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# AN EXAMPLE OF AN ALMOST GREEDY BASIS IN $L^1(0,1)$

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Abstract. We give an explicit construction of an almost greedy basis of $L^1(0,1)$, complementing the results on existence of such a basis. The basis is described in terms of the Haar basis. We construct a quasi-greedy basis in a Banach space which is isomorphic to $L^1(0,1)$, and then we calculate an isomorphic image of a quasi-greedy basis.

## 1. Introduction

Let $\Psi = \{\psi_k\}_{k=1}^{+\infty}$ be a normalized basis in a Banach space $X$. For each $f \in X$ we have

$$f = \sum_{k=1}^{\infty} c_k(f,\Psi)\psi_k, \tag{1}$$

where $\lim_{k\to\infty} c_k(f,\Psi) = 0$. We define $\Lambda_0 = \emptyset$; then for each $m \geq 1$ we inductively define sets $\Lambda_m \subset \mathbb{N}$ to satisfy

$$\#\Lambda_m = m, \quad \Lambda_{m-1} \subset \Lambda_m \quad \text{and} \quad \min_{k\in\Lambda_m} \mid c_k(f,\Psi) \mid \geq \max_{k\notin\Lambda_m} \mid c_k(f,\Psi) \mid .$$

Note that the sets $\Lambda_m$ are not uniquely determined. Denote the set of all such sequences $\{\Lambda_m\}$ by $D(f)$. For any $\Lambda \in D(f)$ we put

$$G_m(f) = G_m(f,\Psi) = G_m(f,\Psi,\Lambda) = \sum_{k\in\Lambda_m} c_k(f,\Psi)\psi_k.$$

This nonlinear method of approximation is known as a Thresholding Greedy Algorithm (TGA) or as a Greedy algorithm (see for example [6]).

In fact $G_m(f)$ can be realized by the following procedure: take the expansion (1) and form a sum of $m$ terms with the biggest $\mid c_k(f,\Psi) \mid$. If the basis $\Psi$ is unconditional, then obviously

$$\|G_m(f,\Psi,\Lambda)\|_X \leq C \cdot \|f\|_X, \tag{2}$$

with a constant $C$ independent of $f, m$ and $\Lambda$.

**Definition.** A basis $\Psi = \{\psi_k\}_{k=1}^{+\infty}$ is called quasi-greedy for $X$ if there exists a constant $C$ such that for any $f \in X$ and for any $\Lambda \in D(f)$ the inequality (2) holds for any $m \in \mathbb{N}$.

The following theorem was proved in [8].

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**Theorem A.** *A basis $\Psi = \{\psi_k\}_{k=1}^{+\infty}$ is quasi-greedy for $X$ if and only if for any $f \in X$ there exists $\Lambda \in D(f)$ such that*

$$\lim_{m\to+\infty} \|f - G_m(f,\Psi,\Lambda)\|_X = 0.$$

**Definition 2.** A basis $\Psi = \{\psi_k\}_{k=1}^{+\infty}$ is called almost greedy for $X$ if there exists a constant $C$ such that for all $f \in X$ and $m \in \mathbb{N}$ we have

$$\|f - G_m(f,\Psi)\|_X \le C \inf_{A\subset\mathbb{N},|A|=m} \left\{ \left\| f - \sum_{j\in A} c_j(f,\Psi)\psi_j \right\| \right\}.$$

It follows from Definition 2 that every almost greedy basis is also a quasi-greedy basis. It was proved in [2] that $\Psi$ is almost greedy if and only if $\Psi$ is quasi-greedy and democratic. Recall that $\Psi$ is democratic if for some $C \ge 1$ we have

$$\left\| \sum_{j\in A} \psi_j \right\| \le C \left\| \sum_{j\in B} \psi_j \right\| \quad \text{whenever} \quad |\ A\ | \le |\ B\ |\ .$$

