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ESSENTIALLY SPECTRALLY BOUNDED LINEAR MAPS

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Abstract. Let $\mathcal{L}(\mathcal{H})$ be the algebra of all bounded linear operators on an infinite dimensional complex Hilbert space $\mathcal{H}$. We characterize essentially spectrally bounded linear maps from $\mathcal{L}(\mathcal{H})$ onto $\mathcal{L}(\mathcal{H})$ itself. As a consequence, we characterize linear maps from $\mathcal{L}(\mathcal{H})$ onto $\mathcal{L}(\mathcal{H})$ itself that compress different essential spectral sets such as the the essential spectrum, the (left, right) essential spectrum, and the semi-Fredholm spectrum.

1. Introduction and statement of the main result

Throughout this paper, $\mathcal{H}$ will denote an infinite dimensional complex Hilbert space and $\mathcal{L}(\mathcal{H})$ will denote the algebra of all bounded linear operators on $\mathcal{H}$. The closed ideal of all compact operators on $\mathcal{H}$ is denoted by $\mathcal{K}(\mathcal{H})$, and the Calkin algebra is denoted, as usual, by $\mathcal{C}(\mathcal{H}) := \mathcal{L}(\mathcal{H})/\mathcal{K}(\mathcal{H})$. For an operator $T \in \mathcal{L}(\mathcal{H})$, let $\sigma_e(T)$, $\sigma_{le}(T)$, $\sigma_{re}(T)$, and $\sigma_{SF}(T)$ denote the essential spectrum, the left essential spectrum, the right essential spectrum, and the semi-Fredholm spectrum, respectively, of T; see for instance [1]. The essential norm of the operator T is given by $\|T\|_e := \operatorname{dist}(T, \mathcal{K}(\mathcal{H}))$, and the essential spectral radius, denoted by $r_e(T)$, is the limit of the convergent sequence $({\|T^n\|_e}^{1/n})_{n\geq 1}$. It coincides with $r(\pi(T))$ the classical spectral radius of $\pi(T)$, where π denotes the canonical quotient map from $\mathcal{L}(\mathcal{H})$ onto $\mathcal{C}(\mathcal{H})$.

New contributions to the study of linear preserver problems in $\mathcal{L}(\mathcal{H})$ have recently been made in [4, 5, 9, 20, 21, 22]. In [22], Mbekhta has treated the problem of characterizing surjective linear maps on $\mathcal{L}(\mathcal{H})$ preserving the set of Fredholm operators in both directions. He proved, in particular, that a surjective linear map on $\mathcal{L}(\mathcal{H})$ preserves the set of Fredholm operators in both directions if and only if it leaves invariant the closed ideal $\mathcal{K}(\mathcal{H})$ of all compact operators and the induced map on the Calkin algebra, $\mathcal{C}(\mathcal{H}) := \mathcal{L}(\mathcal{H})/\mathcal{K}(\mathcal{H})$, is either an automorphism or an anti-automorphism multiplied by an invertible element in $\mathcal{C}(\mathcal{H})$. In [9], Cui and Hou

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established independently similar results and characterized linear maps on $\mathcal{L}(\mathcal{H})$ preserving certain essential spectral sets such as the set of (left, right) Fredholm operators and the set of semi-Fredholm operators. Part of [21] is also devoted to the study of surjective linear maps on $\mathcal{L}(\mathcal{H})$ preserving the set of semi-Fredholm operators in both directions, and analogous results are obtained.

It should be noted that [20, 21] contain many good ideas and elegant results which opened the way for certain authors to generalize the results from [9, 20, 21, 22]. In [4], Bendaoud, Bourhim and Sarih presented one main result with a simple proof which characterizes surjective linear maps on $\mathcal{L}(\mathcal{H})$ preserving the essential spectral radius and recaptured, as consequences, the main results of [9, 20, 21, 22]. While in [5], Boudi and Hadder presented several main results on surjective linear maps on $\mathcal{L}(\mathcal{H})$ preserving, in one direction, generalized invertibility, and Fredholm and semi-Fredholm operators. As a consequence of their main results, they also recaptured the results of [9, 20, 21, 22]. However, neither the main result of [4] nor the main results of [5] can be deduced from each other.

