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GROUND STATES OF NONLINEAR SCHRÖDINGER SYSTEMS

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Abstract. This paper concerns the existence of positive radial ground states of the time-independent Schrödinger system

$$\begin{cases} -\Delta u_1 + \lambda_1 u_1 = \mu_1 u_1^3 + \beta u_2^2 u_1, & \text{in } \mathbb{R}^n, \\ -\Delta u_2 + \lambda_2 u_2 = \mu_2 u_2^3 + \beta u_1^2 u_2, & \text{in } \mathbb{R}^n, \\ u_1(x) \to 0, \quad u_2(x) \to 0, & \text{as } |x| \to \infty, \end{cases}$$

where $n = 1, 2, 3$, $\lambda_j > 0$ and $\mu_j > 0$ for $j = 1, 2$, and $\beta > 0$. A result from Sirakov, *Comm. Math. Phys.* **271** (2007), 199-221, is improved.

Consider the time-independent Schrödinger system

$$\begin{cases} -\Delta u_1 + \lambda_1 u_1 = \mu_1 u_1^3 + \beta u_2^2 u_1, & \text{in } \mathbb{R}^n, \\ -\Delta u_2 + \lambda_2 u_2 = \mu_2 u_2^3 + \beta u_1^2 u_2, & \text{in } \mathbb{R}^n, \\ u_1(x) \to 0, \quad u_2(x) \to 0, & \text{as } |x| \to \infty, \end{cases} \tag{1}$$

where $n = 1, 2, 3$, $\lambda_j > 0$ and $\mu_j > 0$ for $j = 1, 2$, and $\beta > 0$. Solutions $(u_1(x), u_2(x))$ of (1) correspond to standing wave solutions $(e^{i\lambda_1 t} u_1(x), e^{i\lambda_2 t} u_2(x))$ of the time-dependent system of 2 coupled nonlinear Schrödinger equations

$$\begin{cases} -i\frac{\partial}{\partial t}\Phi_1 = \Delta\Phi_1 + \mu_1|\Phi_1|^2\Phi_1 + \beta|\Phi_2|^2\Phi_1, & \text{in } \mathbb{R}^n, \\ -i\frac{\partial}{\partial t}\Phi_2 = \Delta\Phi_2 + \mu_2|\Phi_2|^2\Phi_2 + \beta|\Phi_1|^2\Phi_2, & \text{in } \mathbb{R}^n, \\ \Phi_1(x,t) \to 0, \quad \Phi_2(x,t) \to 0, & \text{as } |x| \to \infty. \end{cases} \tag{2}$$

The system (2) stems from many physical problems, especially in nonlinear optics and in the Hartree-Fock theory for Bose-Einstein condensates; see, for example, [1, 5, 8, 9, 10, 11, 12, 20, 23].

Recently, (1) has attracted tremendous attention and has been studied extensively from the point of view of physics (see [1, 10, 11] for instance) as well as mathematics (see [2, 3, 4, 13, 14, 15, 16, 18, 19, 21, 22, 24, 25]). Solutions of (1) correspond to critical points of the energy functional

$$E(\vec{u}) = \frac{1}{2}\int_{\mathbb{R}^n} (|\nabla u_1|^2 + \lambda_1 u_1^2 + |\nabla u_2|^2 + \lambda_2 u_2^2) - \frac{1}{4}\int_{\mathbb{R}^n} (\mu_1 u_1^4 + 2\beta u_1^2 u_2^2 + \mu_2 u_2^4),$$

defined for $\vec{u} = (u_1, u_2) \in (H^1(\mathbb{R}^n))^2$. A solution (u_1, u_2) of (1) is called a nontrivial solution if $u_1 \neq 0$ and $u_2 \neq 0$, a semitrivial solution if either $u_1 \neq 0$ or $u_2 \neq 0$, a positive solution if $u_1 > 0$ and $u_2 > 0$, and a semipositive solution if $u_1 \geq 0$ and

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$u_2 \geq 0$ and if either $u_1 \neq 0$ or $u_2 \neq 0$. A solution is said to be a ground state if it has the least energy among all semitrivial solutions.

