r^{-1}(X)$, we denote the Banach space of holomorphic functions f on X' such that $f|_{r^{-1}(x)} \in l_{p,\psi,x}(N')$ for any $x \in X$ with norm

$$\sup_{x\in X} |f|_{r^{-1}(x)}|_{p,\psi,x}. \tag{1.9}$$

As an application of Theorem 1.3 we prove a result on extension of holomorphic functions from complex submanifolds.

Let U be a relatively compact open subset of a holomorphically convex domain $V \subset\subset N$ containing C_M, and let $Y \subset V \setminus C_M$ be a closed complex submanifold of V. We set $X := Y \cap U$. Consider a covering $r : N' \to N$.

Theorem 1.5. *For every $f \in \mathcal{H}_{p,\psi}(Y')$, there is a function $F \in \mathcal{H}_{p,\psi}(U')$ such that $F = f$ on X'.*

Remark 1.6. Let $M \subset\subset N$ be a strongly pseudoconvex manifold. As before, we assume that $\pi_1(M) = \pi_1(N)$ and that N is strongly pseudoconvex as well. Then there exist a normal Stein space X_N, a proper holomorphic surjective map $p : N \to X_N$ with connected fibres, and points $x_1, \dots, x_l \in X_N$ such that

$$p : N \setminus \bigcup_{1\le i\le l} p^{-1}(x_i) \to X_N \setminus \bigcup_{1\le i\le l} \{x_i\}$$

is biholomorphic; see [6], [9]. By definition, the domain $X_M := p(M) \subset X_N$ is strongly pseudoconvex (so it is Stein). Without loss of generality we will assume that $x_1, \dots, x_l \in X_M$ so that $\bigcup_{1\le i\le l} p^{-1}(x_i) = C_M$. Next, $X_V := p(V)$ is a Stein subdomain of X_N. Now, we take Y to be the preimage under p of a closed complex submanifold of X_V that does not contain points $x_1, \dots, x_l$.

Another application of Theorem 1.3 is the following approximation result.

Theorem 1.7. *Let $K \subset\subset M \setminus C_M$ be a compact holomorphically convex subset and let $O \subset M \setminus C_M$ be a neighbourhood of K. Then every function $f \in \mathcal{H}_{p,\psi}(O')$ can be uniformly approximated on K' in the norm of $\mathcal{H}_{p,\psi}(K')$ by holomorphic functions from $\mathcal{H}_{p,\psi}(M')$.*

In the case of coverings of Stein manifolds, the results similar to Theorems 1.5 and 1.7 are proved in [5, Theorems 1.8, 1.10].

2. Proof of Theorem 1.3

2.1. We begin the proof with the following auxiliary result.

Proposition 2.1. *For every $z \in M \setminus C_M$ and $p \in [1, \infty]$ there is a linear operator $T_{\psi,z} \in \mathcal{B}(l_{p,\psi,z}(M'), \mathcal{H}_{p,\psi}(M'))$ such that*

$$(T_{\psi,z}h)(x) = h(x) \quad \text{for any} \quad h \in l_{p,\psi,z}(M') \quad \text{and} \quad x \in r^{-1}(z).$$

(In what follows we call such $T_{\psi,z}$ a linear interpolation operator.)

Proof. Let $\widetilde{M} \subset\subset N$ be a strongly pseudoconvex manifold containing $\overline{M}$ and let $\widetilde{M}' := r^{-1}(\widetilde{M}) \subset N'$ be the corresponding covering of $\widetilde{M}$. Then Theorem 1.1(a) of [4] implies that for every function $f \in l_{2,\psi,z}(M')$ there exists $F \in H^2_\psi(\widetilde{M}')$ such that $F|_{r^{-1}(z)} = f$. Thus the operator $R_z : H^2_\psi(\widetilde{M}') \to l_{2,\psi,z}(M')$, $R_z g := g|_{r^{-1}(z)}$, is a linear continuous surjective map of Hilbert spaces. In particular, there is a linear continuous map $S_{\psi,z} : l_{2,\psi,z}(M') \to H^2_\psi(\widetilde{M}')$ such that $R_z \circ S_{\psi,z} = id$.

Remark 2.2. The fact that R_z maps $H^2_\psi(\widetilde{M}')$ into $l_{2,\psi,z}(M')$ and is continuous easily follows from the uniform continuity of $\log\psi$ and the mean value property for plurisubharmonic functions. Similarly one obtains that the restriction operator $R_{M'} : H^2_\psi(\widetilde{M}') \to \mathcal{H}_{2,\psi}(M')$, $g \mapsto g|_{M'}$, is continuous.

