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A WAVELET CHARACTERIZATION FOR THE DUAL OF WEIGHTED HARDY SPACES

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ABSTRACT. We define the weighted Carleson measure space CMO^p_w using wavelets, where the weight function w belongs to the Muckenhoupt class. Then we show that CMO^p_w is the dual space of the weighted Hardy space H^p_w by using sequence spaces. As an application, we give a wavelet characterization of BMO_w.

1. INTRODUCTION

Meyer [4] described the Hardy space H^1 and BMO via wavelets. He offered several characterizations of H^1 in terms of its decompositions with respect to wavelet bases, and characterized BMO in terms of a Carleson condition on wavelet coefficients. A natural extension is to consider their weighted counterparts. In 2001, Garcia-Cuerva and Martell [2] gave a wavelet characterization of weighted Hardy spaces $H^p_w(\mathbb{R})$, $0 < p \le 1$. In this article, we give a wavelet characterization for the dual of $H^p_w(\mathbb{R})$, $0 < p \le 1$. In order to do this, we define the weighted Carleson measure space CMO^p_w and two sequence spaces s^p_w and c^p_w. We first show that c^p_w is the dual of s^p_w and then obtain that CMO^p_w is the dual of H^p_w. As a consequence, CMO^1_w is the same as BMO_w, and hence we succeed by an approach different from the one in [5] for the wavelet characterization of BMO_w.

Let ψ be an *orthonormal wavelet*; that is, $\psi \in L^2(\mathbb{R})$ such that the system

$$\psi_{j,k}(x) := 2^{j/2}\psi(2^j x - k), \qquad j, k \in \mathbb{Z},$$

is an orthonormal basis for $L^2(\mathbb{R})$. We define the operator $\mathcal{W}_\psi$ by

$$\mathcal{W}_\psi f = \left\{ \sum_{j,k\in\mathbb{Z}} |\langle f, \psi_{j,k}\rangle|^2 |I_{j,k}|^{-1} \chi_{I_{j,k}} \right\}^{1/2}, \qquad f \in L^2(\mathbb{R}),$$

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where $I_{j,k} = [2^{-j}k, 2^{-j}(k+1)]$. Denoting by $\mathcal{D}$ the set of all dyadic intervals $I_{j,k}$ with $j,k \in \mathbb{Z}$, and letting $\psi_{I_{j,k}} = \psi_{j,k}$, we can also write

$$\mathcal{W}_\psi f = \left\{ \sum_{I\in\mathcal{D}} |\langle f, \psi_I\rangle|^2 |I|^{-1} \chi_I \right\}^{1/2}.$$

Henceforth, we always use I and J to denote dyadic intervals. In what follows, we shall work exclusively with the one-dimensional case. For $\alpha \geq 1$, we say that ψ belongs to the regularity class $\mathcal{R}^\alpha$ if $\psi \in C^{[\alpha]}$ and there exist positive constants C, r, ε satisfying

(i) $\displaystyle\int_{\mathbb{R}} x^n \psi(x)\,dx = 0$ for all $0 \leq n \leq [\alpha]-1$,

(ii) $\displaystyle|\psi(x)| \leq \frac{C}{(1+|x|)^{1+[\alpha]+r}}$ for all $x \in \mathbb{R}$,

(iii) $\displaystyle|\psi^{(n)}(x)| \leq \frac{C}{(1+|x|)^{\alpha+\varepsilon}}$ for all $x \in \mathbb{R}$ and $0 \leq n \leq [\alpha]$.

Here $[\alpha]$ denotes the greatest integer not greater than α.

The weight functions mentioned in this article refer to the Muckenhoupt A_q weights. A weight $w \geq 0$ belongs to the class A_q, $1 < q < \infty$, if there is a constant $C > 0$ such that

$$\left(\int_I w(x)\,dx\right)\left(\int_I w(x)^{-1/(q-1)}\,dx\right)^{q-1} \leq C|I|^q \qquad \text{for any interval } I \subset \mathbb{R}.$$

The class A_1 consists of weights w satisfying for some $C > 0$ that

$$\frac{1}{|I|}\int_I w(x)\,dx \leq C \cdot \operatorname*{ess\,inf}_{x\in I} w(x) \qquad \text{for any interval } I \subset \mathbb{R},$$

and $A_\infty := \bigcup_{1\leq q<\infty} A_q$. For $w \in A_\infty$, denote by $q_w := \inf\{q > 1 : w \in A_q\}$ the critical index of w. We use $w(E)$ to denote the weighted measure $\int_E w(x)dx$.