Let $H_r^1(\mathbb{R}^n)$ consist of all radial functions in $H^1(\mathbb{R}^n)$. Denote $X = (H^1(\mathbb{R}^n))^2$ and $X_r = (H_r^1(\mathbb{R}^n))^2$. It was proved in [4] that (1) has a semipositive radial ground state which is of mountain pass type and has Morse index 1 when considered as the critical point of E on X and on X_r. As indicated in [4], a semipositive ground state may only be semipositive and therefore there does not exist a positive ground state, and this is the case if, for example, $\lambda_1 \leq \lambda_2$ and $\mu_1 \geq \beta \geq \mu_2$ and at least one inequality among the three is strict. Therefore, a natural problem is when (1) has a positive ground state. In the case $\lambda_1 = \lambda_2$, there is a complete answer from [4] which states that (1) has a positive ground state if and only if $\beta > \max\{\mu_1, \mu_2\}$ or $\beta = \mu_1 = \mu_2$. Explicit positive ground states can be obtained in this case. According to [22], (u_1, u_2) with u_1 and u_2 defined by

$$u_1 = \sqrt{\frac{\lambda(\beta - \mu_2)}{\beta^2 - \mu_1\mu_2}} w(\sqrt{\lambda}\,\cdot) \quad \text{and} \quad u_2 = \sqrt{\frac{\lambda(\beta - \mu_1)}{\beta^2 - \mu_1\mu_2}} w(\sqrt{\lambda}\,\cdot),$$

where w is the unique positive radial solution of

$$-\Delta w + w = w^3 \ \text{ in } \mathbb{R}^n, \quad w(x) \to 0 \ \text{ as } |x| \to \infty, \tag{3}$$

is a positive ground state of (1) if $\lambda_1 = \lambda_2$ (denoted by λ) and $\beta > \max\{\mu_1, \mu_2\}$. Also, a direct computation shows that

$$\big(\cos\theta\sqrt{\lambda/\beta}\, w(\sqrt{\lambda}\,\cdot),\ \sin\theta\sqrt{\lambda/\beta}\, w(\sqrt{\lambda}\,\cdot)\big), \quad \theta \in (0, \pi/2)$$

are positive ground states of (1) if $\lambda_1 = \lambda_2$ (denoted by λ) and $\beta = \mu_1 = \mu_2$. Results similar to [4] were proved in [6], using the bifurcation theorem.

For $\lambda_1 \neq \lambda_2$ the problem of whether (1) has a positive ground state remains largely open. In what follows, we shall always assume $\lambda_2 \geq \lambda_1$ without introducing any essential restriction. In this general case, Bartsch and Wang [4] first proved that there is some $b > 0$ such that the ground state solution for (1) is positive if $\beta > b$. But no information on how large b is was given. Assuming $\lambda_1 = 1$ and $\lambda := \lambda_2 \geq 1$, Sirakov (see [22, Theorem 2(iv) and Section 3.4]) proved that if

$$\beta > \max\left\{\lambda\mu_1,\ \lambda^{\frac{n}{2}-1}\mu_2\right\}, \tag{4}$$

then (1) has a positive ground state. The aim of the present paper is to improve this result of Sirakov.

The argument of this paper is motivated by those in [4]. We shall distinguish nontrivial solutions from those which are only semitrivial (that is, solutions with only one component being nonzero) by comparing the Morse indices of them.

To state our result, we first introduce the notation

$$C_1 = \int_{\mathbb{R}^n} w^2(x)dx, \quad C_2 = \int_{\mathbb{R}^n} w^4(x)dx.$$

Theorem 1. *Assume $\lambda_1 \leq \lambda_2$. If*

$$\beta > \max\left\{\mu_1 + \frac{(\lambda_2 - \lambda_1)C_1}{\lambda_1 C_2}\mu_1,\ \mu_2 - \frac{(\lambda_2 - \lambda_1)C_1}{\lambda_2 C_2}\mu_2\right\}, \tag{5}$$

then (1) *has a positive radial ground state.*

Remark 2. a) We first compare Theorem 1 with [22, Theorem 2(iv)] in the two and three dimensional cases. Since w is a positive solution of (3), we see that $0 < C_1 < C_2$ and therefore

$$\frac{\lambda_2}{\lambda_1}\mu_1 > \mu_1 + \frac{(\lambda_2 - \lambda_1)C_1}{\lambda_1 C_2}\mu_1.$$

Since clearly, for $n = 2, 3$,

$$\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{n}{2}-1}\mu_2 > \mu_2 - \frac{(\lambda_2 - \lambda_1)C_1}{\lambda_2 C_2}\mu_2,$$

the assumption (5) is weaker than (4). Thus our theorem improves [22, Theorem 2(iv)]. Note that other sufficient assumptions were also given in [22] (see [Proposition 3.7] therein) to guarantee existence of a positive radial ground state of (1). Nevertheless, those assumptions involve heavy notation and were described with quite complex inequalities, which are not easy to verify.