We set

$$T_{\psi,z} := R_{M'} \circ S_{\psi,z}.$$

Then $T_{\psi,z}$ is the required interpolation operator for $p = 2$. Let us prove the result for $p \neq 2$.

We will naturally identify $r^{-1}(z)$ with $\{z\}\times S$ where S is the fibre of r. Let $\{e_s\}_{s\in S}$, with $e_s(z,t)=0$ for $t\neq s$ and $e_s(z,s)=(\psi(z,s))^{-1/2}$, be the orthonormal basis of $l_{2,\psi,z}(M')$. We set

$$h_{s,z}:=T_{\psi,z}(e_s)\in\mathcal{H}_{2,\psi}(M').$$

Then for a sequence $a=\{a_s\}_{s\in S}\in l^2(S)$ we have

$$h_a:=\sum_{s\in S}a_sh_{s,z}\in\mathcal{H}_{2,\psi}(M')\quad\text{and}\quad |h_a|_{2,\psi}\le c||a||_{l^2(S)}. \tag{2.1}$$

We define $F_{s,z}\in\mathcal{H}_{1,\psi}(M')$ by the formula

$$F_{s,z}(w):=\psi(z,s)\,h_{s,z}^2(w),\quad w\in M'. \tag{2.2}$$

Then (2.1) yields

$$\sum_{s\in S}\frac{|F_{s,z}(w)|}{\psi(z,s)}\,\psi(w)\le c^2,\quad w\in M'. \tag{2.3}$$

Next, for $a=\{a_s\}_{s\in S}\in l_{p,\psi,z}(M')$ (i.e., $\sum_{s\in S}|a_s|^p\,\psi(z,s):=|a|^p_{p,\psi,z}<\infty$) we define

$$\widetilde{T}_{\psi,z}(a):=\sum_{s\in S}a_sF_{s,z}. \tag{2.4}$$

For $p=1,\infty$ we set $T_{\psi,z}:=\widetilde{T}_{\psi,z}$ and show that $T_{\psi,z}$ is the required interpolation operator. In fact, for $p=\infty$, using (2.3) we obtain

$$\sup_{w\in M'}\{|(T_{\psi,z}(a))(w)|\psi(w)\}:=\sup_{w\in M'}\left\{\left|\sum_{s\in S}a_sF_{s,z}(w)\right|\psi(w)\right\}$$

$$\le\left(\sup_{s\in S}\{|a_s|\psi(z,s)\}\right)\cdot\left(\sup_{w\in M'}\left\{\sum_{s\in S}\frac{|F_{s,z}(w)|}{\psi(z,s)}\,\psi(w)\right\}\right)\le c^2|a|_{\infty,\psi,z}.$$

Also,

$$(T_{\psi,z}a)(z,t):=\sum_{s\in S}a_sF_{s,z}(z,t):=a_t\psi(z,t)\,e_t^2(z,t)=a_t:=a(z,t).$$

Thus $T_{\psi,z}$ is the required interpolation operator for $p=\infty$.

Similarly, for $p=1$ we have, from (2.1) with $h_a:=h_{s,z}$,

$$\sup_{w\in M}\left\{\sum_{y\in r^{-1}(w)}|(T_{\psi,z}(a))(y)|\psi(y)\right\}:=\sup_{w\in M}\left\{\sum_{y\in r^{-1}(w)}\left|\sum_{s\in S}a_sF_{s,z}(y)\right|\psi(y)\right\}$$

$$\le\left(\sum_{s\in S}|a_s|\psi(z,s)\right)\cdot\left(\sup_{w\in M}\left\{\sup_{s\in S}\left\{\sum_{y\in r^{-1}(w)}\frac{|F_{s,z}(y)|}{\psi(z,s)}\,\psi(y)\right\}\right\}\right)\le c^2|a|_{1,\psi,z}.$$

Thus $T_{\psi,z}$ is the required interpolation operator for $p=1$.

