# SIGNED SUMS OF TERMS OF A SEQUENCE 

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#### Abstract

We give a sufficient and necessary condition on the sequence $\left\{a_{n}\right\}$ of integers that for any integer $l \geq 1$, every integer can be represented in the form $\varepsilon_{l} a_{l}+\varepsilon_{l+1} a_{l+1}+\cdots+\varepsilon_{k} a_{k}$, where $\varepsilon_{i} \in\{-1,1\}(i=l, l+1, \ldots, k)$. This generalizes the known result on integral-valued polynomial values. Moreover, we show that such sequences exist with any growth rate. This answers two problems posed by Bleicher. We also pose several problems for further research.


## 1. Introduction

In [1, Bleicher proved the following interesting result: for any given integer $k \geq 2$, every integer can be represented in the form

$$
n=\varepsilon_{1} 1^{k}+\varepsilon_{2} 2^{k}+\cdots+\varepsilon_{t} t^{k}, \quad \varepsilon_{i} \in\{-1,1\}, \quad i=1,2, \ldots, t
$$

Yu 11 generalized the result to integral-valued polynomials with fixed divisor equal to 1. Boulanger and Chabert [2] considered the problem in the ring of algebraic integers $O_{K}$ of a cyclotomic field $K$. For related research, one may refer to [3], 4], [5], 7], 8] and [9]. Starting from the result on $k$ th powers, Bleicher [1] posed the following two problems.
Problem 1. Does there exist a constant $c>1$ and an increasing sequence of integers $\left\{a_{i}\right\}$ with $a_{i}>c^{i}$ for every positive integer $i$ such that for every positive integer $n$, there is a positive integer $m$ and a choice of $\varepsilon_{i}= \pm 1$ for which $n=$ $\sum_{i=1}^{m} \varepsilon_{i} a_{i}$ ?
Problem 2. On the assumption that Problem 1 is answered affirmatively, is there an upper bound for possible choices of $c$ ?

For sequences, Erdős and Surányi [6] proved the following result: Let $0<a_{1}<$ $a_{2}<\cdots$ be a sequence of integers. If (a) the sequence contains infinitely many odd numbers; (b) for some $m$, all positive integers $>m$ can be represented as the sum of different elements of the sequence; (c) $a_{n+1}<2 a_{n}-m$ for $n \geq n_{0}$, then every integer can be represented in the form $\varepsilon_{1} a_{1}+\varepsilon_{2} a_{2}+\cdots+\varepsilon_{t} a_{t}$, where $\varepsilon_{i} \in\{-1,1\}(i=1,2, \ldots, t)$.

In this note we prove the following result. This generalizes the result on integralvalued polynomials (Bleicher [1], Yu [11). Moreover, we answer Problem 1 affirmatively and Problem 2 negatively.

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Theorem 1. Let $a_{1}, a_{2}, \ldots$ be a sequence of integers. Then for any integers $l \geq 1$ and $n$, there exists an integer $t \geq l$ and a choice of $\varepsilon_{i}= \pm 1$ such that

$$
n=\varepsilon_{l} a_{l}+\varepsilon_{l+1} a_{l+1}+\cdots+\varepsilon_{t} a_{t}
$$

if and only if the following two conditions hold:
(i) there exists a nonzero integer $M$ such that for any $l \geq 1$ there exists an integer $r=r_{l} \geq l$ and $\varepsilon_{i} \in\{-1,1\}(l \leq i \leq r)$ with $M=\varepsilon_{l} a_{l}+\cdots+\varepsilon_{r} a_{r}$;
(ii) for any $l \geq 1$ we have $\operatorname{gcd}\left(a_{l}, a_{l+1}, \ldots\right)=1$.