b) It seems that (5) is only a sufficient condition for (1) to have a positive radial ground state in the case $\lambda_1 < \lambda_2$. But if $\lambda_1 = \lambda_2$, then (5) reduces to

$$\beta > \max\{\mu_1, \mu_2\},$$

and in this case (5) is necessary for (1) to have a positive radial ground state provided that $\mu_1 \neq \mu_2$, according to [4].

c) The argument provided here cannot be generalized to systems consisting of more than 2 equations. By comparing energies, a positive radial ground state for systems consisting of N ($N \geq 2$) equations was recently obtained in [17].

Proof of Theorem 1. Denote

$$w_j(x) = \sqrt{\frac{\lambda_j}{\mu_j}}w(\sqrt{\lambda_j}x), \quad j = 1, 2.$$

Then, for $j = 1, 2$, w_j is the unique solution of the equation

$$\begin{cases} -\Delta w + \lambda_j w = \mu_j w^3 \ \text{ and } \ w > 0 \ \text{ in } \mathbb{R}^n, \\ w(0) = \max w(x), \quad w(x) \to 0 \text{ as } |x| \to \infty. \end{cases}$$

Set $U_1 = (w_1, 0)$ and $U_2 = (0, w_2)$. Then U_1 and U_2 are the only two semipositive radial solutions of (1) with only one component being nonzero.

It has been proved in [4] that (1) has a semipositive radial ground state $\vec{u} \in X_r$ and its Morse index is 1. In order to prove that this ground state solution is positive, it suffices to prove that U_1 and U_2 have Morse indices at least 2. We now estimate the Morse indices of U_1 and U_2. For any $(\phi_1, \phi_2) \in (H^1(\mathbb{R}^n))^2$, we have

$$\begin{aligned} &E''(U_1)[(\phi_1, \phi_2), (\phi_1, \phi_2)] \\ &\quad = \int_{\mathbb{R}^n} (|\nabla\phi_1|^2 + \lambda_1\phi_1^2 + |\nabla\phi_2|^2 + \lambda_2\phi_2^2) - \int_{\mathbb{R}^n} (3\mu_1 w_1^2\phi_1^2 + \beta w_1^2\phi_2^2). \end{aligned}$$

Thus, for any $t_1, t_2 \in \mathbb{R}$,

$$\begin{aligned} &E''(U_1)[(t_1w_1, t_2w_1),\ (t_1w_1, t_2w_1)] \\ &\quad = t_1^2\int_{\mathbb{R}^n} (|\nabla w_1|^2 + \lambda_1 w_1^2 - 3\mu_1 w_1^4) + t_2^2\int_{\mathbb{R}^n} (|\nabla w_1|^2 + \lambda_2 w_1^2 - \beta w_1^4) \\ &\quad = t_1^2\int_{\mathbb{R}^n} (-2\mu_1 w_1^4) + t_2^2\int_{\mathbb{R}^n} \big((\mu_1 - \beta)w_1^4 + (\lambda_2 - \lambda_1)w_1^2\big). \end{aligned}$$

Since

$$\int_{\mathbb{R}^n} \left((\mu_1-\beta)w_1^4+(\lambda_2-\lambda_1)w_1^2\right) = -\frac{(\beta-\mu_1)\lambda_1^{2-\frac{n}{2}}C_2}{\mu_1^2}+\frac{(\lambda_2-\lambda_1)\lambda_1^{1-\frac{n}{2}}C_1}{\mu_1}$$
$$= \frac{\lambda_1^{2-\frac{n}{2}}C_2}{\mu_1^2}\left[-\beta+\mu_1+\frac{(\lambda_2-\lambda_1)C_1}{\lambda_1 C_2}\mu_1\right],$$

we see that if

$$\beta > \mu_1 + \frac{(\lambda_2-\lambda_1)C_1}{\lambda_1 C_2}\mu_1, \tag{6}$$

then, for any $(t_1,t_2)\in\mathbb{R}^2\setminus\{(0,0)\}$,

$$E''(U_1)[(t_1w_1,t_2w_1),\ (t_1w_1,t_2w_1)]<0.$$

Therefore, if (6) holds, then the Morse index of U_1 is at least 2. Similarly, for any $(\phi_1,\phi_2)\in(H^1(\mathbb{R}^n))^2$, we have

$$E''(U_2)[(\phi_1,\phi_2),\ (\phi_1,\phi_2)]$$
$$= \int_{\mathbb{R}^n}(|\nabla\phi_1|^2+\lambda_1\phi_1^2+|\nabla\phi_2|^2+\lambda_2\phi_2^2)-\int_{\mathbb{R}^n}(3\mu_2w_2^2\phi_2^2+\beta w_2^2\phi_1^2).$$

