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RESULTS ON WITT KERNELS OF QUADRATIC FORMS FOR MULTI-QUADRATIC EXTENSIONS

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Abstract. In this paper we compute the Witt kernel of quadratic forms for the composition of a purely inseparable multi-quadratic extension with a separable quadratic extension. We also include the case of a multi-quadratic purely inseparable extension by completing the proof given before by the second author for such an extension.

Throughout this paper F denotes a field of characteristic 2. Let $W_q(F)$ (*resp.* $W(F)$) be the Witt group of nonsingular quadratic forms over F (*resp.* the Witt ring of regular bilinear forms over F). The group $W_q(F)$ is endowed with a $W(F)$-module structure as follows: To a bilinear form (V, B) and a nonsingular quadratic form (W, φ) with the polar form B_φ, we associate a nonsingular quadratic form $B \otimes \varphi$ defined on $V \otimes_F W$ by: $B \otimes \varphi(v \otimes w) = B(v, v)\varphi(w)$ for $(v, w) \in V \times W$, whose polar form is $B \otimes B_\varphi$ [B].

For an integer $n \geq 1$, let I^nF denote the n-th power of the fundamental ideal IF of $W(F)$ (set $I^0F = W(F)$), and let $I^nW_q(F)$ denote the group $I^nF \otimes W_q(F)$.

We denote by $\langle a_1, \cdots, a_n\rangle_b$ the n-dimensional diagonal bilinear form $\sum_{i=1}^n a_ix_iy_i$ for $a_1, \ldots, a_n \in F^* := F \setminus \{0\}$. The 2-dimensional quadratic form $ax^2 + xy + by^2$ will be denoted by $[a, b]$.

A given field extension K/F induces natural homomorphisms $i_K : W_q(F) \longrightarrow W_q(K)$ and $j_K : W(F) \longrightarrow W(K)$. Let $W_q(K/F)$ and $W(K/F)$ denote the kernels of i_K and j_K, respectively.

An important problem in the algebraic theory of quadratic (bilinear) forms is the computation of $W_q(K/F)$ and $W(K/F)$. These kernels are known for some field extensions. For example, for K/F purely transcendental, these kernels are trivial. If K/F is separable, then $W(K/F)$ is trivial since an anisotropic F-bilinear form remains anisotropic over a separable extension of F. For K/F inseparable, the kernel $W(K/F)$ was computed by Hoffmann in [H]. For K the function field of a hypersurface (including algebraic simple extensions), the kernel $W(K/F)$ was computed by Dolphin and Hoffmann in [DH]. Before this result the second author computed $W(K/F)$ when K is the function field of a quadric [L1]. Still for K the function field of a quadric, some partial results on $W_q(K/F)$ are given in [L1]. Also, for φ a bilinear Pfister form, the kernel $I^m(F(\varphi)/F)$ was computed by Baeza and the first author in [AB].

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In this paper we are interested in the kernel $W_q(K/F)$ when K/F is a multi-quadratic extension. As the characteristic is 2, there are two kinds of quadratic extensions: the inseparable quadratic extension $F[t]/(t^2-a)$ with $a \in F \setminus F^2$, which we denote by $F(\sqrt{a})$, and the separable quadratic extension $F[t]/(t^2+t+b)$ with $b \notin \wp(F)$, which we denote by $F(\wp^{-1}(b))$ (here $\wp : F \longrightarrow F$ is the Artin-Schreier operator given by: $x \mapsto x^2 - x$).

In the case of a purely inseparable multi-quadratic extension K/F, the kernel $W_q(K/F)$ was computed in [L2], but it turns out that the proof contains a mistake. So our first aim in this paper is to give a complete proof of the following theorem (see comments after Proposition 1):

Theorem 1 ([L2, Th. 1])**.** *For any scalars $a_1, \cdots, a_n \in F$, we have the following:*

$$W_q(F(\sqrt{a_1}, \cdots, \sqrt{a_n})/F) = \sum_{i=1}^{n} \langle 1, a_i \rangle_b \otimes W_q(F).$$

After that, based on Theorem 1, we will compute the kernel of the composition of a purely inseparable extension with a quadratic separable extension:

Theorem 2. *For any scalars $a_1, \ldots, a_n, b \in F$, we have the following:*

$$W_q(F(\wp^{-1}(b), \sqrt{a_1}, \cdots, \sqrt{a_n})/F) = W(F) \otimes [1, b] + \sum_{i=1}^{n} \langle 1, a_i \rangle_b \otimes W_q(F).$$