Remark. For any $l \geq 1$, letting $r=r_{l}$ and $s=r_{r+1}$, we have

$$
0=M-M=\varepsilon_{l} a_{l}+\cdots+\varepsilon_{r} a_{r}-\varepsilon_{r+1} a_{r+1}-\cdots-\varepsilon_{s} a_{s} .
$$

So, for a fixed $l \geq 1$ and a fixed integer $n$, the $t$ in Theorem 1 can take infinitely many values.

Remark. Let $a_{1}=2, a_{2}=3$ and $a_{i+2}=a_{i+1}+a_{i}+1(i=1,2, \ldots)$. Since $1=$ $-a_{i}-a_{i+1}+a_{i+2}$, (i) is true for $M=1$ and (ii) is true by $\left(a_{i}, a_{i+1}, a_{i+2}\right)=1$ for all $i \geq 1$. This verifies Theorem 1. One may easily prove that $a_{i}>(\sqrt{2})^{i}$ for all $i \geq 1$. Thus Problem 1 is answered affirmatively.

Remark. For an integral-valued polynomial $f(x)$ of degree $h$ with fixed divisor equal to 1 , let $a_{i}=f(i)(i=1,2, \ldots)$. For any integers $l$ and $n$, by the Lagrange interpolation formula we know that $f(n)$ is a combination of $f(l), f(l+1), \ldots$, $f(l+h)$ with integral coefficients. Since $f(x)$ has no fixed factors, we have

$$
\operatorname{gcd}(f(l), f(l+1), \ldots, f(l+h))=1
$$

On the other hand, let $f_{1}(x)=f(x+1)-f(x), f_{i+1}(x)=f_{i}\left(x+2^{i}\right)-f_{i}(x)(i=$ $1,2, \ldots)$. Then $f_{i}(x)$ is a polynomial of degree $h-i(1 \leq i \leq h)$. So $f_{h}(x)$ is a nonzero constant. We also have

$$
f_{h}(x)=\sum_{i=0}^{2^{h}-1} \varepsilon_{i} f(x+i), \quad \varepsilon_{i} \in\{-1,1\} .
$$

Since $f(x)$ is integral-valued, $f_{h}(x)=f_{h}(0)$ is an integer. For related information, one may refer to [10. By Theorem 1 for any $l \geq 1$, there exists $t \geq l$ such that every integer can be represented in the form $n=\varepsilon_{l} f(l)+\varepsilon_{l+1} f(l+1)+\cdots+\varepsilon_{t} f(t)$, where $\varepsilon_{i} \in\{-1,1\}(i=l, l+1, \ldots, t)$. That is the main result in [11.

For Problems 1 and 2 we have the following general result. This means that sequences exist with any growth rate.

Theorem 2. For any sequence $1<c_{1}<c_{2}<\cdots$, there exists a sequence $1<$ $a_{1}<a_{2}<\cdots$ of integers with $a_{i}>c_{i}$ for every positive integer $i$ such that for any integers $l \geq 1$ and $n$, there are infinitely many positive integers $m \geq l$ for which there is a choice of $\varepsilon_{i}= \pm 1$ with $n=\sum_{i=l}^{m} \varepsilon_{i} a_{i}$.

Remark. For any given $C>1$, let $c_{i}=C^{i}$. Problems 1 and 2 immediately follow from Theorem 2

In the proof of Theorem 2, some consecutive terms are "near". If we require that consecutive terms are not "near", i.e. $a_{k+1} \geq \alpha a_{k}$ for all $k \geq 1$, what is the largest possible value of $\alpha$ ? We have the following precise result.