Let $\varphi \in \mathcal{S}$ satisfy $\int_{\mathbb{R}} \varphi(x)dx = 1$. The maximal function f^* is defined by

$$f^*(x) = \sup_{r>0} |f * \varphi_r(x)|,$$

where $\varphi_r(x) = r^{-1}\varphi(x/r)$, $r > 0$. The *weighted Hardy spaces* H^p_w consist of those tempered distributions $f \in \mathcal{S}'$ for which $f^* \in L^p_w$ with $\|f\|_{H^p_w} = \|f^*\|_{L^p_w}$. We refer readers to [1, 3] for the details about A_q and H^p_w.

The following theorem was proved by Garcia-Cuerva and Martell [2].

Theorem A. *Let $0 < p \leq 1$ and $w \in A_\infty$. If $\psi \in \mathcal{R}^\alpha$ is an orthonormal wavelet with $\alpha \geq q_w/p$, then there exist two constants $0 < c \leq C < \infty$ such that*

$$c\|f\|_{H^p_w} \leq \|\mathcal{W}_\psi f\|_{L^p_w} \leq C\|f\|_{H^p_w}.$$

Definition. For $0 < p \leq 1$ and $w \in A_\infty$, let $\psi \in \mathcal{R}^\alpha$ be an orthonormal wavelet with $\alpha \geq q_w/p$. The *weighted Carleson measure space* CMO^p_w is the set of all $g \in L^1_{\mathrm{loc}}$ satisfying

$$\|g\|_{CMO^p_w} := \sup_{J\in\mathcal{D}} \left\{ \frac{1}{w(J)^{\frac{2}{p}-1}} \sum_{I\subset J} |\langle g, \psi_I\rangle|^2 \frac{|I|}{w(I)} \right\}^{1/2} < \infty.$$

Remark 1. If $w \equiv$ constant and $p = 1$, then the above definition reduces to the Carleson condition that characterizes BMO (cf. [4, p. 154]). Theorem A implies that the wavelet characterization of H^p_w is independent of the choice of ψ, and hence, by the following Theorem 1, the definition of CMO^p_w is independent of the choice of ψ, too.

We now state our main result as follows.

Theorem 1. *For $0 < p \le 1$ and $w \in A_\infty$, let $\psi \in \mathcal{R}^\alpha$ be an orthonormal wavelet with $\alpha \ge q_w/p$. The dual of H^p_w is CMO^p_w in the following sense.*

(a) *For each $g \in CMO^p_w$, there is a linear functional ℓ_g, initially defined on $H^p_w \cap L^2$, which has a continuous extension to H^p_w and $\|\ell_g\| \le C\|g\|_{CMO^p_w}$.*

(b) *Conversely, every continuous linear functional ℓ of H^p_w can be realized as $\ell = \ell_g$ with some $g \in CMO^p_w$ and $\|g\|_{CMO^p_w} \le C\|\ell\|$.*

It is known that the dual space of H^1_w is

$$BMO_w = \Big\{ f \in L^1_{\rm loc} : \sup_{\text{interval } Q} \frac{1}{w(Q)} \int_Q |f(x) - f_Q| dx < \infty \Big\}$$

for $w \in A_\infty$, and the dual space of H^p_w, $0 < p < 1$, is

$$\left\{ \frac{f(x)}{w(x)} \in L^{r'}_{\rm loc}(w(x)dx) : \left(\int_Q \left| \frac{f(x) - P_Q(x)}{w(x)} \right|^{r'} \frac{w(x)dx}{w(Q)} \right)^{1/r'} \le C w(Q)^{1/p-1} \text{ for any bounded interval } Q \right\} \tag{1}$$

for $w \in A_r$, $1 \le r < \infty$, where $f_Q = \frac{1}{|Q|}\int_Q f(x)dx$ and P_Q is the unique polynomial of degree $\le [q_w/p] - 1$ such that $\int_Q (f(x) - P_Q(x))x^k dx = 0$ for $k = 0, 1, \cdots, [q_w/p] - 1$ (see [1]). Thus, we have a wavelet characterization of BMO_w and a continuous characterization of CMO^p_w as follows.