Previously, Theorem 1 was proved by Mammone and Moresi [MM, Th. 2(i)] for $n = 2$, and by Baeza [B, Lem. 4.3, page 182] for $n = 1$. Theorem 2 is due to Ahmad for $n = 1$ [Ah]. In [B, Cor. 4.16, page 128], Baeza gave the computation in the case of quadratic and bi-quadratic separable extensions: $W_q(F(\wp^{-1}(a), \wp^{-1}(b))/F) = W(F) \otimes [1, a] + W(F) \otimes [1, b]$. But this computation does not extend to tri-quadratic separable extensions as was proved by Mammone and Moresi [MM, Prop. 1].

Our proofs are reduced to computations on differential forms. This is allowed by the well known result of Kato [K], which gives the relation between nonsingular quadratic forms and differential forms (see below).

Let us recall some facts about differential forms. For any integer $n \geq 1$, we denote by $\Omega_F^n = \bigwedge^n \Omega_F^1$ the space of n-differential forms ($\Omega_F^0 = F$), where Ω_F^1 is the F-vector space generated by the symbols dx, $x \in F$, subject to the relations $d(x+y) = dx + dy$ and $d(xy) = xdy + ydx$ for $x, y \in F$. In particular, there is a F^2-linear map $F \longrightarrow \Omega_F^1$, given by: $x \mapsto dx$. This map extends to the differential operator $d : \Omega_F^n \longrightarrow \Omega_F^{n+1}$ as follows: $d(xdx_1 \wedge \cdots \wedge dx_n) = dx \wedge dx_1 \wedge \cdots \wedge dx_n$. The usual Artin-Schreier operator $\wp : F \longrightarrow F$, $\wp(a) = a^2 - a$, extends to the general Artin-Schreier map

$$\wp : \Omega_F^n \longrightarrow \Omega_F^n / d\Omega_F^{n-1}$$

given on generators by

$$\wp(x \frac{dx_1}{x_1} \wedge \cdots \wedge \frac{dx_n}{x_n}) = (x^2 - x) \frac{dx_1}{x_1} \wedge \cdots \wedge \frac{dx_n}{x_n} + d\Omega_F^{n-1}.$$

Let $\nu_F(n) = \mathrm{Ker}(\wp)$ and $H_2^{n+1}(F) = \mathrm{Coker}(\wp)$. By a result of Kato [K], there exists an isomorphism

$$e^n : \overline{I^n} W_q(F) \longrightarrow H_2^{n+1}(F)$$

given on generators as follows:

$$e^n(\langle\langle a_1,\cdots,a_n\rangle\rangle_b\otimes[1,c]+I^{n+1}W_q(F))=c\frac{da_1}{a_1}\wedge\cdots\wedge\frac{da_n}{a_n}+\wp(\Omega_F^n)+d\Omega_F^{n-1},$$

where $\overline{I^n}W_q(F)$ denotes the quotient $I^nW_q(F)/I^{n+1}W_q(F)$ and $\langle\langle a_1,\cdots,a_n\rangle\rangle_b$ denotes the n-fold bilinear Pfister form $\langle 1,a_1\rangle_b\otimes\cdots\otimes\langle 1,a_n\rangle_b$. In [K], Kato also established a relation between bilinear forms and differentials forms, but we will not need this fact here.

Let $\{a_i\mid i\in I\}$ be a 2-basis of F; i.e., the set $\{\prod_{i\in I}a_i^{\epsilon_i}\mid\epsilon_i\in\{0,1\},$ and $\epsilon_i=0$ for almost all $i\in I\}$ is an F^2-basis of F. We fix some ordering on I and consider the set $\sum_n:=\{\sigma:\{1,\cdots,n\}\longrightarrow I\mid\sigma(i)<\sigma(j)$ whenever $i<j\}$. This set $\sum_n$ is equipped with the lexicographic ordering. Hence, the set $\{\frac{da_\sigma}{a_\sigma}\mid\sigma\in\sum_n\}$ is an F-basis of Ω_F^n, where $\frac{da_\sigma}{a_\sigma}=\frac{da_{\sigma(1)}}{a_{\sigma(1)}}\wedge\cdots\wedge\frac{da_{\sigma(n)}}{a_{\sigma(n)}}$ for any $\sigma\in\sum_n$. For $\alpha\in\sum_n$, let $\Omega_{F,\alpha}^n$ (*resp.* $\Omega_{F,<\alpha}^n$) be the subspace of Ω_F^n generated by $\frac{da_\sigma}{a_\sigma}$ with $\sigma\le\alpha$ (*resp.* the subspace of Ω_F^n generated by $\frac{da_\sigma}{a_\sigma}$ with $\sigma<\alpha$). Hence, we get a filtration of Ω_F^n given by $\Omega_{F,\alpha}^n$, $\alpha\in\sum_n$ (and $\Omega_{F,<\alpha}^n$, $\alpha\in\sum_n$). For any $i\in I$, let F_i be the subfield $F^2(a_j\mid j\le i)$.