Theorem 3. (i) If $\left\{a_{i}\right\}$ is a sequence of positive integers such that, for some integer $n_{0}$ and infinitely many positive integers $l$, there is an integer $m>l$ and a choice of $\varepsilon_{i}= \pm 1$ with

$$
\sum_{i=l}^{m} \varepsilon_{i} a_{i}=n_{0}
$$

then there are infinitely many positive integers $k$ such that $a_{k+1} \leq 2 a_{k}-1$.
(ii) Let $a_{1}$ be any positive integer, and define the sequence $\left\{a_{i}\right\}$ by $a_{k+1}=2 a_{k}-1$ for all $k \geq 1$. Then for any integers $l \geq 1$ and $n$, there are infinitely many positive integers $m>l$ for which there is a choice of $\varepsilon_{i}= \pm 1$ with

$$
n=\sum_{i=l}^{m} \varepsilon_{i} a_{i} .
$$

Now we consider only representations of the form

$$
n=\sum_{i=1}^{m} \varepsilon_{i} a_{i} .
$$

For this purpose we introduce the following definition.
Definition 1. Let $M$ be a positive integer. The subsequence $\left\{a_{l}, a_{l+1}, \ldots, a_{k}\right\}(k \geq$ $l$ ) is said to be $M$-coprime if $\operatorname{gcd}\left(M, a_{l}, a_{l+1}, \ldots, a_{k}\right)=1$.

The condition (ii) in Theorem implies that there are infinitely many disjoint $M$-coprime subsequences.

Theorem 4. Let $a_{1}, a_{2}, \ldots$ be a sequence of integers satisfying
(i) there exists a positive integer $M$ such that for any $l \geq 1$ there exists an integer $r=r_{l} \geq l$ and $\varepsilon_{i} \in\{-1,1\}(l \leq i \leq r)$ with

$$
M=\varepsilon_{l} a_{l}+\cdots+\varepsilon_{r} a_{r}
$$

(ii) there are at least $M$ disjoint $M$-coprime subsequences of $\left\{a_{1}, a_{2}, \ldots\right\}$.

Then every integer can be represented in the form

$$
n=\varepsilon_{1} a_{1}+\varepsilon_{2} a_{2}+\cdots+\varepsilon_{t} a_{t}, \quad \varepsilon_{i} \in\{-1,1\}, \quad i=1,2, \ldots, t .
$$

## 2. Proof of theorems

Proof of Theorem 1. First we assume that for any integers $l \geq 1$ and $n$, there exists an integer $t \geq l$ and a choice of $\varepsilon_{i}= \pm 1$ such that

$$
n=\varepsilon_{l} a_{l}+\varepsilon_{l+1} a_{l+1}+\cdots+\varepsilon_{t} a_{t}
$$

We take $n=1$. Then (i) holds with $M=1$. (ii) follows from

$$
\operatorname{gcd}\left(a_{l}, a_{l+1}, \ldots, a_{t}\right)=1
$$

Now we assume that both (i) and (ii) hold.
For any $l \geq 1$, let $r=r_{l}$ and $s=r_{r+1}$. We have

$$
2 M=\varepsilon_{l} a_{l}+\cdots+\varepsilon_{r} a_{r}+\varepsilon_{r+1} a_{r+1}+\cdots+\varepsilon_{s} a_{s}
$$

So we may assume that $M$ is a positive even number.

Fix an integer $l \geq 1$. Let

$$
A_{k}=\left\{\varepsilon_{l} a_{l}+\varepsilon_{l+1} a_{l+1}+\cdots+\varepsilon_{k} a_{k}: \varepsilon_{i} \in\{-1,1\}(l \leq i \leq k)\right\}, \quad k=l, l+1, \ldots
$$

and

$$
\bar{A}_{k}=\left\{b: 0 \leq b \leq M-1, b \equiv a(\bmod M), a \in A_{k}\right\}, \quad k=l, l+1, \ldots
$$

Let $m_{k}=\left|\bar{A}_{k}\right|$. Since the numbers in $A_{k}$ have the same parity, we know that the numbers in $\bar{A}_{k}$ have the same parity. Thus $1 \leq m_{k} \leq M / 2$ for all $k \geq l$. It is clear that $m_{l} \leq m_{l+1} \leq \cdots$. So there exist two positive integers $m$ and $i_{0}>l$ such that $m_{i}=m$ for all $i \geq i_{0}$. Let

$$
\bar{A}_{i}=\left\{b_{i 1}, b_{i 2}, \ldots, b_{i m}\right\}, \quad i \geq i_{0}
$$