In this paper, we unify and extend all the results from [4, 5, 9, 20, 21, 22] by a characterization of surjective essentially spectrally bounded linear maps on $\mathcal{L}(\mathcal{H})$. Recall that a linear map ϕ from $\mathcal{L}(\mathcal{H})$ into itself is said to be *surjective up to compact operators* if $\mathcal{L}(\mathcal{H}) = \mathrm{range}(\phi) + \mathcal{K}(\mathcal{H})$, and it is called *essentially spectrally bounded* if there exists a positive constant M such that $r_e(\phi(T)) \leq M r_e(T)$ for all $T \in \mathcal{L}(\mathcal{H})$. This paper presents a complete characterization of such maps. Its proof below is considerably simpler than the ones given in [5, 9, 20, 21, 22] for closely related results.

Theorem 1.1. *Assume that $\phi\, \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H})$ is a linear map which is surjective up to compact operators. Then ϕ is essentially spectrally bounded if and only if $\phi(\mathcal{K}(\mathcal{H})) \subseteq \mathcal{K}(\mathcal{H})$ and the induced map $\widehat{\phi} : \mathcal{C}(\mathcal{H}) \to \mathcal{C}(\mathcal{H})$ defined by $\widehat{\phi}(\pi(T)) := \pi(\phi(T))$ $(T \in \mathcal{L}(\mathcal{H}))$ is either a continuous epimorphism or a continuous anti-epimorphism multiplied by a nonzero scalar.*

Of course the "if" part is obvious. For the "only if" part, we first use Vesentini's theorem, on subharmonity of the spectral radius when composed with a holomorphic function, together with Liouville's theorem to show that if ϕ is a surjective essentially spectrally bounded linear map on $\mathcal{L}(\mathcal{H})$, then ϕ leaves invariant the ideal of all compact operators on $\mathcal{H}$, and therefore the induced mapping on the Calkin algebra $\mathcal{C}(\mathcal{H})$ is a well defined spectrally bounded linear map. To conclude the proof, we apply Lemma 2.1, which is a variant of several results that characterize spectrally bounded linear maps from a purely infinite C^*-algebra with real rank zero onto a semisimple Banach algebra; see [7, 10, 12, 15, 16, 17, 18, 19, 24].

It is very well known that the Calkin algebra $\mathcal{C}(\mathcal{H})$ is a simple algebra provided that $\mathcal{H}$ is a separable Hilbert space; see for instance [8]. In this case, every epimorphism (resp. anti-epimorphism) on $\mathcal{C}(\mathcal{H})$ is an automorphism (resp. anti-automorphism). It should also be noted that, unlike $\mathcal{L}(\mathcal{H})$, the Calkin algebra $\mathcal{C}(\mathcal{H})$ has, in general, outer automorphisms as shown by Phillips and Weaver [23], who answered negatively a long-standing problem which asks whether every automorphism of $\mathcal{C}(\mathcal{H})$ is inner. However, Farah showed in [11] that there are complex Hilbert spaces $\mathcal{H}$ such that all the automorphisms of the corresponding Calkin algebra $\mathcal{C}(\mathcal{H})$ are inner.

2. Proof of the main result

In this section, we first state and prove the promised auxiliary lemma which will be needed for the proof of Theorem 1.1. After proving the main result, we present some consequences which were discovered in [5].