Choosing $(\phi_1,\phi_2)=(t_1w_2,t_2w_2)$, one then obtains

$$E''(U_2)[(t_1w_2,t_2w_2),\ (t_1w_2,t_2w_2)]$$
$$= t_1^2\int_{\mathbb{R}^n}(|\nabla w_2|^2+\lambda_1w_2^2-\beta w_2^4)+t_2^2\int_{\mathbb{R}^n}(|\nabla w_2|^2+\lambda_2w_2^2-3\mu_2w_2^4)$$
$$= t_1^2\int_{\mathbb{R}^n}\left((\mu_2-\beta)w_2^4+(\lambda_1-\lambda_2)w_2^2\right)+t_2^2\int_{\mathbb{R}^n}(-2\mu_2w_2^4).$$

But

$$\int_{\mathbb{R}^n}((\mu_2-\beta)w_2^4+(\lambda_1-\lambda_2)w_2^2) = \frac{\lambda_2^{2-\frac{n}{2}}(\mu_2-\beta)C_2}{\mu_2^2}+\frac{\lambda_2^{1-\frac{n}{2}}(\lambda_1-\lambda_2)C_1}{\mu_2}$$
$$= \frac{\lambda_2^{2-\frac{n}{2}}C_2}{\mu_2^2}\left[\mu_2-\beta-\frac{(\lambda_2-\lambda_1)C_1}{\lambda_2C_2}\mu_2\right].$$

Therefore, if

$$\beta > \mu_2 - \frac{C_1(\lambda_2-\lambda_1)}{C_2\lambda_2}\mu_2, \tag{7}$$

then, for any $(t_1,t_2)\in\mathbb{R}^2\setminus\{(0,0)\}$,

$$E''(U_2)[(t_1w_2,t_2w_2),\ (t_1w_2,t_2w_2)]<0,$$

which implies that if (7) holds, then the Morse index of U_2 is at least 2. Now if β satisfies both (6) and (7), then the Morse indices of U_1 and U_2 are at least 2. Since U_1 and U_2 are the only two semipositive radial solutions of (1) with only one component being nonzero and since the semipositive radial ground state obtained in [4] has Morse index 1, the semipositive radial ground state is positive. □

Remark 3. In estimating the lower bound of the Morse index of E at U_1, we have used the two-dimensional subspace spanned by $(w_1,0)$ and $(0,w_1)$ as a test subspace, other than span$\{(w_2,0),\ (0,w_1)\}$ or span$\{(w_1,0),\ (0,w_2)\}$ or span$\{(w_2,0),\ (0,w_2)\}$. The reason is that the subspace span$\{(w_1,0),\ (0,w_1)\}$ among the four

yields the most transparent estimate. All the other subspaces do not give an estimate which can be described as simply as (5). The same remark applies to the argument of estimating the lower bound of the Morse index of E at U_2. Nevertheless, if we use all four subspaces as test subspaces to estimate the Morse indices of E at U_1 and U_2, then we obtain the following result.

Theorem 4. *Assume* $\lambda_1 \le \lambda_2$. *If*

$$\beta > \max\{m_1(n,\lambda_1,\lambda_2)\mu_1,\ m_2(n,\lambda_1,\lambda_2)\mu_2\},$$

where

$$m_1(n,\lambda_1,\lambda_2) = \min\left\{1+(\lambda_2-\lambda_1)\lambda_1^{-1}C_1C_2^{-1},\ \lambda_2^{1-\frac{n}{2}}\lambda_1^{-1}C_2C^{-1}(\lambda_1,\lambda_2)\right\},$$
$$m_2(n,\lambda_1,\lambda_2) = \min\left\{1-(\lambda_2-\lambda_1)\lambda_2^{-1}C_1C_2^{-1},\ \lambda_1^{1-\frac{n}{2}}\lambda_2^{-1}C_2C^{-1}(\lambda_1,\lambda_2)\right\},$$