By $\left|\bar{A}_{i+1}\right|=m$, we have

$$
\begin{aligned}
& \left(b_{i 1}-a_{i+1}\right)+\left(b_{i 2}-a_{i+1}\right)+\cdots+\left(b_{i m}-a_{i+1}\right) \\
& \quad \equiv\left(b_{i 1}+a_{i+1}\right)+\left(b_{i 2}+a_{i+1}\right)+\cdots+\left(b_{i m}+a_{i+1}\right)(\bmod M)
\end{aligned}
$$

Thus $2 m a_{i+1} \equiv 0(\bmod M)$ for all $i \geq i_{0}$. By (ii) we have $2 m \equiv 0(\bmod M)$. Since $1 \leq m \leq M / 2$, we have $m=M / 2$. Again, by (ii), there exists an integer $j \geq i_{0}$ such that $a_{j+1}$ is odd. Hence

$$
\bar{A}_{j} \cup \bar{A}_{j+1}=\{0,1, \ldots, M-1\} .
$$

Thus, for any integer $n$, there exists $k=j$ or $j+1, \varepsilon_{i} \in\{-1,1\}(i=l, l+1, \ldots, k)$ and an integer $u$ such that

$$
n=\varepsilon_{l} a_{l}+\varepsilon_{l+1} a_{l+1}+\cdots+\varepsilon_{k} a_{k}+u M
$$

By (i) we obtain a proof of Theorem 1 .
Proof of Theorem 2. Let

$$
a_{2 i-1}=\left[c_{2 i}\right]+2 i+1, \quad a_{2 i}=\left[c_{2 i}\right]+2 i+2, \quad i=1,2, \ldots
$$

Then $a_{2 i+1}>a_{2 i}>a_{2 i-1}>\left[c_{2 i}\right]+1>c_{2 i}>c_{2 i-1}$ for all $i \geq 1$.
Let $l, n$ be two integers with $l \geq 1$. For any integer $j>l+|n|$ we have $n+a_{l}+$ $\cdots+a_{2 j}>n+2 j>0$. Let $t=n+\sum_{i=l}^{2 j} a_{i}$. Then

$$
n=\sum_{i=l}^{2 j}\left(-a_{i}\right)+\sum_{i=j+1}^{j+t}\left(-a_{2 i-1}+a_{2 i}\right)
$$

Hence, for every integer $n$ there are infinitely many positive integers $m>l$ for which there is a choice of $\varepsilon_{i}= \pm 1$ with $n=\sum_{i=l}^{m} \varepsilon_{i} a_{i}$. This completes the proof of Theorem 2

Proof of Theorem 3. (i) Suppose that $\left\{a_{n}\right\}$ is a sequence of positive integers satisfying the condition, but $a_{k+1} \geq 2 a_{k}$ for all $k \geq k_{0}$. Then $a_{k} \rightarrow+\infty$ as $k \rightarrow+\infty$. Take an integer $l \geq k_{0}$ with $a_{l}>\left|n_{0}\right|$ for which there is an integer $m>l$ and a choice of $\varepsilon_{i}= \pm 1$ with

$$
\sum_{i=l}^{m} \varepsilon_{i} a_{i}=n_{0}
$$

Thus

$$
\begin{aligned}
a_{m} & =\varepsilon_{m} n_{0}+\sum_{i=l}^{m-1}-\varepsilon_{m} \varepsilon_{i} a_{i} \leq\left|n_{0}\right|+\sum_{i=l}^{m-1} a_{i} \\
& <a_{l}+\sum_{i=l}^{m-1} a_{i}=2 a_{l}+\sum_{i=l+1}^{m-1} a_{i} \\
& \leq a_{l+1}+\sum_{i=l+1}^{m-1} a_{i}=2 a_{l+1}+\sum_{i=l+2}^{m-1} a_{i} \\
& \leq \cdots \\
& \leq 2 a_{m-2}+a_{m-1} \leq 2 a_{m-1},
\end{aligned}
$$