A linear map $\phi : \mathcal{A} \to \mathcal{B}$ between two unital Banach algebras $\mathcal{A}$ and $\mathcal{B}$ is called *unital* if $\phi(\mathbf{1}) = \mathbf{1}$, and it is said to be a *Jordan homomorphism* if $\phi(a^2) = \phi(a)^2$ for all $a \in \mathcal{A}$. Equivalently, the map ϕ is a Jordan homomorphism if and only if $\phi(ab + ba) = \phi(a)\phi(b) + \phi(b)\phi(a)$ for all a and b in $\mathcal{A}$. It is called a *Jordan isomorphism* provided that it is a bijective Jordan homomorphism. Clearly, every homomorphism and every anti-homomorphism is a Jordan homomorphism. Recall also that the map ϕ is called *spectrally bounded* if there exists a constant $M > 0$ such that $r(\phi(a)) \leq Mr(a)$ holds for every $a \in \mathcal{A}$, where $r(.)$ denotes, as usual, the spectral radius.

Recall that a C^*-algebra $\mathcal{A}$ is said to have *real rank zero* if the set of all real combinations of orthogonal projections is dense in the set of all selfadjoint elements of $\mathcal{A}$. It said to be *purely infinite* if it has no characters and if for every pair of positive elements a and b in $\mathcal{A}$ with $a \in \overline{\mathcal{A}b\mathcal{A}}$ there is a sequence $(x_n)_{n\in\mathbb{N}}$ in $\mathcal{A}$ such that $a = \lim_n x_n^* b x_n$; see [14]. Denote by $\sim$ the usual Murray-von Neumann equivalence relation on the set of all projections of $\mathcal{A}$, and recall that a nonzero projection p in $\mathcal{A}$ is said to be *properly infinite* if there exist mutually orthogonal subprojections p_1 and p_2 of p such that $p_1 \sim p \sim p_2$. If the identity of $\mathcal{A}$ is properly infinite, then $\mathcal{A}$ itself is called *properly infinite*.

Lemma 2.1. *Let $\mathcal{A}$ be a unital purely infinite C^*-algebra with real rank zero and let $\mathcal{B}$ is a semisimple unital Banach algebra. If $\varphi : \mathcal{A} \to \mathcal{B}$ is a surjective spectrally bounded linear map, then there exist a central invertible element c, viz., $\varphi(\mathbf{1})$, and a Jordan epimorphism $J : \mathcal{A} \to \mathcal{B}$ such that $\varphi(x) = cJ(x)$ for all $x \in \mathcal{A}$.*

Unlike [17, Theorem 3.6 2.5], [19, Theorem B] and [15, Corllary 2.5], we do not assume that φ is unital, which is new and which is, in fact, needed for our purposes. However, the proof of this lemma is on the straightforward side and is included here for the sake of completeness.

Proof of Lemma 2.1. Let p be a nonzero projection in $\mathcal{A}$ and note that, since $\mathcal{A}$ is purely infinite, p is properly infinite; see [14, Theorem 4.16]. Thus $p\mathcal{A}p$ is a properly infinite C^*-algebra, and each of its elements can be written as a finite sum of elements of $p\mathcal{A}p$ with square zero; see [15, proposition 2.1]. By [17, Lemma 3.3], we have

$$\varphi(p)\varphi(q) + \varphi(q)\varphi(p) = 0 \tag{2.1}$$

for all projections $q \in \mathcal{A}$ which are orthogonal to p. Now, apply (2.1) to p and $q = \mathbf{1} - p$ to see that

$$\varphi(p)\varphi(\mathbf{1}) + \varphi(\mathbf{1})\varphi(p) = 2(\varphi(p))^2. \tag{2.2}$$

Multiplying this identity separately on the left and then on the right by $\varphi(p)$ and comparing the obtained equations, we get that

$$\varphi(\mathbf{1})(\varphi(p))^2 = (\varphi(p))^2\varphi(\mathbf{1}). \tag{2.3}$$