and

$$C(\lambda_1,\lambda_2) = \int_{\mathbb{R}^n} w^2(\sqrt{\lambda_1}\,x)w^2(\sqrt{\lambda_2}\,x)dx,$$

then (1) *has a positive radial ground state.*

Remark 5. a) One may think of giving a comparison between the numbers $1+(\lambda_2-\lambda_1)\lambda_1^{-1}C_1C_2^{-1}$ and $\lambda_2^{1-\frac{n}{2}}\lambda_1^{-1}C_2C^{-1}(\lambda_1,\lambda_2)$ as well as a comparison between $1-(\lambda_2-\lambda_1)\lambda_2^{-1}C_1C_2^{-1}$ and $\lambda_1^{1-\frac{n}{2}}\lambda_2^{-1}C_2C^{-1}(\lambda_1,\lambda_2)$. We do not know how to do this at the present stage. Therefore, we do not know if the result in Theorem 4 is better than the result in Theorem 1.

b) But in the $n=1$ dimensional case, w has an explicit expression:

$$w(x) = \frac{2\sqrt{2}}{e^x+e^{-x}}.$$

Using this expression, we easily see that $C_1 = 4$ and $C_2 = \frac{16}{3}$. If, for example, $\lambda_2 = 4\lambda_1$ or $\lambda_2 = 9\lambda_1$, then after a long but elementary calculation, we obtain

$$C(\lambda_1,4\lambda_1) = \frac{4(4-\pi)}{\sqrt{\lambda_1}},\quad C(\lambda_1,9\lambda_1) = \frac{16}{3\sqrt{\lambda_1}} - \frac{128\pi}{81\sqrt{3\lambda_1}}.$$

Then the four numbers mentioned in a) can be estimated in the following table.

	$\lambda_2 = 4\lambda_1$	$\lambda_2 = 9\lambda_1$
$1+(\lambda_2-\lambda_1)\lambda_1^{-1}C_1C_2^{-1}$	$\frac{13}{4} = 3.25$	7
$\lambda_2^{1-\frac{n}{2}}\lambda_1^{-1}C_2C^{-1}(\lambda_1,\lambda_2)$	$\frac{8}{3(4-\pi)} \approx 3.11$	$\frac{81\sqrt{3}}{27\sqrt{3}-8\pi} \approx 6.49$
$1-(\lambda_2-\lambda_1)\lambda_2^{-1}C_1C_2^{-1}$	$\frac{7}{16} \approx 0.44$	$\frac{1}{3} \approx 0.33$
$\lambda_1^{1-\frac{n}{2}}\lambda_2^{-1}C_2C^{-1}(\lambda_1,\lambda_2)$	$\frac{1}{3(4-\pi)} \approx 0.39$	$\frac{3\sqrt{3}}{27\sqrt{3}-8\pi} \approx 0.24$

From this table it can be seen that, for $n=1$ and $\lambda_2 = 4\lambda_1$ or $\lambda_2 = 9\lambda_1$, Theorem 4 is better than Theorem 1. One may calculate $C(\lambda_1,\lambda_2)$ and compare the two theorems in the case $\lambda_2 = k^2\lambda_1$ with an even larger k, say $k = 4, 5, \cdots$, which is more complicated. If λ_2/λ_1 is not a square power of an integer, it seems hard to calculate $C(\lambda_1,\lambda_2)$ and to compare the results of Theorems 1 and 4.

c) Also in the one dimensional case, if $\lambda_2 > \lambda_1$, then

$$C(\lambda_1, \lambda_2) > \frac{C_2}{\sqrt{\lambda_2}},$$

and we have

$$m_1(1, \lambda_1, \lambda_2) \leq 1 + (\lambda_2 - \lambda_1)\lambda_1^{-1} C_1 C_2^{-1} = \frac{1}{4} + \frac{3\lambda_2}{4\lambda_1} < \frac{\lambda_2}{\lambda_1}$$

and

$$m_2(1, \lambda_1, \lambda_2) \leq \sqrt{\lambda_1}\lambda_2^{-1} C_2 C^{-1}(\lambda_1, \lambda_2) < \sqrt{\frac{\lambda_1}{\lambda_2}}.$$

Therefore, Theorem 4 improves [22, Theorem 2(iv)] in the one dimensional case.

d) Extensions of Sirakov's results were also obtained, in particular, in the one dimensional case in [7].

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