**Definition 3.** A basis $\Psi = \{\psi_k\}_{k=1}^{+\infty}$ is called greedy for $X$ if there exists a constant $C$ such that for all $f \in X$ and $m \in \mathbb{N}$ we have

$$\|f - G_m(f,\Psi)\|_X \le C \inf \left\{ \left\| f - \sum_{j\in A} a_j\psi_j \right\| \quad : \quad a_j \in \mathbb{R}, |\ A\ | = m \right\}.$$

It was proved in [6] that a basis is greedy if and only if it is unconditional and democratic. So we have the following relations:

Greedy $\Longrightarrow$ Unconditional $\Longrightarrow$ Quasi-greedy,
Greedy $\Longrightarrow$ Almost greedy $\Longrightarrow$ Quasi-greedy,
Greedy = Unconditional + Democratic,
Almost greedy = Quasi-greedy + Democratic.

It was proved by V. N. Temlyakov (see [7]) that the Haar system is a greedy basis in $L^p(0,1), 1 < p < \infty$.

It is known that there is no unconditional basis in $L^1(0,1)$. But it was proved in [1] that there is an almost greedy basis in $L^1(0,1)$. That proof was not constructive, and that basis was not constructed. In [3] it was shown that the Haar system is not a quasi-greedy basis in $L^1(0,1)$. The same result for general Haar systems and for the Franklin system was proved in [4] and [5] correspondingly.

In this paper we give an example of an almost greedy basis in $L^1(0,1)$. We denote

$$M_i = 2^i - i - \sum_{p:2^p+1<i} \left(2^{2^p+1} - 1\right),\ i = 0,1,\dots. \tag{3}$$

It is easy to check that

$$M_0 = 1 \text{ and } M_{i+1} - M_i = \begin{cases} 0, & \text{if } i = 2^p + 1 \text{ for some natural } p, \\ 2^i - 1, & \text{otherwise.} \end{cases}$$

Let $\{h_n\}$ be a classical Haar system normalized in $L^1(0,1)$ and let $\psi_i = h_{2^i+2}$ (i.e. the Haar function with support $[2^{-i}, 2^{1-i})$).

Let $i \geq 0$ and $i \neq 2^p + 1$ for any $p \geq 0$. For any $0 \leq k \leq M_{i+1} - M_i = 2^i - 1$ we define

$$f_{(i,k)} = \begin{cases} h_{i+1} - \frac{1}{2^i} \sum_{j=0}^{2^i-2} \psi_{M_i+j} + \psi_{M_i+k}, & \text{if } k < 2^i - 1, \\ \\ h_{i+1} - \frac{1}{2^i} \sum_{j=0}^{2^i-2} \psi_{M_i+j}, & \text{if } k = 2^i - 1. \end{cases}$$

**Theorem 1.** *The system $\{f_{(i,k)}\}$ is an almost greedy basis in $L^1(0,1)$.*

We construct a quasi-greedy basis in an auxiliary Banach space and show that it is an isomorphic image of $\{f_{(i,k)}\}$.

## 2. Preliminaries and auxiliary results

We need results from [1]. Let $X$ be a Banach space with a basis $\{b_n\}$. By passing to an equivalent norm we may assume that $\{b_n\}$ is normalized and bimonotone. Let $S$ be a 1-symmetric and 1-unconditional sequence space with basis $\{e_i\}$, $e_i = \{\delta_{ij}\}$. Let $\{e_i^*\}$ be the sequence of biorthogonal functionals in $S^*$. Define

$$f(n) = \|e_1 + \ldots + e_n\|_S$$

and

$$g(n) = \frac{n}{f(n)} = \|e_1^* + \ldots + e_n^*\|_{S^*}$$

We assume that $\{e_i\}$ is not equivalent to the unit vector basis of $c_0$. Thus,

$$f(n) \to \infty \quad \text{as} \quad n \to \infty.$$

For $n \geq 1$, let $\sigma_n = [2^{n-1}; 2^n - 1]$,

$$v_n = \frac{1}{f(2^{n-1})} \sum_{k \in \sigma_n} e_k \quad \text{and} \quad v_n^* = \frac{1}{g(2^{n-1})} \sum_{k \in \sigma_n} e_k^*.$$