Now, using the M. Riesz interpolation theorem (see, e.g., [10]) and arguing as in the proof of Lemma 3.2(a) of [5] from the cases considered above, we obtain that the operator $\widetilde{T}_{\psi,z}$ maps $l_{p,\psi^p,z}(M')$ continuously into $\mathcal{H}_{p,\psi^p}(M')$, $1<p<\infty$, and its norm is bounded by c^2. For such p we define

$$T_{\psi,z}:=\widetilde{T}_{\psi^{1/p},z}.$$

(Note that $\psi^{1/p}$ satisfies the conditions of Theorem 1.3; see section 1.3.) Then $T_{\psi,z} \in \mathcal{B}(l_{p,\psi,z}(M'), \mathcal{H}_{p,\psi}(M'))$ is the required interpolation operator. □

2.2. We proceed with the proof of the theorem.

Let $E_0(M) := M \times \mathcal{H}_{p,\psi}(M')$ be the trivial holomorphic Banach vector bundle on M with fibre $\mathcal{H}_{p,\psi}(M')$. Also, we denote by $E_{p,\phi}(M)$ the Banach vector bundle associated with the natural action of $\pi_1(M)$ on the fibre S of $r : M' \to M$. For the construction of such a bundle see [5, Example 2.2(b)]. Let us recall that the fibre of $E_{p,\phi}(M)$ is the Banach space $l_{p,\phi}(S)$ of complex functions f on S with norm

$$||f||_{p,\phi} := \left(\sum_{s\in S} |f(s)|^p \phi(s)\right)^{1/p}$$

for $p \in [1, \infty)$ and

$$||f||_{\infty,\phi} := \sup_{s\in S}\{|f(s)|\phi(s)\}.$$

Also, ϕ is defined by the formula

$$\phi := \psi|_{r^{-1}(z_0)}$$

for a fixed point $z_0 \in M$. (Here $\log\psi$ is uniformly continuous on M' with respect to the path metric induced by a Riemannian metric pulled back from N.)

It was proved in [5, Proposition 2.4] that the Banach space $B_{p,\phi}(M)$ of bounded holomorphic sections of $E_{p,\phi}(M)$ is naturally isomorphic to $\mathcal{H}_{p,\psi}(M')$.

Further, for $z \in M$ let R_z be the restriction map of functions from $\mathcal{H}_{p,\psi}(M')$ to the fibre $r^{-1}(z)$. If we identify $\mathcal{H}_{p,\psi}(M')$ with the Banach space $B_{p,\phi}(M)$, then R_z will be the evaluation map of sections from $B_{p,\phi}(M)$ at $z \in M$. In particular one can define a (holomorphic) homomorphism of bundles $R : E_0(M) \to E_{p,\phi}(M)$ which maps $z \times v \in E_0(M)$ to the vector $R_z(v)$ in the fibre over z of the bundle $E_{p,\phi}(M)$. Since for every $z \in M \setminus C_M$ the space $l_{p,\psi,z}(M')$ is isomorphic to $l_{p,\phi}(S)$, see inequality (2.5) of [5], Proposition 2.1 implies that every R_z is surjective and, moreover, there exists a linear continuous map C_z of the fibre $E_{p,\phi,z}$ of $E_{p,\phi}(M)$ over z to the fibre $E_{0,z}(M)$ of $E_0(M)$ over z such that $R_z \circ C_z = id$. Repeating exactly the arguments of [5, section 3.2], we thus obtain that *for every $z \in M \setminus C_M$ there is a neighbourhood $U_z \subset\subset M \setminus C_M$ of z such that $KerR|_{U_z}$ is biholomorphic to $U_z \times KerR_z$ and this biholomorphism is linear on every $KerR_x$ and maps this space onto $x \times KerR_z$, $x \in V_z$.*

The latter shows that the bundle $E_{p,\phi}(M)$ is locally complemented in $E_0(M)$ over $M \setminus C_M$. Now, if $\Omega \subset M \setminus C_M$ is an open Stein manifold, then by the Bungart theorem [1] based on the previous statement one obtains that $E_{p,\phi}(M)$ is complemented in $E_0(M)$ over Ω; see [5, section 3.2] for details. This means that there is a (holomorphic) homomorphism of bundles $F : E_{p,\phi}(M)|_\Omega \to E_0(M)|_\Omega$ such that $R \circ F = id$. Moreover, by definition $F|_K$ is bounded over $K \subset\subset \Omega$.

Finally, we set

$$L_z := F(z), \quad z \in M.$$

Then by definition, every L_z is a linear continuous map of $l_{p,\psi,z}(M')$ into $\mathcal{H}_{p,\psi}(M')$, the family $\{L_z\}$ is holomorphic in $z \in M$, $R_z \circ L_z = id$, and $\sup_{z\in K} ||L_z|| < \infty$.