a contradiction with $a_{k+1} \geq 2 a_{k}$ for all $k \geq k_{0}$. This completes the proof of Theorem 3 (i).
(ii) Let $l \geq 1$ and $n$ be two integers. For any integer $t>l+|n|$, let $s=$ $n+a_{l}+a_{l+1}+\cdots+a_{t}$. Then by $a_{i} \geq 1$ for all $i \geq 1$ we have $s \geq n+t-l+1>0$. By $a_{k+1}=2 a_{k}-1$ we have

$$
\begin{aligned}
a_{t+s} & =a_{t+s-1}+a_{t+s-1}-1=a_{t+s-1}+a_{t+s-2}+a_{t+s-2}-2 \\
& =\cdots \\
& =a_{t+s-1}+a_{t+s-2}+\cdots+a_{t}+a_{t}-s \\
& =a_{t+s-1}+a_{t+s-2}+\cdots+a_{t}-a_{t-1}-a_{t-2}-\cdots-a_{l}-n .
\end{aligned}
$$

Thus

$$
n=-\sum_{i=l}^{t-1} a_{i}+\sum_{i=t}^{t+s-1} a_{i}-a_{t+s}
$$

This completes the proof of Theorem 3 (ii).
Proof of Theorem 4. If $M=1$, then Theorem 4 is clear. Now we assume that $M>1$. Let the notation be as in the proof of Theorem (the fixed $l$ is equal to 1 and we do not assume that $M$ is even). Let $i_{0}=0$ and let $\left\{a_{i_{j-1}+1}, \ldots, a_{i_{j}}\right\}(j=$ $1,2, \ldots, M)$ be $M$ disjoint $M$-coprime subsequences of the sequence $\left\{a_{i}\right\}$. It is clear that $m_{i_{1}} \geq 1$. If $m_{i_{2}}=m_{i_{1}}$, then, similarly to the proof of Theorem [1, we have

$$
2 m_{i_{1}} a_{j} \equiv 0(\bmod M), \quad j=i_{1}+1, \ldots, i_{2} .
$$

Since $\left\{a_{i_{1}+1}, \ldots, a_{i_{2}}\right\}$ is an $M$-coprime subsequence, we have $2 m_{i_{1}} \equiv 0(\bmod M)$. If $M$ is odd, then $m_{i_{1}}=M$. If $M$ is even, then, similarly to the proof of Theorem [1, we have $m_{i_{1}}=M / 2$. Hence, if $m_{i_{1}} \neq M, M / 2$, then $m_{i_{2}}>m_{i_{1}}$. Continuing these arguments, we have that if $M$ is odd, then $m_{i_{M}}=M$. Thus $\bar{A}_{i_{M}}=\{0,1, \ldots, M-1\}$. If $M$ is even, then $m_{i_{M / 2}}=M / 2$ and $m_{i} \leq M / 2$ for all $i$. Since $\left\{a_{i_{M / 2}+1}, \ldots, a_{i_{(M / 2)+1}}\right\}$ is an $M$-coprime subsequence, there exists $j_{0} \in$ $\left\{i_{M / 2}+1, \ldots, i_{(M / 2)+1}\right\}$ such that $a_{j_{0}}$ is odd. Since $m_{i_{M / 2}} \leq m_{j_{0}-1} \leq m_{j_{0}} \leq M / 2$, we have $m_{j_{0}-1}=m_{j_{0}}=M / 2$. Since the parities of the numbers in $\bar{A}_{j_{0}-1}$ and in $\bar{A}_{j_{0}}$ are distinct, we have

$$
\bar{A}_{j_{0}-1} \cup \bar{A}_{j_{0}}=\{0,1, \ldots, M-1\}
$$

Now the following arguments are similar to those of Theorem This completes the proof of Theorem [4]