Let $P$ be the norm-one projection on $S$ defined by

$$P\xi = \sum_{n=1}^{\infty} \langle \xi, v_n^* \rangle v_n,$$

and let $Q = I - P$. Define a norm on $c_{00}$ by

$$\|\xi\|_Y = \left\| \sum_{n=1}^{\infty} \langle \xi, v_n^* \rangle b_n \right\|_X + \|Q\xi\|_S$$

and then complete it to obtain a sequence space $Y$.

The following statements were proved in [1].

**Statement 1** (Proposition 6.1)**.** *Suppose that $\{b_n\}$ is a bimonotone basis for $X$. Then $\{e_n\}$ is a basis for $Y$ such that*

$$\frac{1}{8} \sup_n \eta_n f(n) \leq \|\xi\|_Y \leq 6 \sum_{n=1}^{\infty} \frac{f(n)}{n} \eta_n$$

*for all real scalars $\xi = (\xi_n)$ in $c_{00}$, where $\{\eta_i\}$ is the nonincreasing rearrangement of $\{| \xi_i |\}$.*

**Statement 2.** *Suppose that $X$ is a Banach space with a basis that contains a complemented subspace isomorphic to $S$. Then $Y \sim X$.*

**Statement 3** (Theorem 7.1)**.** *Suppose that $\{b_n\}$ is a basis for $X$ and $S = \ell_1$. Then $\{e_n\}$ is a quasi-greedy basis for $Y$.*

Let us apply Statements 1-3 when $X = L^1(0,1)$ and $S = \ell_1, \{b_i\} = \{h_i\}$ (the Haar system).

**Lemma 1.** *The system $\{e_i\}$ is an almost greedy basis in $Y$ and*

$$Q(e_{2^i+k}) = \Big(\underbrace{0,\dots,0}_{2^i-1}, \underbrace{-2^{-i},\dots,-2^{-i}}_{k}, 1-2^{-i}, \underbrace{-2^{-i},\dots,-2^{-i}}_{2^i-k-1}, 0, 0\dots\Big) \tag{4}$$

*for all $0 \le k < 2^i$.*

*Proof.* We have $f(n) = n$ and $g(n) = 1$. Let $\xi = \sum_{k\in A} e_k$ with $\mid A \mid= m$. Then

$$\eta_n = \begin{cases} 1, & n \le m, \\ 0, & n > m. \end{cases}$$

Hence, according to Statement 1 we conclude that

$$\frac{m}{8} \le \| \sum_{k\in A} e_k \|_Y \le 6m.$$

Hence $\| \sum_{k\in A} e_k \|_Y \asymp |A|$. So $\{e_n\}$ is democratic in $Y$. Combined with Statement 3 we conclude that $\{e_n\}$ is an almost greedy basis. Equality (4) immediately follows from the definition of $Q$. Lemma 1 is proved. □

According to the definition of $Y$ there exists an isomorphic operator $\mathrm{R} : Y \mapsto L^1(0,1) \oplus Q(\ell_1)$ such that

$$\mathrm{R}(e_{2^i+k}) = (h_{i+1}, Q(e_{2^i+k})) \tag{5}$$

for every $k = 0,\dots,2^i-1;\ i = 0,1,\dots$.

