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ON THE WEAKER FORMS OF THE SPECIFICATION PROPERTY AND THEIR APPLICATIONS

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Abstract. We show the following two results, which are derived from the weaker forms of the specification property: Firstly, if an automorphism of a compact metric abelian group with finite topological entropy is ergodic under the Haar measure, then it satisfies the level 2 large deviation principle. Secondly, the topological pressure formula for periodic orbits is given under the expansiveness and the almost product property.

1. Introduction

The specification property was introduced by Bowen [4] to construct the equilibrium states. After that several authors introduced the weaker forms of this property, including the *almost specification property* ([7], [13]), the *approximate product property* ([15]) and the *almost product property* ([16]). The aim of this paper is to study the relation among them and consider two applications.

As a first application, we consider the large deviation principle for group automorphisms which satisfy the almost specification property. Let (X, d) be a compact metric space and f be a continuous map from X to itself. We denote by $\mathcal{M}(X)$ the set of all Borel probability measures on X with the weak topology. We say that f satisfies the *level* 2 *large deviation principle* with a reference measure $m \in \mathcal{M}(X)$ and a rate function $q\colon \mathcal{M}(X) \to [-\infty, 0]$ if q is upper semicontinuous,

$$\liminf_{n\to\infty} \frac{1}{n} \log m \left(\left\{ x \in X : \frac{1}{n} \sum_{j=0}^{n-1} \delta_{f^j x} \in G \right\} \right) \geq \sup_{\mu \in G} q(\mu)$$

holds for any open set $G \subset \mathcal{M}(X)$ and

$$\limsup_{n\to\infty} \frac{1}{n} \log m \left(\left\{ x \in X : \frac{1}{n} \sum_{j=0}^{n-1} \delta_{f^j x} \in F \right\} \right) \leq \sup_{\mu \in F} q(\mu)$$

holds for any closed set $F \subset \mathcal{M}(X)$, where δ_y stands for the δ-measure at the point $y \in X$. We refer to [9] for a general theory of large deviations and its background in statistical mechanics.

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In this paper, we show the following:

Theorem 1.1. *Let f be an automorphism of a compact metric abelian group X. If f is ergodic under the Haar measure m and the topological entropy $h(f)$ of f is finite, then f satisfies the level 2 large deviation principle for m and the rate function q is defined by*

$$q(\mu) = \begin{cases} h_\mu(f) - h(f) & (\mu \text{ is } f\text{-invariant}); \\ -\infty & (\text{otherwise}), \end{cases}$$

where $h_\mu(f)$ means the metric entropy of μ.

We have known that there are examples of ergodic automorphisms which do not satisfy the specification property (see [2]). By Theorem 1.1 we can find a group automorphism which does not have the specification property but satisfies the large deviation principle.

Secondly we compute the topological pressure by using periodic points for some dynamical systems with the almost product property, including all β-shifts (see §2). Historically several authors studied deep connections between topological pressure (or topological entropy) and periodic points such as [4], [11] (uniformly-hyperbolic case) and [6], [10] (non-uniformly-hyperbolic case). It is known by Bowen [4] that if an expansive homeomorphism $f\colon X \to X$ of a compact metric space (X, d) has the strong specification property (see §2 for definition) and a continuous function φ satisfies the Bowen condition (see [4]), then the topological pressure $P(f, \varphi)$ of φ holds the following:

$$P(f,\varphi) = \lim_{n\to\infty} \frac{1}{n} \log \sum_{x\in P_n(f)} \exp\left\{ \sum_{j=0}^{n-1} \varphi(f^j x) \right\}, \tag{1.1}$$

where $P_n(f) := \{x \in X : f^n x = x\}$. After that He, Li and Sun [11] extended the above result for non-invertible maps.

In this paper we generalize the above results as follows:

Theorem 1.2. *Let $f\colon X \to X$ be a positively expansive continuous map or an expansive homeomorphism of a compact metric space (X, d). If f satisfies the strong almost product property, then for any continuous function φ, the equation* (1.1) *holds. In particular, the topological entropy $h(f)$ satisfies*

$$h(f) = \lim_{n\to\infty} \frac{1}{n} \log \sharp P_n(f),$$

where $\sharp A$ denotes the cardinality of A.