A crucial result that we will use in our proofs is due to Kato. We will refer to it as Kato's lemma:

Proposition 1 ([K]). *Let $\alpha\in\sum_n$ and $y\in F$ be such that $\wp(y\frac{da_\alpha}{a_\alpha})\in\Omega_{F,<\alpha}^n+d\Omega_F^{n-1}$. Then, there exist $v\in\Omega_{F,<\alpha}^n$ and $c_i\in F_{\alpha(i)}^*$, $1\le i\le n$, such that*

$$y\frac{da_\alpha}{a_\alpha}=\frac{dc_1}{c_1}\wedge\cdots\wedge\frac{dc_n}{c_n}+v.$$

Clearly, this proposition implies that any element $\eta\in\nu_F(n)$ can be written as $\eta=\sum_{\beta\le\alpha}u_\beta\frac{dl_\beta}{l_\beta}$ for some $\alpha\in\sum_n$, $l_{\beta(i)}\in F_{\beta(i)}^*$, $u_\beta=0,1$, and $\frac{dl_\beta}{l_\beta}=\frac{dl_{\beta(1)}}{l_{\beta(1)}}\wedge\cdots\wedge\frac{dl_{\beta(n)}}{l_{\beta(n)}}$.

The following proposition is incorporated in [L2, Prop. 1], and it is the basis of Theorem 1. When trying to prove Theorem 2 we noticed that some details in the proof of [L2, Prop. 1] should be corrected (see the explanations in the following footnote). This is the reason why we reproduce the proof here.

Proposition 2. *For any integer $n\ge 1$ and scalars $\alpha_1,\cdots,\alpha_m\in F$, we have*

$$\mathrm{Ker}(H_2^{n+1}(F)\longrightarrow H_2^{n+1}(F(\sqrt{\alpha_1},\cdots,\sqrt{\alpha_m})))=\sum_{i=1}^m\overline{d\alpha_i\wedge\Omega_F^{n-1}(F)}.$$

Proof. Let $L=F(\sqrt{\alpha_1},\cdots,\sqrt{\alpha_m})$ and $M=F(\sqrt{\alpha_1})$. Without loss of generality, we may suppose that $[L:F]=2^m$. So we can choose a 2-basis $\mathcal{B}=\{c_1,\cdots,c_m,c_{m+1},\cdots\}$ of F such that $c_i=\alpha_i$ for $i=1,\cdots,m$.

We proceed by induction on m. The case $m=1$ is done in [AB, Lem. 2.18]. Suppose that the proposition is true for any field F' of characteristic 2 and any purely inseparable multi-quadratic extension L'/F' such that $[L':F']<2^m$.

Since $[L:M]=2^{m-1}$, we get by the induction hypothesis that $\mathrm{Ker}(H_2^{n+1}(M)\longrightarrow H_2^{n+1}(L))=\sum_{i=2}^m\overline{d\alpha_i\wedge\Omega_M^{n-1}}$.

Let $\omega \in \Omega_F^n$ be such that $\overline{\omega} \in \mathrm{Ker}(H_2^{n+1}(F) \longrightarrow H_2^{n+1}(L))$. Then, over M, we have $\overline{\omega} \in \mathrm{Ker}(H_2^{n+1}(M) \longrightarrow H_2^{n+1}(L))$. Hence, $\overline{\omega} \in \sum_{i=2}^m \overline{d\alpha_i \wedge \Omega_M^{n-1}}$. This means that there exist $u \in \Omega_M^n$ and $v \in \Omega_M^{n-1}$ such that

$$w = \sum_{i=2}^m \frac{d\alpha_i}{\alpha_i} \wedge \sigma_i + \wp(u) + d(v), \tag{1}$$

for suitable $\sigma_2, \cdots, \sigma_m \in \Omega_M^{n-1}$. For the sequel we use a descent argument[1] in order to reduce to the case where $\sigma_1, \cdots, \sigma_m, u, v$ are defined over F.