## 3. Isomorphic operators

Let $\{M_i\}$ be the sequence defined by (3) and let $\Lambda_n = [2^{n-1}, 2^n - 2]$. Denote

$$\Lambda = \{n_1, n_2, n_3, \dots\} = \bigcup_{p=0}^{\infty} \Lambda_{2^p+2} \ \text{ and } \{m_1, m_2, \dots\} = \big(\bigcup_{n=1}^{\infty} \Lambda_n\big) \setminus \Lambda,$$

where sequences $\{n_i\}$ and $\{m_i\}$ are increasing. For every $\{a_i\} \in Q(\ell_1)$ we define the operator $\mathrm{S} : Q(\ell_1) \mapsto \ell_1 \oplus \ell_1$ by

$$\mathrm{S}(\{a_i\}) = \Big(\{a_{m_1}, a_{m_2}, \dots\}, \{a_{n_1}, a_{n_2}, \dots\}\Big).$$

Since $\{a_i\} \in Q(\ell_1)$ if and only if $\{a_i\} \in \ell_1$ and $a_{2^n-1} = -\sum_{i\in\Lambda_n} a_i$ for all natural $n$, we conclude that S is an isomorphic operator. The next lemma follows from the definitions of $Q$ and S.

**Lemma 2.** *1) Let $i \neq 2^p+1$ for any $p \geq 0$. Then*

$$\mathrm{S}\big(Q(e_{2^i+k})\big) = \begin{cases} \Big(\big(\underbrace{0,\dots,0}_{M_i-1}, \underbrace{-2^{-i},\dots,-2^{-i}}_{k}, 1-2^{-i}, \underbrace{-2^{-i},\dots,-2^{-i}}_{2^i-k-2}, 0\dots\big), \{0\}\Big), & \text{if } k<2^i-1, \\ \Big(\big(\underbrace{0,\dots,0}_{M_i-1}, \underbrace{-2^{-i},\dots,-2^{-i}}_{2^i-1}, 0\dots\big), \{0\}\Big), & \text{if } k=2^i-1. \end{cases}$$

*2) Let $i = 2^p+1$ for some $p \geq 0$ and $0 \leq k < 2^i$. Then*

$$\mathrm{S}\big(Q(e_{2^i+k})\big) = \begin{cases} \Big(\{0\}, \big(\underbrace{0,\dots,0}_{2^i-i-M_i}, \underbrace{-2^{-i},\dots,-2^{-i}}_{k}, 1-2^{-i}, \underbrace{-2^{-i},\dots,-2^{-i}}_{2^i-k-2}, 0\dots\big)\Big), & \text{if } k<2^i-1, \\ \Big(\{0\}, \big(\underbrace{0,\dots,0}_{2^i-i-M_i}, \underbrace{-2^{-i},\dots,-2^{-i}}_{2^i-1}, 0, 0\dots\big)\Big), & \text{if } k=2^i-1. \end{cases}$$

Functions $\psi_i$ have disjoint supports ($supp(\psi_i) = (2^{-i}, 2^{1-i})$). Therefore we have

$$\|\sum_{i=1}^{\infty} a_i\psi_i\|_{L^1(0,1)} = \|\{a_i\}\|_{\ell_1}. \tag{6}$$

Denote

$$Z = \{f \in L^1(0,1) \quad : \quad \int_0^1 f(t)\psi_n(t)dt = 0 \text{ for all } n \in \mathbb{N}\}.$$

Let us define the operator $\mathrm{T} : L^1(0,1) \mapsto (Z, \ell_1)$ by the formula

$$\mathrm{T}(f) = \mathrm{T}\Big(\sum_{n=1}^{\infty} a_n h_n\Big) = \Big(\sum_{n=1, n\neq 2^i+2}^{\infty} a_n h_n, \{a_{2^i+2}\}\Big).$$

Note that

$$\mathrm{T}(h_n) = \begin{cases} (0, \{\delta_{p+1,j}\}), & \text{if } n = 2^p+2, \\ (h_n, \{0\}), & \text{otherwise.} \end{cases} \tag{7}$$

The next lemma is obvious.