In §2 we study several types of the specification property, and give a proof of Theorem 1.1 in §3. We also prove Theorem 1.2 in §4.

2. Preliminaries

2.1. **Specification properties.** Let (X, d) be a compact metric space and $f\colon X \to X$ be a continuous map. We denote by $C(X, \mathbb{R})$ the set of all continuous real-valued functions of X and by $\mathcal{M}(X)$ the set of all Borel probability measures on X with the weak topology. We also denote by $\mathcal{M}_f(X)$ the set of all f-invariant Borel probability measures on X.

In this subsection we study several types of the specification property. A continuous map $f\colon X \to X$ is said to satisfy the *specification property* (abbreviated *SP*) if for any $\epsilon > 0$ there is an integer M_ϵ such that for any $k \geq 1$ and k points $x_1, \cdots, x_k \in X$ and for any sequence of integers $0 \leq a_1 \leq b_1 < a_2 \leq b_2 < \cdots < a_k \leq b_k$ with $a_i - b_{i-1} \geq M_\epsilon$ $(2 \leq i \leq k)$, there is an $x \in X$ with $d(f^{a_i+j}x, f^j x_i) \leq \epsilon$ $(0 \leq j \leq b_i - a_i, 1 \leq i \leq k)$. If such a point x can be chosen as a periodic point, then we say that f has the *strong specification property* (*S-SP*).

Historically the property *SP* for group automorphisms has been studied by several authors, including [1] (in the zero-dimensional case), [12] (in the toral case) and [2] (in the solenoidal case). According to [2], an automorphism of a solenoidal group has *SP* iff it is ergodic under the Haar measure and has central spin (see [2] for a precise definition), and so ergodic group automorphisms do not always satisfy *SP*. In [13] Marcus introduced a weaker form of *SP* which holds for every ergodic group automorphism as follows:

Definition 2.1. We say that f has the *almost specification property* (*ASP*) if for any $\epsilon > 0$ there is a function $M_\epsilon\colon \mathbb{Z}_+ \to \mathbb{Z}_+$ (where $\mathbb{Z}_+$ denotes the set of non-negative integers) with $M_\epsilon(n)/n \to 0$ as $n \to \infty$ such that for any $k \geq 1$ and k points $x_1, \cdots, x_k \in X$ and for any sequence of integers $0 \leq a_1 \leq b_1 < a_2 \leq b_2 < \cdots < a_k \leq b_k$ with $a_i - b_{i-1} \geq M_\epsilon(b_i - a_i)$ $(2 \leq i \leq k)$, there is an $x \in X$ with $d(f^{a_i+j}x, f^j x_i) \leq \epsilon$ $(0 \leq j \leq b_i - a_i, 1 \leq i \leq k)$.

On the other hand it is known that the β-shift has *SP* only for a set of β of Lebesgue measure 0 (see [17]). But Pfister and Sullivan [15] introduced the weaker form of *SP*, which holds for all β-shifts as follows:

Definition 2.2. We say that f has the *approximate product property* (*ApPP*) if for any $\epsilon > 0$, $\delta_1 > 0$ and $\delta_2 > 0$ there exists an integer $N > 0$ such that for any $n \geq N$ and any sequence $\{x_i\}_{i=1}^\infty$ of X there exist a sequence of integers $\{h_i\}_{i=1}^\infty$ and a point $x \in X$ satisfying $h_1 = 0,\ n \leq h_{i+1} - h_i \leq n(1 + \delta_2)$ and

$$\sharp\{0 \leq j \leq n-1 : d(f^{h_i+j}x, f^j x_i) > \epsilon\} \leq \delta_1 n$$

for any $i \geq 1$.

ApPP is a sufficient condition of the entropy density, which plays an important role in getting the large deviation estimates (see §3). As an application of *ApPP*, Pfister and Sullivan proved the large deviation principle for β-shifts in [15]. After that they introduced the almost product property.