Corollary 2. *Let $0 < p \le 1$ and $w \in A_r$, $1 \le r \le \infty$. Also let $\psi \in \mathcal{R}^\alpha$ be an orthonormal wavelet with $\alpha \ge q_w/p$.*

(a) *For $p = 1$ and $w \in A_\infty$, $f \in BMO_w$ if and only if its wavelet coefficients $\langle f, \psi_I \rangle$ satisfy Carleson's condition:*

$$\sup_{J \in \mathcal{D}} \frac{1}{w(J)} \sum_{I \subset J} |\langle f, \psi_I \rangle|^2 \frac{|I|}{w(I)} \le C.$$

(b) *For $0 < p < 1$ and $w \in A_r$, $1 \le r < \infty$, f satisfies (1) if and only if $f \in CMO^p_w$.*

Remark 2. When the wavelet ψ has compact support, the above characterization of BMO_w was given by Wu [5]. Here we offer a different but simpler approach.

2. Sequence spaces

In this section, we introduce two sequence spaces s^p_w and c^p_w, $0 < p \le 1$.

Definition. Let $0 < p \le 1$ and $w \ge 0$ be a weight function. The sequence space s^p_w is defined to be the collection of all complex-valued sequences

$$s^p_w = \left\{ \{s_I\} : \|\{s_I\}\|_{s^p_w} := \left\| \left(\sum_I |s_I|^2 |I|^{-1} \chi_I \right)^{1/2} \right\|_{L^p_w} < \infty \right\}.$$

Similarly, c_w^p is defined to be the collection of all complex-valued sequences

$$c_w^p = \left\{ \{t_I\} : \|\{t_I\}\|_{c_w^p} := \sup_{J\in\mathcal{D}} \left(\frac{1}{w(J)^{\frac{2}{p}-1}} \sum_{I\subset J} |t_I|^2 \frac{|I|}{w(I)} \right)^{1/2} < \infty \right\}.$$

Theorem 3. *Let $0 < p \le 1$ and $w \in A_\infty$. The dual of s_w^p is c_w^p in the following sense.*

(a) *For each $\{t_I\} \in c_w^p$, the linear functional $\{s_I\} \mapsto \sum_I s_I \cdot \overline{t_I}$ is continuous on s_w^p.*

(b) *Conversely, every continuous linear functional on s_w^p arises as in* (a) *with a unique element $\{t_I\}$ of c_w^p.*

Moreover, the norm of $\{t_I\}$ as a linear functional on s_w^p is equivalent to its c_w^p-norm.

Proof. (a) Given $\{t_I\} \in c_w^p$, it suffices to show that

$$\left| \sum_I s_I \cdot \overline{t_I} \right| \le C \|\{s_I\}\|_{s_w^p} \|\{t_I\}\|_{c_w^p} \qquad \text{for all } \{s_I\} \in s_w^p.$$

For $\{s_I\} \in s_w^p$, write

$$\Omega_k = \left\{ x \in \mathbb{R} : S(x) := \left(\sum_I |s_I|^2 |I|^{-1} \chi_I(x) \right)^{1/2} > 2^k \right\}$$

and

$$B_k = \left\{ I : w(I \cap \Omega_k) > \frac{1}{2} w(I) \quad \text{and} \quad w(I \cap \Omega_{k+1}) \le \frac{1}{2} w(I) \right\}.$$

