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ERRATUM TO "A NOTE ON RESOLUTION OF RATIONAL AND HYPERSURFACE SINGULARITIES"

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ABSTRACT. Our earlier paper contains a lemma on degeneration of a spectral sequence whose proof is incorrect. In this note we explain the mistake and provide a correction to it.

In my paper "A note on resolution of rational and hypersurface singularities" [3], in the proof of Lemma 2.4, I claim that on a compact Kähler variety a restriction of a harmonic differential form onto a subvariety is harmonic. However, this is not correct. Here is a sketch of a counterexample due to D. Arapura.

Example. Let X be a non-hyperelliptic curve of genus $g > 2$. X embeds into its Jacobian J. Let us give J a flat metric and X the induced metric. The harmonic forms on J are precisely the forms with constant coefficients, so they form an algebra. In particular, $\gamma_{ij} = \alpha_i \wedge \bar{\alpha}_j$ give a basis for the harmonic $(1,1)$-forms on J, where $\alpha_i = dz_i$ and z_i are the coordinates on $\mathbb{C}^g$. The restrictions of α_i to X give a basis for the holomorphic 1-forms on X, and these separate points. Therefore we can find at least 2 linearly independent restrictions $\gamma_{ij}|_X$. However, the space of harmonic $(1,1)$-forms on X is 1-dimensional. Thus one of these restrictions is not harmonic.

On the other hand, our Lemma 2.4 is itself correct. In their recent preprint D. Arapura, P. Bakhtari, and J. Włodarczyk prove it as a consequence of their more general result ([1], 2.1, 2.2, 2.3). Our original argument can also be corrected, and here we present such a correction. The last paragraph of the proof of Lemma 2.4 from [3], starting with "Our aim is to show that $d_r = 0$, $r \geq 2$," should now be read as follows:

Our aim is to show that $d_r = 0$, $r \geq 2$. The differential d_2 is trivial if the representative $a \in K^{p,q}$ can be chosen in such a way that $\delta(a)$ is exactly 0 but not only 0 modulo $\bar{\partial}K^{p+1,q-1}$. But this is true because there are harmonic differential forms in the class $\bar{a} \in H^q_{\bar{\partial}}(K^{p,*})$ and we can take a to be a harmonic form of pure type $(0,q)$. Notice that since we work on a compact Kähler variety, the form a is not only $\bar{\partial}$-closed but also $\partial + \bar{\partial}$-closed, where $\partial + \bar{\partial}$ is the complex de Rham differential. The form δa is defined by means of restrictions onto subvarieties and linear operations. We cannot claim that it is also harmonic, but it remains $\partial + \bar{\partial}$-closed and of pure type $(0,q)$. But it is $0 \mod (\bar{\partial}K^{p+1,q-1})$, i.e., $\bar{\partial}$-exact. It follows from the $\partial\bar{\partial}$-lemma [2, Ch. 1, sec. 2] that δa is also ∂-exact and thus is exactly 0.

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Further, this $\delta a = 0$ can be lifted to $a' = 0$ in $K^{p+1,q-1}$; thus $\delta(a') = 0$ and so forth. This shows that $d_r = 0$ also for all $r \geq 3$.

References

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