Definition 2.3. We say that f has the *almost product property* (*AlPP*) if there exist $g\colon \mathbb{N} \to \mathbb{N}$ with $\lim_{n\to\infty} \frac{g(n)}{n} = 0$ and $m\colon \mathbb{R}^+ \to \mathbb{N}$ such that for any $k \geq 1$, any $x_1, \cdots, x_k$, any $\epsilon_1 > 0, \cdots, \epsilon_k > 0$ and any $n_1 \geq m(\epsilon_1), \cdots, n_k \geq m(\epsilon_k)$, there exists $x \in X$ such that

$$\sharp\{0 \leq j \leq n_i - 1 : d(f^{j+n_1+\cdots+n_{i-1}}x, f^j x_i) > \epsilon_i\} \leq g(n_i)$$

holds for any $1 \leq i \leq k$. If such a point x can be chosen as a periodic point (i.e. $f^{n_1+\cdots+n_k}x = x$), then we say that f has the *strong almost product property* (*S-AlPP*).

We note that *AlPP* is slightly stronger than *ApPP*, but all β-shifts still satisfy *S-AlPP* (see Example in [16]).

Finally, we summarize the relation between *SP*, *ASP*, *ApPP* and *AlPP*. In [16] Pfister and Sullivan proved that *SP* implies *AlPP*. The other implications (*SP* implies *ASP*, *ASP* implies *ApPP*, and *AlPP* implies *ApPP*) in the following diagram are obvious by the definitions:

$$\begin{array}{ccc} SP & \Longrightarrow & ASP \\ \Downarrow & & \Downarrow \\ AlPP & \Longrightarrow & ApPP. \end{array}$$

2.2. **Separated sets.** Let $\epsilon > 0$ and $n \geq 1$. A subset $E \subset X$ is called (n,ϵ)-*separated* if for any two distinct points $x, y \in E$ there exists $0 \leq j \leq n-1$ such that $d(f^j x, f^j y) > \epsilon$. For $\varphi \in C(X, \mathbb{R})$, $\epsilon > 0$ and $n \geq 1$, put

$$P_n(f, \varphi, \epsilon) := \sup \left\{ \sum_{x \in E} e^{S_n \varphi(x)} : E \text{ is an } (n,\epsilon)\text{-separated set in } X \right\},$$

where $S_n\varphi(x) := \sum_{j=0}^{n-1} \varphi(f^j x)$, and define the *topological pressure* of φ by

$$P(f, \varphi) := \lim_{\epsilon \to 0} \limsup_{n \to \infty} \frac{1}{n} \log P_n(f, \varphi, \epsilon).$$

The topological pressure of φ corresponding to the function $\varphi \equiv 0$ is called the *topological entropy* of f, which we denote by $h(f)$.

For dynamical systems with *SP* or *ASP*, an (n,ϵ)-separated set is very useful for estimating the topological pressure (see [4] and [8] for example). However to treat *AlPP* or *ApPP*, it is not useful because they only guarantee the existence of an orbit which partially ϵ-shadows the specified orbit segments. So Pfister and Sullivan [15] introduced a (δ, n, ϵ)-separated set.

Definition 2.4. Let $\epsilon > 0$, $\delta > 0$ and $n \geq 1$. A subset E is called (δ, n, ϵ)-*separated* if for any two distinct points $x, y \in E$,

$$\sharp\{0 \leq j \leq n-1 : d(f^j x, f^j y) > \epsilon\} > \delta n$$

holds.

The following proposition, which is important to show the topological pressure formula for (δ, n, ϵ)-separated sets, is proved by Pfister and Sullivan in [15].

Proposition 2.5 ([15], Proposition 2.1)**.** *Let μ be ergodic and $h < h_\mu(f)$. Then there exist $\delta > 0$ and $\epsilon > 0$ such that for any neighborhood F of μ in $\mathcal{M}(X)$ there exists $N \in \mathbb{N}$ so that for any $n \geq N$ there exists a (δ, n, ϵ)-separated set $\Gamma \subset X_{n,F}$ such that*

$$\sharp\Gamma \geq e^{nh},$$

where $X_{n,F} := \{x \in X : \frac{1}{n}\sum_{j=0}^{n-1} \delta_{f^j x} \in F\}$.