Then

$$\left| \sum_I s_I \cdot \overline{t_I} \right| = \left| \sum_k \sum_{\tilde I \in B_k} \sum_{\substack{I\subset \tilde I \\ I \in B_k}} s_I \cdot \overline{t_I} \right| \le \sum_k \sum_{\tilde I \in B_k} \sum_{\substack{I\subset \tilde I \\ I \in B_k}} |s_I| |\overline{t_I}|,$$

where $\tilde I$'s are the maximal dyadic intervals in B_k. Applying the inequality $\|\cdot\|_{\ell^1} \le \|\cdot\|_{\ell^p}$, we get

$$\begin{aligned} \sum_k \sum_{\tilde I \in B_k} \sum_{\substack{I\subset \tilde I \\ I \in B_k}} |s_I| |\overline{t_I}| &\le \sum_k \sum_{\tilde I \in B_k} \Bigg(\sum_{\substack{I\subset \tilde I \\ I \in B_k}} |s_I|^2 \frac{w(I)}{|I|} \Bigg)^{1/2} \Bigg(\sum_{\substack{I\subset \tilde I \\ I \in B_k}} |t_I|^2 \frac{|I|}{w(I)} \Bigg)^{1/2} \\ &\le \Bigg\{ \sum_k \sum_{\tilde I \in B_k} \Bigg(\sum_{\substack{I\subset \tilde I \\ I \in B_k}} |s_I|^2 \frac{w(I)}{|I|} \Bigg)^{p/2} \Bigg(\sum_{\substack{I\subset \tilde I \\ I \in B_k}} |t_I|^2 \frac{|I|}{w(I)} \Bigg)^{p/2} \Bigg\}^{1/p} \\ &\le \|\{t_I\}\|_{c_w^p} \Bigg\{ \sum_k \sum_{\tilde I \in B_k} w(\tilde I)^{1-p/2} \Bigg(\sum_{\substack{I\subset \tilde I \\ I \in B_k}} |s_I|^2 \frac{w(I)}{|I|} \Bigg)^{p/2} \Bigg\}^{1/p}. \end{aligned}$$

Write

$$\widetilde{\Omega}_k = \{ x \in \mathbb{R} : M_w(\chi_{\Omega_k})(x) > 1/2 \},$$

where M_w is the weighted Hardy-Littlewood maximal function defined by

$$M_w f(x) = \sup_{\text{interval } Q \ni x} \frac{1}{w(Q)} \int_Q |f(x)| w(x) dx.$$

Then $I \subset \widetilde{\Omega}_k$ for any $I \in B_k$. Since the $\tilde{I}$'s are mutually disjoint dyadic intervals, $\sum_{\tilde{I}\in B_k} w(\tilde{I}) \le w(\widetilde{\Omega}_k)$. We then apply Hölder's inequality to obtain

$$\left|\sum_I s_I \cdot \overline{t_I}\right| \le \|\{t_I\}\|_{c^p_w}\left\{\sum_k w(\widetilde{\Omega}_k)^{1-p/2}\left(\sum_{I\in B_k}|s_I|^2\frac{w(I)}{|I|}\right)^{p/2}\right\}^{1/p}.$$

We claim that

$$\sum_{I\in B_k}|s_I|^2\frac{w(I)}{|I|} \le C2^{2k}w(\widetilde{\Omega}_k).$$

M_w is of weak type $(1,1)$ with respect to $w(x)dx$, so $w(\widetilde{\Omega}_k) \le Cw(\Omega_k)$ and the claim gives

$$\begin{aligned}\left|\sum_I s_I \cdot \overline{t_I}\right| &\le C\|\{t_I\}\|_{c^p_w}\left\{\sum_k 2^{kp}w(\widetilde{\Omega}_k)\right\}^{1/p}\\ &\le C\|\{t_I\}\|_{c^p_w}\left\{\sum_k 2^{kp}w(\Omega_k)\right\}^{1/p}\\ &\le C\|\{t_I\}\|_{c^p_w}\|S\|_{L^p_w}\\ &= C\|\{t_I\}\|_{c^p_w}\|\{s_I\}\|_{s^p_w}.\end{aligned}$$

To prove the claim, by the definitions of $S(x)$ and B_k, we have

$$\int_{\widetilde{\Omega}_k\setminus\Omega_{k+1}} S^2(x)w(x)\,dx \le 2^{2k+2}w(\widetilde{\Omega}_k)$$

and

$$\begin{aligned}\int_{\widetilde{\Omega}_k\setminus\Omega_{k+1}} S^2(x)w(x)\,dx &\ge \int_{\widetilde{\Omega}_k\setminus\Omega_{k+1}}\sum_{I\in B_k}|s_I|^2|I|^{-1}\chi_I(x)w(x)\,dx\\ &= \sum_{I\in B_k}|s_I|^2\frac{w\big(I\cap(\widetilde{\Omega}_k\setminus\Omega_{k+1})\big)}{|I|}\\ &\ge \frac{1}{2}\sum_{I\in B_k}|s_I|^2\frac{w(I)}{|I|}.\end{aligned}$$

