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ERRATUM TO "SOME GEOMETRIC PROPERTIES OF HYPERSURFACES WITH CONSTANT r-MEAN CURVATURE IN EUCLIDEAN SPACE"

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ABSTRACT. An erratum to the paper [D. Impera, L. Mari, and M. Rigoli, *Some geometric properties of hypersurfaces with constant r-mean curvature in Euclidean space*, Proc. Amer. Math. Soc. **139** (2011), no. 6, 2207-2215] is presented.

In computation (13) of our previous paper [3] there are some inaccuracies concerning the constants. The computation should read as follows:

$$S_1S_{j+1} - (j+2)S_{j+2} = m\binom{m}{j+1}H_1H_{j+1} - (j+2)\binom{m}{j+2}H_{j+2}$$
$$= \binom{m}{j+1}(mH_1H_{j+1} - (m-j-1)H_{j+2})$$
$$\geq \binom{m}{j+1}(j+1)H_1H_{j+1} \geq 0,$$

where the inequality is a consequence of (12). Hence,

$$A(r)v_j(r) \geq (j+1)\binom{m}{j+1}H_{j+1}\int_{\partial B_r} H_1 = \frac{\binom{m-2}{j}H_{j+1}}{m-j-1}v_1(r).$$

Condition (2) of Theorem 1.1 should then read as follows:

$$(ii) \quad v_j(r)^{-1} \in L^1(+\infty) \qquad \text{and}$$

$$\liminf_{r\to+\infty} \sqrt{v_1(r)v_j(r)}\int_r^{+\infty}\frac{\mathrm{d}s}{v_j(s)} > \frac{1}{2}\left[\frac{\binom{m-2}{j}H_{j+1}}{m-j-1}\right]^{-1/2}.$$

Moreover, Remark 1.5 should be restated in this way:

> As we will see later, condition $S_{j+1} \equiv 0$ together with $\mathrm{rank}(A) > j$ at every point of M implies the ellipticity of the operator L_j. Moreover, if we assume the additional hypothesis that there exists $p \in M$ such that $H_i(p) > 0$ for every $1 \leq i \leq j$, it can be proved that each P_i is positive definite for every $1 \leq i \leq j$.

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Similarly, Proposition 2.2(ii) has to be replaced by:

(ii) $S_{j+1} \equiv 0$, $\mathrm{rank}(A) > j$ at every point of M, and there exists $p \in M$ such that $H_i(p) > 0$ for every $1 \le i \le j$.

The next remark should be added after Proposition 2.2.

Remark. We stress that, by [10], when $S_{j+1} \equiv 0$, the sole condition $\mathrm{rank}(A) > j$ is equivalent to the requirement that L_j be elliptic.

Taking into account the previous observations, Theorem 1.4 has to be replaced by the following:

Theorem 1.4. *Let $f : M \to \mathbb{R}^{m+1}$ be a complete, connected orientable hypersurface with $H_{j+1} \equiv 0$, for some $j \in \{0, \dots, m-2\}$. If $j \ge 1$, assume that* $\mathrm{rank}(A) > j$ *at every point. Furthermore, if j is even, suppose that there exists $p \in M$ such that $H_j(p) > 0$. Set*

$$v_j(r) = (m-j)\int_{\partial B_j} |S_j|, \qquad v_{j+2}(r) = \int_{\partial B_j} |S_{j+2}|.$$

If either

$$(1) \qquad \begin{aligned} &(i) \quad |v_j(r)|^{-1} \notin L^1(+\infty) \qquad \textit{and} \qquad H_{j+2} \notin L^1(M) \quad \textit{or} \\ &(ii) \quad |v_j(r)|^{-1} \in L^1(+\infty) \qquad \textit{and} \\ &\qquad \liminf_{r\to+\infty} \sqrt{s_{j+2}(r)v_j(r)} \int_r^{+\infty} \frac{\mathrm{d}s}{|v_j(s)|} > \frac{1}{2}\sqrt{\frac{1}{j+2}}, \end{aligned}$$

then for every compact set $K \subset M$ we have

$$\bigcup_{p\in M\setminus K} T_pM \equiv \mathbb{R}^{m+1};$$

that is, the tangent envelope of $M\setminus K$ coincides with $\mathbb{R}^{m+1}$.

Proof. We start by observing that we can assume that v_j is positive on $\mathbb{R}^+$. Indeed, in our assumptions, by the remark after Proposition 2.2 the operator L_j is elliptic; that is, P_j is either positive definite or negative definite everywhere. Thus, (3) of Lemma 2.1 implies that either $H_j > 0$ or $H_j < 0$ on M. If j is odd, we can change the orientation of M in such a way that H_j is positive, whence $v_j > 0$ on $\mathbb{R}^+$. On the other hand, if j is even, this trick cannot be used and we have to rely on the existence of $p \in M$ with $H_j(p) > 0$ to deduce that $v_j > 0$ on $\mathbb{R}^+$. Applying (5) of Lemma 2.1 we obtain

$$0 < \mathrm{Tr}(A^2P_j) = -(j+2)S_{j+2};$$

hence $S_{j+2} < 0$ on M, and then $v_{j+2} < 0$ on $\mathbb{R}^+$. Now, suppose by contradiction that for some K the tangent envelope of $M\setminus K$ does not coincide with $\mathbb{R}^{m+1}$. By choosing Cartesian coordinates appropriately, we can assume that the origin 0 satisfies

$$0 \notin \bigcup_{p\in M\setminus K} T_pM.$$

Then, the function $u = \langle f, \nu\rangle$ is nowhere vanishing and smooth on $M\backslash K$. Up to changing the sign of u on each connected component, we can assume that $u > 0$ on $M\backslash K$. By Proposition 2.4, $T_j u = 0$ and hence $\lambda_1^{-T_j}(M\backslash K) \geq 0$. Note that here $H_{j+1} \equiv 0$ is essential. Defining

$$0 < A(r) = \frac{1}{v_j(r)} \int_{\partial B_r} \mathrm{Tr}(A^2 P_j) = -(j+2)\frac{1}{v_j(r)} \int_{\partial B_r} S_{j+2} = (j+2)\frac{s_{j+2}(r)}{v_j(r)},$$

under assumptions (i) or (ii) the ODE $(v_j z')' + A v_j z = 0$ is oscillatory. To show this fact, we rest upon the same oscillation criteria used in the proof of Theorem 1.1. The rest of the proof is identical to that of Theorem 1.1. □

Further references

We would like to add the paper [2] (for (ii) of Proposition 2.2) and two foundational works which have been of inspiration for the research on higher-order mean curvature hypersurfaces. The first one is the classic [1], which contains the original proof of Gärding's inequality, and the second one, [4], characterizes hypersurfaces with H_j constant in space forms from the variational point of view.

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