In this paper, for more precise analysis, we introduce a $(g; n, \epsilon)$-separated set, which is useful for proving Theorem 1.2.

Definition 2.6. Let $\epsilon > 0$, $n \geq 1$ and $g \colon \mathbb{N} \to \mathbb{N}$ be a function. A subset E is called $(g; n, \epsilon)$-*separated* if for any two distinct points $x, y \in E$,

$$\sharp\{0 \leq j \leq n-1 : d(f^j x, f^j y) > \epsilon\} > g(n)$$

holds.

3. Proof of Theorem 1.1

In this section we give a proof of Theorem 1.1 by using the following criterion, proved by Pfister and Sullivan in [15]:

Theorem 3.1 ([15], Theorems 3.1 and 3.2)**.** *Let (X, d) be a compact metric space and let $f\colon X \to X$ be a continuous map, let $m \in \mathcal{M}(X)$, let φ be a continuous function on X and assume that the four conditions (C1)-(C4) hold:*

(C1) *The entropy map $\mu \mapsto h_\mu(f)$ is upper semicontinuous on $\mathcal{M}_f(X)$.*
(C2) *Entropy dense condition: for any $\mu \in \mathcal{M}_f(X)$, $h < h_\mu(f)$ and any neighborhood G of μ, there exists an ergodic measure $\nu \in G$ such that $h_\nu(f) > h$.*
(C3) $$\lim_{\epsilon\to 0} \liminf_{n\to\infty} \inf_{x\in X} \left(\frac{1}{n} \log m(B_n(x,\epsilon)) + \frac{1}{n} \sum_{j=0}^{n-1} \varphi(f^j x) \right) \geq 0.$$
(C4) $$\lim_{\epsilon\to 0} \limsup_{n\to\infty} \sup_{x\in X} \left(\frac{1}{n} \log m(B_n(x,\epsilon)) + \frac{1}{n} \sum_{j=0}^{n-1} \varphi(f^j x) \right) \leq 0.$$

Here $B_n(x,\epsilon) := \{y \in X : d(f^j x, f^j y) \leq \epsilon\}$. Then f satisfies the level 2 large deviation principle for m and the rate function q defined by

$$q(\mu) = \begin{cases} h_\mu(f) - \int \varphi d\mu & (\mu \in \mathcal{M}_f(X)); \\ -\infty & (\textit{otherwise}). \end{cases}$$

Now we prove Theorem 1.1. Since f is a group automorphism with $h(f) < \infty$, it follows from [14] that f is asymptotically h-expansive, which implies the condition (C1) (see [8], Theorem 20.9). By the ergodicity of f and Theorem 1 in [7], f satisfies *ASP*, and so *ApPP* holds. Thus the condition (C2) follows from Theorem 2.1 in [15].

Now we show the conditions (C3) and (C4) for the constant function $\varphi \equiv h(f)$. Fix any $\epsilon > 0$, $n > 0$ and $x \in X$. Since X is compact there exists a finite partition ξ with $diam A < \epsilon$ $(A \in \xi)$, where $diam A$ denotes the diameter of the set A. Taking a point $x_A \in A$ for each element $A \in \xi^n$, where $\xi^n := \bigvee_{j=0}^{n-1} f^{-j}\xi$, we can easily see that $A \subset B_n(x_A, \epsilon)$. Thus we have

$$\sum_{A\in\xi^n} m(A) \log m(A) \leq \sum_{A\in\xi^n} m(A) \log m(B_n(x_A,\epsilon)) = \log m(B_n(x,\epsilon)),$$

where the last equality follows from the translation invariance of m. Therefore we get

$$\liminf_{n\to\infty} \inf_{x\in X} \frac{1}{n} \log m(B_n(x,\epsilon)) \geq -h_m(f) = -h(f),$$

which implies (C3).