The set $\mathcal{B}' := \{c_2, \cdots, c_m, c_{m+1}, \cdots, \sqrt{\alpha_1}\}$ is a 2-basis of M. We may suppose that $\sqrt{\alpha_1}$ is the last element in $\mathcal{B}'$. Let δ be the maximal multi-index in (1).

Suppose that $\delta(1) > m$. Then we get

$$w_\delta \frac{dc_\delta}{c_\delta} = \wp(u_\delta \frac{dc_\delta}{c_\delta}) + t\frac{dc_\delta}{c_\delta} \tag{2}$$

such that $t\frac{dc_\delta}{c_\delta} + d(v) := w' \in \Omega_{M,<\delta}^n$.

(i) Suppose that $c_{\delta(n)} = \sqrt{\alpha_1}$: In this case, we have $w_\delta = 0$, and thus $\wp(u_\delta \frac{dc_\delta}{c_\delta}) \in d\Omega_M^{n-1} + \Omega_{M,<\delta}^n$. By Kato's lemma, $u_\delta \frac{dc_\delta}{c_\delta} = \frac{de_{\delta(1)}}{e_{\delta(1)}} \wedge \cdots \wedge \frac{de_{\delta(n)}}{e_{\delta(n)}} + u'$, where $e_{\delta(j)} \in M_{\delta(j)}$ and $u' \in \Omega_{M,<\delta}^n$. By substituting the expression $u_\delta \frac{dc_\delta}{c_\delta}$ in (2) and using the fact that $\wp(\frac{de_{\delta(1)}}{e_{\delta(1)}} \wedge \cdots \wedge \frac{de_{\delta(n)}}{e_{\delta(n)}}) \in d\Omega_M^{n-1}$, we are reduced to the case where $c_{\delta(n)} \in F$.

(ii) Suppose that $c_{\delta(n)} \in F$: In this case, we have $v \in \Omega_F^{n-1}$. Otherwise, $v = v_0 + \sqrt{\alpha_1} v_1$ with $v_0, v_1 \in \Omega_F^{n-1}$ and $v_1 \neq 0$. Then, $d(v) = d(v_0) + \sqrt{\alpha_1} d(v_1) + v_1 \wedge d(\sqrt{\alpha_1})$. Consequently, $\frac{dc_\delta}{c_\delta}$ contains the slot $d(\sqrt{\alpha_1})$, which is not possible. Hence, $t\frac{dc_\delta}{c_\delta}$ and w' belong to Ω_F^n. In particular, $u_\delta \frac{dc_\delta}{c_\delta} \in \Omega_F^n$. Now substituting $w_\delta \frac{dc_\delta}{c_\delta}$ in w, we get

$$\begin{aligned} w &= w_\delta \tfrac{dc_\delta}{c_\delta} + w'', \quad \text{with } w'' \in \Omega_{F,<\delta}^n \\ &= \wp(u_\delta \tfrac{dc_\delta}{c_\delta}) + d(v) + w' + w'' \\ &\equiv w''' \mod \wp(\Omega_F^n) + d\Omega_F^{n-1}, \end{aligned}$$

with $w''' = w' + w'' \in \Omega_{F,<\delta}^n$. By iterating the procedure whenever $\delta(1) > m$, we are reduced to an element w''' whose maximal multi-index λ satisfies $\lambda(1) \leq m$. This means that $w''' \in \sum_{i=2}^m \frac{d\alpha_i}{\alpha_i} \wedge \Omega_F^{n-1}$. Hence, there exist $\tau_2, \cdots, \tau_m \in \Omega_F^{n-1}$, $r \in \Omega_F^{n-1}$ and $s \in \Omega_F^n$ such that, over the field M, we have

$$w = \sum_{i=2}^m \frac{d\alpha_i}{\alpha_i} \wedge \tau_i + \wp(r) + d(s).$$

Since $\mathrm{Ker}(H_2^{n+1}(F) \longrightarrow H_2^{n+1}(M)) = \overline{\frac{d\alpha_1}{\alpha_1} \wedge \Omega_F^{n-1}}$, there exists $\tau_1 \in \Omega_F^{n-1}$ such that $\overline{w} = \overline{\sum_{i=1}^m \frac{d\alpha_i}{\alpha_i} \wedge \tau_i}$. □

The proof of Theorem 2 will be based on the following proposition:

[1] A mistake in the proof of [L2, Prop. 1] has been made when trying to make this descent. More precisely, it happens in [L2, page 2484, lines 7-8] when we tried to apply the Artin-Schreier operator.