Now, let $a = \sum_{i=1}^{i=n} \lambda_i p_i$ be a linear combination of orthogonal projections $p_1, \ldots, p_n$ of $\mathcal{A}$. In view of (2.1), we have $\varphi(a)^2 = \sum_{i=1}^{i=n} \lambda_i^2 (\varphi(p_i))^2$ and $(\varphi(a))^2\varphi(\mathbf{1}) = \varphi(\mathbf{1})(\varphi(a))^2$ by (2.3). As φ is continuous (see [2, Theorem 5.5.2]) and $\mathcal{A}$ has real rank zero, we get that $(\varphi(h))^2\varphi(\mathbf{1}) = \varphi(\mathbf{1})(\varphi(h))^2$ for all selfadjoint elements $h \in \mathcal{A}$. From this, it follows easily that $(\varphi(x))^2\varphi(\mathbf{1}) = \varphi(\mathbf{1})(\varphi(x))^2$ for all $x \in \mathcal{A}$. Since every element y in $\mathcal{B}$ can be written as a finite sum of square elements in $\mathcal{B}$ and φ is surjective, we see that $\varphi(\mathbf{1})$ is a central element of $\mathcal{B}$. Therefore, it follows from (2.2) that

$$(\varphi(p))^2 = \varphi(p)\varphi(\mathbf{1}) = \varphi(p^2)\varphi(\mathbf{1}) \tag{2.4}$$

for all projections $p \in \mathcal{A}$. Just as before and employing (2.4) instead of (2.3), we get that $(\varphi(x))^2 = \varphi(x^2)\varphi(\mathbf{1})$ for all $x \in \mathcal{A}$. As φ is surjective, there is an element $u \in \mathcal{A}$ such that $\varphi(u) = \mathbf{1}$, and $\mathbf{1} = (\varphi(u))^2 = \varphi(u^2)\varphi(\mathbf{1}) = \varphi(\mathbf{1})\varphi(u^2)$. This shows that $\varphi(\mathbf{1})$ is invertible.

Finally, set $J(x) := \varphi(\mathbf{1})^{-1}\varphi(x)(x \in \mathcal{A})$, and note that

$$(J(x))^2 = \varphi(\mathbf{1})^{-2}(\varphi(x))^2 = \varphi(\mathbf{1})^{-1}\varphi(x^2) = J(x^2)$$

for all $x \in \mathcal{A}$. Thus J is a Jordan epimorphism, and the proof is therefore complete. □

We are now in a position to prove the main result of this paper.

Proof of Theorem 1.1. Assume that there is a positive constant M such that $r_e(\phi(T)) \leq M r_e(T)$ for all $T \in \mathcal{L}(\mathcal{H})$, and let us establish the "only if" part of Theorem 1.1. We first show that ϕ leaves $\mathcal{K}(\mathcal{H})$ invariant. So pick a compact operator $K \in \mathcal{K}(\mathcal{H})$, and let us prove that $\phi(K)$ is compact as well. Let $S \in \mathcal{L}(\mathcal{H})$ be an arbitrary operator on $\mathcal{H}$ and note that, since ϕ is surjective up to compact operators, there exist $T \in \mathcal{L}(\mathcal{H})$ and $K_0 \in \mathcal{K}(\mathcal{H})$ such that $S = \phi(T) + K_0$. For every $\lambda \in \mathbb{C}$, we have

$$\begin{aligned} r(\lambda\pi(\phi(K)) + \pi(S)) &= r(\pi(\lambda\phi(K) + S)) = r_e(\lambda\phi(K) + S) \\ &= r_e(\phi(\lambda K + T) + K_0) = r_e(\phi(\lambda K + T)) \\ &\leq M r_e(\lambda K + T) = M r_e(T). \end{aligned}$$

Since $\lambda \mapsto r(\lambda\pi(\phi(K)) + \pi(S))$ is a subharmonic function on $\mathbb{C}$, Liouville's Theorem implies that $r(\pi(\phi(K)) + \pi(S)) = r(\pi(S))$. As S is arbitrary in $\mathcal{L}(\mathcal{H})$, it follows from semi-simplicity of $\mathcal{C}(\mathcal{H})$ and the Zemánek's characterization of the radical [2, Theorem 5.3.1] that $\pi(\phi(K)) = 0$ and $\phi(K) \in \mathcal{K}(\mathcal{H})$.