**Lemma 3.** *The operator* T *represents an isomorphism between $L^1(0,1)$ and $Z \oplus \ell_1$.*

## 4. An almost greedy basis in $L^1(0,1)$

In the proof of Statement 2 the following isomorphic chain was used:

$$\begin{aligned} Y \sim X \oplus Q(S) &\sim Z \oplus S \oplus Q(S) \\ &\sim Z \oplus S \oplus P(S) \oplus Q(S) \\ &\sim Z \oplus S \oplus S \sim Z \oplus S \sim X. \end{aligned} \tag{8}$$

In the proof of Theorem 1 we will use the following:

$$\begin{aligned} Y &\sim L^1(0,1) \oplus Q(\ell_1) \\ &\sim Z \oplus \ell_1 \oplus Q(\ell_1) \qquad \text{(by operator T)} \\ &\sim Z \oplus \ell_1 \oplus \ell_1 \oplus \ell_1 \qquad \text{(by operator S)} \\ &\sim L^1(0,1) \oplus \ell_1 \oplus \ell_1. \qquad \text{(by operator T).} \end{aligned}$$

*Proof of Theorem* 1. For $y \in Y$ denote

$\mathrm{R}(y) = (y_1, q_1)$, where $y_1 \in L^1(0,1),\ q_1 \in Q(\ell_1)$;

$\mathrm{T}(y_1) = (z_1, a_1)$, where $z_1 \in Z,\ a_1 \in \ell_1$;

$\mathrm{S}(q_1) = (a_2, a_3)$, where $a_2, a_3 \in \ell_1$;

$\mathrm{T}^{-1}(z_1, a_2) = y_2$, where $y_2 \in L^1(0,1)$;

and set

$$\mathrm{D}(y) = \{y_2, a_1, a_3\}.$$

It is clear that D is an isomorphic operator from $Y$ to $L^1(0,1) \oplus \ell_1 \oplus \ell_1$. Now, let us calculate $\mathrm{D}(e_n)$. Let $n = 2^i + k$ with $0 \le k < 2^i$. According to (5) we have

$$\mathrm{R}(e_{2^i+k}) = \big(h_{i+1}, Q(e_{2^i+k})\big),$$

which means that $y_1 = h_{i+1}$ and $q_1 = Q(e_{2^i+k})$. We consider four cases.

*Case* 1. $i = 2^p + 1$ for some $p \ge 0$ and $k < 2^i - 1$. According to (7)

$$\begin{aligned} \big(z_1, a_1\big) &= \mathrm{T}(h_{i+1}) = (0, \{\delta_{p+1,j}\}), \\ (a_2, a_3) &= \mathrm{S}(Q(e_{2^i+k})) \\ &= \Big(\{0\}, \{\underbrace{0, \dots, 0}_{2^i-i-M_i}, \underbrace{-2^{-i}, \dots, -2^{-i}}_{k}, 1 - 2^{-i}, \underbrace{-2^{-i}, \dots, -2^{-i}}_{2^i-k-2}, 0 \dots\}\Big), \\ y_2 &= \mathrm{T}^{-1}(z_1, a_2) = \mathrm{T}^{-1}\big(0, \{0\}\big) = 0. \end{aligned}$$

Hence

$$\mathrm{D}(e_{2^i+k}) = \Big(0, \{\delta_{p+1,j}\}, \{\underbrace{0, \dots, 0}_{2^i-i-M_i}, \underbrace{-2^{-i}, \dots, -2^{-i}}_{k}, 1 - 2^{-i}, \underbrace{-2^{-i}, \dots, -2^{-i}}_{2^i-k-2}, 0 \dots\}\Big).$$

*Case* 2. $i = 2^p + 1,\ k = 2^i - 1$.

$$\begin{aligned} \big(z_1, a_1\big) &= \mathrm{T}(h_{i+1}) = (0, \{\delta_{p+1,j}\}), \\ (a_2, a_3) &= \mathrm{S}(Q(e_{2^i+k})) = \Big(\{0\}, \{\underbrace{0, \dots, 0}_{2^i-i-M_i}, \underbrace{-2^{-i}, \dots, -2^{-i}}_{2^i-1}, 0 \dots\}\Big), \\ y_2 &= \mathrm{T}^{-1}(z_1, a_2) = \mathrm{T}^{-1}\big(0, \{0\}\big) = 0. \end{aligned}$$