This completes the proof of the theorem. □

3. Proofs

3.1. **Proof of Theorem 1.5.** Let $f \in \mathcal{H}_{p,\psi}(Y')$ be a function where Y' satisfies assumptions of the theorem. These assumptions imply that there is a strongly pseudoconvex manifold $\widetilde{M} \subset\subset N$ such that $V \subset\subset \widetilde{M}$. We apply Theorem 1.3 with M' substituted for $\widetilde{M}'$. Then we consider the function

$$h(z) := L_z(f|_{r^{-1}(z)}), \quad z \in Y.$$

By [5, Proposition 2.4] and the properties of $\{L_z\}$ we obtain that h is a $\mathcal{H}_{p,\psi}(\widetilde{M}')$-valued holomorphic function on Y. (It can be written as the scalar function of the variables $(z,w) \in Y \times \widetilde{M}'$.) Thus it suffices to prove the extension theorem for the Banach-valued holomorphic function h on Y extending it to V. Evaluating the extended Banach-valued function at the points $(r(y), y)$, $y \in U'$, we get the required function F (cf. arguments in [5, section 4]). Now the above Banach-valued extension theorem follows directly from the Banach-valued version of the classical Cartan B theorem for Stein manifolds due to Bungart [1]. □

3.2. **Proof of Theorem 1.7.** We retain the notation of Remark 1.6. By the conditions of the theorem we obtain that $X_K := p(K)$ is a holomorphically convex compact subset of X_M that does not contain points x_i, $1 \le i \le l$. Then there is a non-degenerate analytic polyhedron $P \subset\subset X_O$ containing K and formed by holomorphic functions on X_M. Now, for $f \in \mathcal{H}_{p,\psi}(O')$ we consider the function

$$h(z) := L_z(f|_{r^{-1}(z)}), \quad z \in O,$$

with $\{L_z\}$ as in Theorem 1.3. Then h is a $\mathcal{H}_{p,\psi}(M')$-valued holomorphic function on O. Next, we apply to $h|_{p^{-1}(P)}$ the Weil integral formula (also valid for Banach-valued holomorphic functions). Expanding the kernel in this formula in an analytic series, we obtain as in the classical case that h can be approximated uniformly on K by $\mathcal{H}_{p,\psi}(M')$-valued holomorphic functions on M. Taking restrictions of these functions to the set $\{(r(y), y) \ : \ y \in M'\}$, we obtain the required approximation theorem. □

References

1. L. Bungart, *On analytic fibre bundles I. Holomorphic fibre bundles with infinite dimensional fibres.* Topology, **7** (1) (1968), 55–68. MR0222338 (36:5390)
2. A. Brudnyi, *On holomorphic L^2-functions on coverings of strongly pseudoconvex manifolds.* Publications of RIMS, Kyoto University, **43** (2007), 963–976. MR2389789
3. A. Brudnyi, *Hartogs type theorems for CR L^2-functions on coverings of strongly pseudoconvex manifolds.* Nagoya Math. J., **189** (2008), 27–47. MR2396582
4. A. Brudnyi, *On holomorphic functions of slow growth on coverings of strongly pseudoconvex manifolds.* J. Funct. Anal., **249** (2007), no. 2, 354–371. MR2345336
5. A. Brudnyi, *Representation of holomorphic functions on coverings of pseudoconvex domains in Stein manifolds via integral formulas on these domains.* J. Funct. Anal. **231** (2006), 418–437. MR2195338 (2006m:32001)
6. H. Cartan, *Sur les fonctions de plusieurs variables complexes. Les espaces analytiques.* Proc. Intern. Congress Mathematicians Edinburgh 1958, Cambridge Univ. Press, 1960, pp. 33–52. MR0117763 (22:8537)
7. M. Gromov, G. Henkin and M. Shubin, *Holomorphic L^2-functions on coverings of pseudoconvex manifolds.* GAFA, **8** (1998), 552–585. MR1631263 (2000d:32058)
8. J. Leiterer, *Holomorphic vector bundles and the Oka-Grauert principle.* Several complex variables. IV. Algebraic aspects of complex analysis. A translation of Sovremennye problemy matematiki. Fundamental´nye napravleniya, Tom 10, Akad. Nauk SSSR, Vsesoyuz. Inst.

Nauchn. i Tekn. Inform., Moscow 1986. Encyclopaedia of Mathematical Sciences, 10. Springer-Verlag, Berlin, 1990. MR894263 (88k:32072)

9. R. Remmert, *Sur les espaces analytiques holomorphiquement séparables et holomorphiquement convexes.* C. R. Acad. Sci. Paris, **243** (1956), 118–121. MR0079808 (18:149c)

10. W. Rudin, *Real and complex analysis.* Second edition. McGraw-Hill Series in Higher Mathematics, New York, 1974. MR0344043 (49:8783)

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