Therefore $\phi(\mathcal{K}(\mathcal{H})) \subseteq \mathcal{K}(\mathcal{H})$, and ϕ induces a surjective spectrally bounded linear map $\widehat{\phi} : \mathcal{C}(\mathcal{H}) \to \mathcal{C}(\mathcal{H})$ defined by $\widehat{\phi}(\pi(T)) := \pi(\phi(T))$ for all $T \in \mathcal{L}(\mathcal{H})$. As $\mathcal{C}(\mathcal{H})$ is a purely infinite C^*-algebra with real rank zero, Lemma 2.1 tells us that $\widehat{\phi}$ is a continuous Jordan epimorphism J multiplied by an invertible central element of $\mathcal{C}(\mathcal{H})$, viz., $\widehat{\phi}(\pi(\mathbf{1}))$. However, since the centre of $\mathcal{C}(\mathcal{H})$ is trivial, $\widehat{\phi}(\pi(\mathbf{1}))$ must be a nonzero complex number c. As $\mathcal{C}(\mathcal{H})$ is prime, the well known theorem of Herstein [13] tells us that J must be an epimorphism or an anti-epimorphism. The proof is therefore complete. □

As a consequence, we describe linear maps compressing certain essential spectral sets. Recall that a linear map $\phi : \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H})$ is said to compress the essential spectrum if $\sigma_e(\phi(T)) \subseteq \sigma_e(T)$ for all $T \in \mathcal{L}(\mathcal{H})$. The linear maps compressing the

left essential spectrum, the right essential spectrum, or semi-Fredholm spectrum are defined in a similar way.

Corollary 2.2. *Let $\phi : \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H})$ be a linear map surjective up to compact operators. Then ϕ compresses the essential spectrum (the semi-Fredholm spectrum) if and only if $\phi(\mathcal{K}(\mathcal{H})) \subseteq \mathcal{K}(\mathcal{H})$ and the induced map $\widehat{\phi} : \mathcal{C}(\mathcal{H}) \to \mathcal{C}(\mathcal{H})$ is either a continuous epimorphism or a continuous anti-epimorphism.*

Proof. It suffices to prove the "only if" part. Assume that ϕ compresses the essential spectrum (the semi-Fredholm spectrum), and note that, in this case, ϕ is an essentially spectrally bounded linear map. By Theorem 1.1, $\phi(\mathcal{K}(\mathcal{H})) \subseteq \mathcal{K}(\mathcal{H})$, and the induced map $\widehat{\phi} : \mathcal{C}(\mathcal{H}) \to \mathcal{C}(\mathcal{H})$ is either a continuous epimorphism or a continuous anti-epimorphism multiplied by a nonzero scalar c. Of course, c is equal to 1 because $\{c\} = \sigma_e(\phi(\mathbf{1})) \subset \sigma_e(\mathbf{1}) = \{1\}$.

The proof of the other case when ϕ compresses the semi-Fredholm spectrum holds in a similar way. □

The second consequence characterizes surjective linear maps on $\mathcal{L}(\mathcal{H})$ compressing the left and right essential spectrum. Its proof proceeds along the same lines as the one for the above corollary.

Corollary 2.3. *Let $\phi : \mathcal{L}(\mathcal{H}) \to \mathcal{L}(\mathcal{H})$ be a linear map surjective up to compact operators. Then ϕ compresses the left essential spectrum (the right essential spectrum) if and only if $\phi(\mathcal{K}(\mathcal{H})) \subseteq \mathcal{K}(\mathcal{H})$ and the induced map $\widehat{\phi} : \mathcal{C}(\mathcal{H}) \to \mathcal{C}(\mathcal{H})$ is a continuous epimorphism.*

We finally close this paper with the following remark.

Remark 2.4. As in [3, Section 5] and [6, Section 4], the main result of this paper and its corollaries can be stated in a more general situation where one considers essentially spectrally bounded linear maps from a unital purely infinite C^*-algebra with real rank zero onto a semisimple Banach algebra.

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