(b) Clearly, every $\ell \in (s^p_w)'$ is of the form

$$\ell(\{s_I\}) = \sum_I s_I\overline{t_I}, \qquad \{s_I\}\in s^p_w,$$

where $\{t_I\}$ is a certain sequence. Fix a dyadic interval J. Let $S_J = \{I \in \mathcal{D} : I \subset J\}$ and define a measure ν on S_J by

$$d\nu(I) = \frac{|I|}{w(J)^{\frac{2}{p}-1}} \qquad \text{for } \ I \in S_J.$$

By duality,

$$\begin{aligned}
\left(\frac{1}{w(J)^{\frac{2}{p}-1}}\sum_{I\subset J}|t_I|^2\frac{|I|}{w(I)}\right)^{1/2} &= \left\|\left\{t_I\frac{1}{w(I)^{\frac12}}\right\}\right\|_{\ell^2(S_J,d\nu)}\\
&= \sup_{\|\{s_I\}\|_{\ell^2(S_J,d\nu)}\le 1}\left|\sum_{I\subset J}s_I\overline{t_I}\frac{|I|}{w(J)^{\frac{2}{p}-1}w(I)^{\frac12}}\right| \qquad (2)\\
&\le \|\ell\|\sup_{\|\{s_I\}\|_{\ell^2(S_J,d\nu)}\le 1}\left\|\left\{s_I\frac{|I|}{w(J)^{\frac{2}{p}-1}w(I)^{\frac12}}\right\}\right\|_{s^p_w}.
\end{aligned}$$

For $\{s_I\}\in\ell^2(S_J,d\nu)$, Hölder's inequality yields

$$\begin{aligned}
\left\|\left\{s_I\frac{|I|}{w(J)^{\frac{2}{p}-1}w(I)^{\frac12}}\right\}\right\|_{s^p_w} &= \frac{1}{w(J)^{\frac{2}{p}-1}}\left\{\int_J\left(\sum_{I\subset J}|s_I|^2\frac{|I|}{w(I)}\chi_I(x)\right)^{p/2}w(x)\,dx\right\}^{1/p}\\
&\le\left\{\frac{1}{w(J)^{\frac{2}{p}-1}}\int_J\sum_{I\subset J}|s_I|^2\frac{|I|}{w(I)}\chi_I(x)w(x)\,dx\right\}^{1/2}\\
&=\|\{s_I\}\|_{\ell^2(S_J,d\nu)},
\end{aligned}$$

and hence

$$\sup_{\|\{s_I\}\|_{\ell^2(S_J,d\nu)}\le 1}\left\|\left\{s_I\frac{|I|}{w(J)^{\frac{2}{p}-1}w(I)^{\frac12}}\right\}\right\|_{s^p_w}\le 1.$$

Taking the supremum over $J\in\mathcal{D}$ in (2), we obtain $\|\{t_I\}\|_{c^p_w}\le\|\ell\|$. □

3. Proof of the main theorem

In this section we show that Theorem 1 follows as a consequence of Theorem 3. Let $\psi\in\mathcal{R}^\alpha$, $\alpha\ge 1$, be an orthonormal wavelet. Define a map P from the family of complex sequences into $\mathcal{S}'$ by

$$P(\{s_I\})=\sum_I s_I\psi_I.$$

Define another map L from function space into the family of complex sequences by

$$L(f)=\big\{\langle f,\psi_I\rangle\big\}$$

such that all $\langle f,\psi_I\rangle$'s are well defined. Figure 1 illustrates the relationship among s^p_w, c^p_w, H^p_w, and CMO^p_w. Then $P\circ L\big|_{L^2}$ is the identity on L^2. For $0<p\le 1$ and