Proposition 3. *Let $\alpha_1, \cdots, \alpha_m, a \in F$, and let $L = F(\sqrt{\alpha_1}, \cdots, \sqrt{\alpha_m})$, $M = F(\wp^{-1}(a))$. Then we have the following:*

$$\mathrm{Ker}(H_2^{n+1}(F) \longrightarrow H_2^{n+1}(L \cdot M)) = a\nu_F(n) + \sum_{i=1}^{m} \overline{d\alpha_i \wedge \Omega_F^{n-1}}.$$

Proof. Without loss of generality, we may suppose that $[L : F] = 2^m$, $a \in F \setminus \wp(F)$ and $a \in F^2$. Put $\beta = \wp^{-1}(a)$. Let $\mathcal{B} := \{c_1, \cdots, c_m, c_{m+1}, \cdots\}$ be a 2-basis of F such that $c_i = \alpha_i$ for $i = 1, \cdots, m$. Since M/F is separable, $\mathcal{B}$ remains a 2-basis of M. Then $\Omega^n_{M,<\delta} = \Omega^n_{F,<\delta} + \beta\Omega^n_{F,<\delta}$.

Let $\omega \in \Omega^n_F$ be such that $\overline{w} \in \mathrm{Ker}(H_2^{n+1}(F) \longrightarrow H_2^{n+1}(L \cdot M))$. Then, over M, $\overline{\omega} \in \mathrm{Ker}(H_2^{n+1}(M) \longrightarrow H_2^{n+1}(L \cdot M))$. It follows from Proposition 2 that

$$\omega = \sum_{i=1}^{m} \sigma_i \wedge d\alpha_i + \wp(u) + d(v) \tag{3}$$

with $u \in \Omega^n_M$, $v \in \Omega^{n-1}_M$, and $\sigma_1, \cdots, \sigma_m \in \Omega^{n-1}_M$. Let δ be the maximal multi-index in relation (3).

Suppose that $\delta(1) > m$. Then, as in the relation (2), there exists $w' \in \Omega^n_{M,<\delta}$ such that

$$w_\delta \frac{dc_\delta}{c_\delta} = \wp(u_\delta \frac{dc_\delta}{c_\delta}) + d(v) + w'. \tag{4}$$

Let $r_\delta, s_\delta \in F$, $v_0, v_1 \in \Omega^{n-1}_F$, and $w'_0, w'_1 \in \Omega^n_{F,<\delta}$ be such that $u_\delta = r_\delta + \beta s_\delta$, $v = v_0 + \beta v_1$ and $w' = w'_0 + \beta w'_1$. A simple computation shows that the relation (4) gives the following:

$$\begin{cases} w_\delta \frac{dc_\delta}{c_\delta} = \wp(r_\delta \frac{dc_\delta}{c_\delta}) + d(v_0) + as_\delta^2 \frac{dc_\delta}{c_\delta} + w'_0, \\ \wp(s_\delta \frac{dc_\delta}{c_\delta}) = d(v_1) + w'_1. \end{cases} \tag{5}$$

By applying Kato's lemma to the second relation in (5), we get

$$s_\delta \frac{dc_\delta}{c_\delta} = \frac{da_{\delta(1)}}{a_{\delta(1)}} \wedge \cdots \wedge \frac{da_{\delta(n)}}{a_{\delta(n)}} + w'_2,$$

for some $w'_2 \in \Omega^n_{F,<\delta}$ and $a_{\delta(j)} \in F_{\delta(j)}$. Again with the second relation in (5), we obtain

$$\begin{aligned} as_\delta^2 \tfrac{dc_\delta}{c_\delta} &= as_\delta \tfrac{dc_\delta}{c_\delta} + ad(v_1) + aw'_1 \\ &= a\tfrac{da_{\delta(1)}}{a_{\delta(1)}} \wedge \cdots \wedge \tfrac{da_{\delta(n)}}{a_{\delta(n)}} + d(av_1) + aw'_1 + aw'_2. \end{aligned}$$