For any $\epsilon > 0$ and $n > 0$, let E_n be an (n,ϵ)-separated set with maximal cardinality. Then $B_n(y, \frac{\epsilon}{2}) \cap B_n(y', \frac{\epsilon}{2}) = \emptyset$ holds for any two distinct points $y, y' \in E_n$, and so we have

$$\sum_{y\in E_n} m\left(B_n\left(y, \frac{\epsilon}{2}\right)\right) \leq 1,$$

which implies $\sharp E_n m\left(B_n\left(x, \frac{\epsilon}{2}\right)\right) \leq 1$ for any $x \in X$. Since $\epsilon > 0$, $n > 0$ and $x \in X$ are arbitrary, we get (C4). So Theorem 1.1 follows by Theorem 3.1.

4. Proof of Theorem 1.2

The aim of this section is to give a proof of Theorem 1.2. First we prove the pressure formula for (δ, n, ϵ)-separated sets and $(g; n, \epsilon)$-separated sets, which play important roles in proving Theorem 1.2. For $\varphi \in C(X, \mathbb{R})$, $n \geq 1$, $\epsilon > 0$, $\delta > 0$ and $g \colon \mathbb{N} \to \mathbb{N}$ we set

$$P_n(f, \varphi, \epsilon, \delta) := \sup \left\{ \sum_{x \in E} e^{S_n \varphi(x)} : E \text{ is a } (\delta, n, \epsilon)\text{-separated set in } X \right\}$$

and

$$P_n(f, \varphi, g; \epsilon) := \sup \left\{ \sum_{x \in E} e^{S_n \varphi(x)} : E \text{ is a } (g; n, \epsilon)\text{-separated set in } X \right\}.$$

Proposition 4.1. *For any $\varphi \in C(X, \mathbb{R})$,*

$$\begin{aligned} P(f, \varphi) &= \lim_{\epsilon, \delta \to 0} \limsup_{n \to \infty} \frac{1}{n} \log P_n(f, \varphi, \epsilon, \delta) \\ &= \lim_{\epsilon, \delta \to 0} \liminf_{n \to \infty} \frac{1}{n} \log P_n(f, \varphi, \epsilon, \delta). \end{aligned}$$

Proof. For any $\epsilon > 0$, $\delta > 0$ and $n \geq 1$, we have $P_n(f, \varphi, \epsilon, \delta) \leq P_n(f, \varphi, \epsilon)$ since every (δ, n, ϵ)-separated set is also (n, ϵ)-separated. This implies

$$\lim_{\epsilon, \delta \to 0} \limsup_{n \to \infty} \frac{1}{n} \log P_n(f, \varphi, \epsilon, \delta) \leq P(f, \varphi).$$

For any $t > 0$ and any ergodic measure μ, we choose a neighborhood F of μ so that $\nu \in F$ implies $|\int \varphi d\mu - \int \varphi d\nu| < t$. Then by Proposition 2.5, there exist $\epsilon > 0$, $\delta > 0$ and $N \in \mathbb{N}$ such that for any $n \geq N$ there exists a (δ, n, ϵ)-separated set $\Gamma \subset X_{n,F}$ with $\sharp\Gamma \geq e^{n(h_\mu(f) - t)}$. Since $\Gamma \subset X_{n,F}$, $S_n\varphi(x) > n(\int \varphi d\mu - t)$ holds for any $x \in \Gamma$, and so we have

$$\begin{aligned} \frac{1}{n} \log P_n(f, \varphi, \epsilon, \delta) &\geq \frac{1}{n} \log \sum_{x \in \Gamma} e^{S_n \varphi(x)} \\ &\geq \frac{1}{n} \log \sum_{x \in \Gamma} e^{n(\int \varphi d\mu - t)} \\ &= \frac{1}{n} \log \sharp\Gamma e^{n(\int \varphi d\mu - t)} \\ &\geq h_\mu(f) + \int \varphi d\mu - 2t \end{aligned}$$

for any $n \geq N$. Since the quantity $\liminf_{n \to \infty} \frac{1}{n} \log P_n(f, \varphi, \epsilon, \delta)$ increases if ϵ and δ decrease, Proposition 4.1 follows from the variational principle (see [18], Corollary 8.6.1). $\square$

Proposition 4.2. *Let $f \colon X \to X$ be a positively expansive continuous map or an expansive homeomorphism of a compact metric space (X, d) with an expansive constant ϵ. Then for any $\varphi \in C(X, \mathbb{R})$ and any $g \colon \mathbb{N} \to \mathbb{N}$ with $\lim_{n \to \infty} \frac{g(n)}{n} = 0$,*

$$P(f, \varphi) = \lim_{n \to \infty} \frac{1}{n} \log P_n(f, \varphi, g; \epsilon)$$

holds.