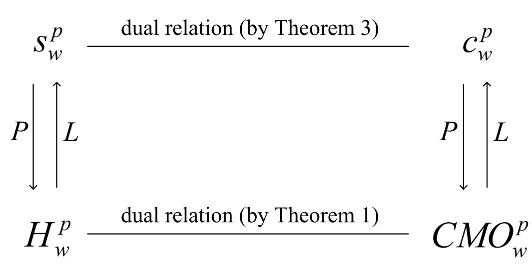

Figure 1. Diagram for spaces and maps

$w\in A_\infty$ with critical index q_w, if $\alpha\ge q_w/p$, then Theorem A yields

$$\|\{L(f)\}\|_{s^p_w}\le C\|f\|_{H^p_w}\qquad\text{for } f\in H^p_w\cap L^2 \qquad (3)$$

and

$$\|P(\{s_I\})\|_{H^p_w} \le C\|\mathcal{W}_\psi P(\{s_I\})\|_{L^p_w} = C\|\{s_I\}\|_{s^p_w} \qquad \text{for } \{s_I\} \in s^p_w. \tag{4}$$

By the definitions of c^p_w and CMO^p_w,

$$\|\{L(g)\}\|_{c^p_w} = \|g\|_{CMO^p_w} \qquad \text{for } g \in CMO^p_w \tag{5}$$

and

$$\|P(\{t_I\})\|_{CMO^p_w} = \|\{t_I\}\|_{c^p_w} \qquad \text{for } \{t_I\} \in c^p_w. \tag{6}$$

Proof of Theorem 1. For $g \in CMO^p_w$, define a linear functional $\tilde{\ell}_g$ by

$$\tilde{\ell}_g(f) = \langle L(f), L(g)\rangle \qquad \text{for } f \in H^p_w \cap L^2.$$

By (3), (5), and Theorem 3,

$$|\tilde{\ell}_g(f)| \le C\|L(f)\|_{s^p_w}\|L(g)\|_{c^p_w} \le C\|f\|_{H^p_w}\|g\|_{CMO^p_w} \qquad \text{for } f \in H^p_w \cap L^2.$$

Since $H^p_w \cap L^2$ is dense in H^p_w, the map $\tilde{\ell}_g$ can be extended to a continuous linear functional ℓ_g on H^p_w satisfying $\|\ell_g\| \le C\|g\|_{CMO^p_w}$.

Conversely, let $\ell \in (H^p_w)'$ and set $\ell_1 = \ell \circ P$ on s^p_w. It follows from (4) that $\ell_1 \in (s^p_w)'$. By Theorem 3, there exists $\{t_I\} \in c^p_w$ such that

$$\ell_1(\{s_I\}) = \sum_I s_I \cdot \overline{t_I} \qquad \text{for } \{s_I\} \in s^p_w,$$

and $\|\{t_I\}\|_{c^p_w} \approx \|\ell_1\| \le C\|\ell\|$. For $f \in H^p_w \cap L^2$, we have

$$\ell(f) = \ell_1 \circ L(f) = \sum_I \langle f, \psi_I\rangle \overline{t_I} = \langle L(f), L(g)\rangle,$$

where $g = \sum_I t_I \psi_I$. This shows that $\ell = \ell_g$, and (6) gives

$$\|g\|_{CMO^p_w} = \|\{t_I\}\|_{c^p_w} \le C\|\ell\|.$$

Hence, the proof is finished. □

References

1. J. Garcia-Cuerva, Weighted H^p spaces, *Dissertationes Math.* **162** (1979), 1-63. MR549091 (82a:42018)
2. J. Garcia-Cuerva and J. M. Martell, Wavelet characterization of weighted spaces, *J. Geom. Anal.* **11** (2001), 241-264. MR1856178 (2002h:42039)
3. J. García-Cuerva and J. L. Rubio de Francia, *Weighted Norm Inequalities and Related Topics*, North-Holland, Amsterdam, 1985. MR807149 (87d:42023)
4. Y. Meyer, *Wavelets and Operators*, Cambridge Studies in Advanced Mathematics, Vol. 37, Cambridge University Press, Cambridge, 1992. MR1228209 (94f:42001)
5. S. Wu, A wavelet characterization for weighted Hardy spaces, *Rev. Mat. Iberoamericana* **8** (1992), 329-349. MR1202414 (94i:42027)

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