If we substitute the expression $as_\delta^2 \frac{dc_\delta}{c_\delta}$ in the first relation in (5), we get

$$w_\delta \frac{dc_\delta}{c_\delta} = \wp(r_\delta \frac{dc_\delta}{c_\delta}) + d(v') + a\eta_\delta + w'',$$

where $v' = v_0 + av_1$, $\eta_\delta = \frac{da_{\delta(1)}}{a_{\delta(1)}} \wedge \cdots \wedge \frac{da_{\delta(n)}}{a_{\delta(n)}}$, and $w'' = aw'_1 + aw'_2 + w'_0 \in \Omega^n_{F,<\delta}$.

Since $a\eta_\delta \in \wp(\Omega^n_M) \subset \wp(\Omega^n_M) + d(\Omega^{n-1}_M)$, the multi-index has been reduced. By iterating the procedure whenever $\delta(1) > m$, we obtain, over M, the following:

$$w = \wp(\gamma_1) + d(\gamma_2) + a\gamma_3 + \gamma_4,$$

for $\gamma_1 \in \Omega^n_F$, $\gamma_2 \in \Omega^{n-1}_F$, $\gamma_3 \in \nu_F(n)$ and $\gamma_4 \in \Omega^n_F$, with the condition that the maximal multi-index λ in γ_4 satisfies $\lambda(1) \leq m$. Then $\gamma_4 \in \sum_{i=1}^m d\alpha_i \wedge \Omega^{n-1}_F$. Now,

since the kernel $\mathrm{Ker}(H_2^{n+1}(F) \longrightarrow H_2^{n+1}(M)) = \overline{a\nu_F(n)}$ [AB], we get the desired conclusion. $\square$

As a corollary, we get:

Corollary 1. *Let $n \geq 1$ be an integer, and let $\alpha_1, \cdots, \alpha_m, a \in F$. Let $L = F(\sqrt{\alpha_1}, \ldots, \sqrt{\alpha_m})$ and $M = L(\wp^{-1}(a))$. Then we have:*
(1) $\mathrm{Ker}(\overline{I^n}W_q(F) \longrightarrow \overline{I^n}W_q(L)) = \sum_{i=1}^m \overline{\langle 1, \alpha_i \rangle_b \otimes I^{n-1}W_q(F)}$.
(2) $\mathrm{Ker}(\overline{I^n}W_q(F) \longrightarrow \overline{I^n}W_q(L \cdot M)) = \overline{I^n F \otimes [1, a]} + \sum_{i=1}^m \overline{\langle 1, \alpha_i \rangle_b \otimes I^{n-1}W_q(F)}$.

Proof. We use Propositions 2 and 3 with the commutativity of the diagram

$$\begin{array}{ccc} \overline{I^n}W_q(F) & \longrightarrow & \overline{I^n}W_q(K) \\ \downarrow & & \downarrow \\ H_2^{n+1}(F) & \longrightarrow & H_2^{n+1}(K) \end{array}$$

where $K = L$ (*resp.* $K = L \cdot M$), the vertical arrows are given by the isomorphism e^n, and the horizontal arrows are induced by the inclusion $F \subset K$. $\square$

Proof of Theorems 1 *and* 2. We use Corollary 1, and we follow the same argument used for the proof of [L2, Theorem 1]. $\square$

We also have the following corollary, whose proof is the same as for Theorems 1 and 2:

Corollary 2. *Let $n \geq 1$ be an integer, and let $\alpha_1, \cdots, \alpha_m, a \in F$. Let $L = F(\sqrt{\alpha_1}, \ldots, \sqrt{\alpha_m})$ and $M = L(\wp^{-1}(a))$. Then we have:*
(1) $\mathrm{Ker}(I^n W_q(F) \longrightarrow I^n W_q(L)) = \sum_{i=1}^m \langle 1, \alpha_i \rangle_b \otimes I^{n-1}W_q(F)$.
(2) $\mathrm{Ker}(I^n W_q(F) \longrightarrow I^n W_q(L \cdot M)) = I^n F \otimes [1, a] + \sum_{i=1}^m \langle 1, \alpha_i \rangle_b \otimes I^{n-1}W_q(F)$.

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