## 3. Final Remarks

For a sequence $A=\left\{a_{i}\right\}$ (finite or infinite) and an integer $n$, let $\sigma_{A}(n)$ be the ways of representation of $n=\varepsilon_{1} a_{1}+\varepsilon_{2} a_{2}+\cdots+\varepsilon_{t} a_{t}, \quad \varepsilon_{i} \in\{-1,1\}, \quad i=1,2, \ldots, t$. In the previous arguments all $A$ satisfy $\sigma_{A}(n)=+\infty$. It is natural to ask: is there $a$ sequence $A=\left\{a_{i}\right\}$ such that $1 \leq \sigma_{A}(n)<+\infty$ for all integers $n$ ? Now we construct such a sequence.

For $A_{k}=\left\{a_{1}, a_{2}, \ldots, a_{k}\right\}$, let

$$
\Delta\left(A_{k}\right)=\left\{\varepsilon_{1} a_{1}+\varepsilon_{2} a_{2}+\cdots+\varepsilon_{k} a_{k}: \varepsilon_{i} \in\{-1,1\}, \quad i=1,2, \ldots, k\right\} .
$$

Let $a_{1}=1, a_{2}=3, a_{3}=4$. Then $\Delta\left(A_{1}\right)=\{ \pm 1\}, \Delta\left(A_{2}\right)=\{ \pm 2, \pm 4\}$ and $\Delta\left(A_{3}\right)=\{0, \pm 2, \pm 6, \pm 8\}$. Suppose that we have $a_{1}, \ldots, a_{k}(k \geq 3)$. Let $n_{k}$ be the least positive integer which is not in $\Delta\left(A_{1}\right) \cup \cdots \cup \Delta\left(A_{k}\right)$, and let $a_{k+1}=a_{1}+$ $\cdots+a_{k}+n_{k}$. Then $n_{k} \in \Delta\left(A_{k+1}\right)$ and each positive integer in $\Delta\left(A_{k+1}\right)$ is at least $a_{k+1}-\left(a_{1}+\cdots a_{k}\right)=n_{k}$. Thus we obtain two sequences, $A=\left\{a_{i}\right\}_{i=1}^{\infty}$ and $\left\{n_{i}\right\}_{i=3}^{\infty}$, such that $3=n_{3}<n_{4}<\cdots$ and for every $k \geq 3$ we have $\sigma_{A}(n)=\sigma_{A_{k}}(n) \geq 1$ for all $n<n_{k}$.

Now we have proved the following theorem.
Theorem 5. There exists a strictly increasing sequence $A=\left\{a_{i}\right\}_{i=1}^{\infty}$ of positive integers such that $1 \leq \sigma_{A}(n)<+\infty$ for all integers $n$.

We pose the following problems here for further research.
Problem 3. Is there any (strictly increasing) sequence $A=\left\{a_{i}\right\}$ of positive integers such that $\sigma_{A}(0)=2$ and $\sigma_{A}(n)=1$ for all integers $n$ ?

Problem 4. Is there a constant $c>1$ and a (strictly increasing) sequence $A=\left\{a_{i}\right\}$ of positive integers such that $1 \leq \sigma_{A}(n) \leq c$ for all integers $n$ ?

If $A=\left\{1,3,3^{2}, \ldots\right\}$, then $\sigma_{A}(n) \in\{0,1\}$ for all integers $n$. We pose the following problem.

Problem 5. Is there any (strictly increasing) sequence $A=\left\{a_{i}\right\}$ of positive integers such that $\sigma_{A}(n) \geq 1$ for all integers $n$ and $\sigma_{A}(n)=1$ for infinitely many integers $n$ ?

Added in proof. We have known that Problem 3 is negative and that Problems 4 and 5 are affirmative.

Added after posting. We find that Problem 4 is still open.

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