Proof. We treat the non-invertible case because the invertible case can be shown by similar arguments. Since every $(g;n,\epsilon)$-separated set is (n,ϵ)-separated, we have

$$P(f,\varphi) \geq \limsup_{n\to\infty} \frac{1}{n} \log P_n(f,\varphi,g;\epsilon).$$

Take any $0 < \epsilon' \leq \epsilon$ and $\delta > 0$. Since f is expansive, there exists an integer $M > 0$ such that $\sup_{x\in X} diam(B_M(x,\epsilon)) \leq \epsilon'$. If we choose an $N > M$ with $\sup_{n\geq N} \frac{Mg(n)}{n} \leq \delta$, then every (δ,n,ϵ')-separated set is $(g;n,\epsilon)$-separated for any $n \geq N$. Indeed, let E be a (δ,n,ϵ')-separated set and take any two points $x,y \in E$ with $\sharp\{0 \leq j \leq n-1 : d(f^j x, f^j y) > \epsilon\} \leq g(n)$. If we set

$$\Lambda := \{0 \leq k \leq n-1 : \text{there exists } k \leq j \leq k+M-1 \text{ such that } d(f^j x, f^j y) > \epsilon\},$$

then it is easy to see that $\sharp\Lambda \leq Mg(n)$ and $d(f^k x, f^k y) \leq \epsilon'$ for any $k \notin \Lambda$. So we have

$$\sharp\{0 \leq j \leq n-1 : d(f^j x, f^j y) > \epsilon'\} \leq n\delta,$$

which implies $x = y$ since E is (δ,n,ϵ')-separated.

Thus we get

$$\liminf_{n\to\infty} \frac{1}{n} \log P_n(f,\varphi,\epsilon',\delta) \leq \liminf_{n\to\infty} \frac{1}{n} \log P_n(f,\varphi,g;\epsilon).$$

Since ϵ' and δ are arbitrary, Proposition 4.2 follows from Proposition 4.1. □

Now we give a proof of Theorem 1.2. We treat the non-invertible case because the invertible case can be shown in a similar way. Since f is positively expansive, $P_n(f)$ is an (n,ϵ)-separated subset for any n and sufficiently small $\epsilon > 0$. So we have

$$\limsup_{n\to\infty} \frac{1}{n} \log \sum_{x\in P_n(f)} e^{S_n\varphi(x)} \leq P(f,\varphi).$$

Let $\epsilon > 0$ be an expansive constant, let $\delta > 0$ be an arbitrary number, let n be a sufficiently large integer and let E be a $(2g;n,\epsilon)$-separated subset, where $g\colon \mathbb{N} \to \mathbb{N}$ is a function as in Definition 2.3. Since f satisfies *S-AlPP*, for any $z \in E$ there exists a point $\sigma(z) \in P_n(f)$ such that

$$\sharp\{0 \leq j \leq n-1 : d(f^j z, f^j \sigma(z)) > \frac{\epsilon}{2}\} \leq g(n).$$

Since E is $(2g;n,\epsilon)$-separated, the map $\sigma\colon E \to P_n(f)$ is injective. Moreover, it follows from Lemma 5.2 in [16] that $S_n\varphi(z) \leq S_n\varphi(\sigma(z)) + n\delta$. So we have

$$\frac{1}{n} \log \sum_{z\in E} e^{S_n\varphi(z)} \leq \frac{1}{n} \log \sum_{z\in E} e^{S_n\varphi(\sigma(z))} + \delta \leq \frac{1}{n} \log \sum_{x\in P_n(f)} e^{S_n\varphi(x)} + \delta.$$

Thus Theorem 1.2 follows from Proposition 4.2.

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