Hence $\mathrm{D}(e_{2^i+k}) = \Big(0, \{\delta_{p+1,j}\}, \{\underbrace{0, \dots, 0}_{2^i-i-M_i}, \underbrace{-2^{-i}, \dots, -2^{-i}}_{2^i-1}, 0 \dots\}\Big).$

*Case* 3. $i \neq 2^p+1,\ k \neq 2^i-1$.

$$\begin{aligned}
\big(z_1, a_1\big) &= \mathrm{T}(h_{i+1}) = \ (h_{i+1}, \{0\}),\\
(a_2, a_3) &= \mathrm{S}(Q(e_{2^i+k}))\\
&= \Big(\big(\underbrace{0,\dots,0}_{M_i-1}, \underbrace{-2^{-i},\dots,-2^{-i}}_{k}, 1-2^{-i}, \underbrace{-2^{-i},\dots,-2^{-i}}_{2^i-k-2}, 0\dots\big), \{0\}\Big),\\
y_2 &= \mathrm{T}^{-1}(z_1, a_2)\\
&= \mathrm{T}^{-1}\Big(h_{i+1}, \big(\underbrace{0,\dots,0}_{M_i-1}, \underbrace{-2^{-i},\dots,-2^{-i}}_{k}, 1-2^{-i}, \underbrace{-2^{-i},\dots,-2^{-i}}_{2^i-k-2}, 0\dots\big)\Big)\\
&= h_{i+1} - \frac{1}{2^i}\sum_{j=M_i}^{M_i+2^i-2}\psi_j + \psi_{M_i+k} = h_{i+1} - \frac{1}{2^i}\sum_{j=M_i}^{M_{i+1}-1}\psi_j + \psi_{M_i+k}.
\end{aligned}$$

Therefore $\mathrm{D}(e_{2^i+k}) = \Big(f_{(i,k)}, \{0\}, \{0\}\Big)$.

*Case* 4. $i \neq 2^p+1,\ k = 2^i-1$.

$$\begin{aligned}
\big(z_1, a_1\big) &= \mathrm{T}(h_{i+1}) = \ (h_{i+1}, \{0\}),\\
(a_2, a_3) &= \mathrm{S}(Q(e_{2^i+k})) = \Big(\big(\underbrace{0,\dots,0}_{M_i-1}, \underbrace{-2^{-i},\dots,-2^{-i}}_{2^i-1}, 0\dots\big), \{0\}\Big),\\
y_2 &= \mathrm{T}^{-1}(z_1, a_2) = \mathrm{T}^{-1}\Big(h_{i+1}, \big(\underbrace{0,\dots,0}_{M_i-1}, \underbrace{-2^{-i},\dots,-2^{-i}}_{2^i-1}, 0\dots\big)\Big)\\
&= h_{i+1} - \frac{1}{2^i}\sum_{j=M_i}^{M_{i+1}-1}\psi_j.
\end{aligned}$$

Therefore $\mathrm{D}(e_{2^i+k}) = \Big(f_{(i,k)}, \{0\}, \{0\}\Big)$.

Finally we have

$$\mathrm{D}(e_{2^i+k}) = \begin{cases} \big(0, \{\dots\}, \{\dots\}\big) & ,\ \text{if } i = 2^p+1,\\ \big(f_{(i,k)}, \{0\}, \{0\}\big) & ,\ \text{if } i \neq 2^p+1. \end{cases}$$

Since $\{\mathrm{D}(e_{2^i+k})\}$ is an almost-greedy basis in $L^1(0,1) \oplus \ell_1 \oplus \ell_1$, then the system $\{f_{(i,k)}\}$ is an almost greedy basis in $L^1(0,1)$. Theorem 1 is proved. $\square$

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