PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 140, Number 2, February 2012, Pages 741–743
S 0002-9939(2011)10909-2
Article electronically published on June 2, 2011

CORRIGENDUM TO "EXISTENCE OF SOLUTIONS FOR SEMILINEAR ELLIPTIC PROBLEMS WITHOUT (PS) CONDITION"

JIANFU YANG

(Communicated by Ken Ono)

Abstract. In this corrigendum we give a correct proof for equation (3.17), as well as the assertion on line 15, page 1364, of "Existence of solutions for semilinear elliptic problems without (PS) condition", *Proc. Amer. Math. Soc.* 132(5) (2004), 1355-1366.

In the paper [1], the use of Hospital's rule in (3.17), page 1363, line 15, and page 1364, line 15, is incorrect. In order to avoid the use of Hospital's rule, let $\sigma = -\lim_{t\to\infty} \frac{t\bar{h}'(t)}{\bar{h}(t)}$, we need to assume further that

$$\lim_{t\to\infty} \frac{th'_t(x,t) - \sigma h(x,t)}{\bar{h}(t)} = 0 \tag{0.1}$$

uniformly with respect to x and that for any $a > 0$ there exist $T_a > 0$ and $C_a > 0$ such that for every $t \geq T_a$,

$$\bar{h}(at) \leq C_a \bar{h}(t). \tag{0.2}$$

An example which satisfies (0.1), (0.2) and $(f_1) - (f_4)$ in [1] is given for $t \geq 0$ by

$$f(t) = (\ln(1+t))^\alpha(\gamma + \sin((\ln\ln t)^\beta))t^p + g(x,t), \tag{0.3}$$

with the lower term $g(x,t)$ as the example in [2], where $\alpha < 1$ and $\beta > 0$. Observe that in this example, we have $h(x,t) = (\ln(1+t))^\alpha(\gamma + \sin((\ln\ln t)^\beta))$, $\bar{h}(t) = (\gamma+1)(\ln(1+t))^\alpha$ and $\sigma = 0$. Moreover, such a function f does not satisfy the Ambrosetti-Rabinowitz condition.

Now, we use (0.1) and (0.2) to show that

$$\lim_{n\to\infty} \lambda_n M_n \frac{f'_t(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n)}{\alpha(M_n)} = (p-\sigma)Q(x)u^{p-1}, \tag{0.4}$$

where Hospital's rule was used. We have as on page 1363 of [1] that

$$|\frac{1}{2}\phi^{-\frac{3}{2}}(M_n)\phi'(M_n)\alpha'^{-1}(M_n)\nabla_x f(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n)x| \to 0$$

and $\frac{\alpha(M_n)}{M_n\alpha'(M_n)} \to \frac{1}{p-\sigma}$ as $n\to\infty$. Hence, to prove (0.4), it suffices to prove that

$$\lim_{n\to\infty} T_n(x) = 0 \tag{0.5}$$

Received by the editors November 19, 2010.
2010 *Mathematics Subject Classification.* Primary 35J20, 35J25, 35J60.

for all x such that $u(x) > 0$, where

$$T_n(x) = \frac{(p-\sigma)f(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n) - M_n\tilde{u}_n f_t'(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n)}{\alpha(M_n)}.$$

Here we deal only with the case

$$\lim_{t\to\infty} \bar{h}(t)t^{p-q} = \infty. \tag{0.6}$$

By (0.1), for $\varepsilon > 0$ there exists $t_\varepsilon > 0$ such that for $t \geq t_\varepsilon$ and all $x \in \Omega$,

$$\Big|\frac{th_t'(x,t) - \sigma h(x,t)}{\bar{h}(t)}\Big| < \varepsilon, \tag{0.7}$$

which implies

$$\begin{aligned}|T_n(x)| \leq & \frac{|\sigma h(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n) - M_n\tilde{u}_n h_t'(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n)|}{\bar{h}(M_n)} \\ & + \frac{M_n^q}{M_n^p\bar{h}(M_n)}.\end{aligned}$$

Since $M_n\tilde{u}_n(x) \to \infty$ as $n \to \infty$, by (0.6) and (0.7) we may find $N_\varepsilon > 0$ such that for $n \geq N_\varepsilon$, $|T_n(x)| < \varepsilon$. Equation (0.5) follows.

Next, we prove the formula on page 1364, line 15, that is,

$$(M_n\alpha(M_n))^{-1}\lambda_n F(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n) \to \frac{1}{p+1-\sigma}Q(x)|u(x)|^{p+1}. \tag{0.8}$$

By (f_3), there exists $\varepsilon > 0$ such that $\sigma+\varepsilon < p$ and $-t\bar{h}'(t) \leq (\sigma+\varepsilon)\bar{h}(t)$. Integrating by part, we obtain

$$\int_0^t \bar{h}(s)s^p\,ds \leq C\bar{h}(t)t^{p+1}$$

for t large enough. This inequality, together with (0.1), yields

$$\lim_{t\to+\infty} \frac{tf(x,t) - (p-\sigma+1)F(x,t)}{t^{p+1}\bar{h}(t)} = 0 \tag{0.9}$$

uniformly with respect to x. Therefore, we may deduce as above that

$$\begin{aligned}\lim_{n\to\infty} & \frac{(p-\sigma+1)F(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n) - M_n\tilde{u}_n f(x_n + \phi^{-\frac{1}{2}}(M_n)x, M_n\tilde{u}_n)}{M_n^{p+1}\bar{h}(M_n\tilde{u}_n)} \\ & = 0,\end{aligned}$$

which implies (0.8).

Acknowledgment

The author would like to thank Professor Abdelaziz Harrabi for bringing the error to his attention and for suggesting the correction to the problem.

References

[1] Jianfu Yang, Existence of solutions for semilinear elliptic problems without (PS) condition, *Proc. Amer. Math. Soc.* 132(5) (2004), 1355-1366. MR2053340 (2005b:35048)

[2] D.G. de Figueiredo and Jianfu Yang, On a semilinear elliptic problem without (PS) condition, *J. Differential Equations* 187 (2003), 412-428. MR1949448 (2004c:35074)

Department of Mathematics, Jiangxi Normal University, Nanchang, Jiangxi 330022, People's Republic of China

E-mail address: jfyang_2000@yahoo.com