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PREDUALS OF H^∞ OF FINITELY CONNECTED DOMAINS

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Abstract. A well known result of Ando says that $H^\infty(\mathbb{D})$ has a unique predual. There have been two natural extensions of this result to non-commutative algebras: Ueda showed that finite maximal subdiagonal algebras have unique preduals. In a second direction, Davidson and Wright showed that free semigroup algebras have unique preduals. In these notes, we explore a different natural generalization of this result: Let A be a finitely connected domain in the plane. We show that $H^\infty(A)$ has a unique isometric predual. We also prove a couple of theorems about the structure of the unique predual.

1. Introduction

A dual Banach space $X = Y^*$ is said to have a unique (isometric) predual if $X \cong (Z)^*$ isometrically implies that $Y \cong Z$ isometrically. A classical result of Grothendieck [10] says that the dual Banach space $L^\infty(X, \mu)$ has a unique predual for any measure space (X, μ). Over the years this result has spawned a host of extensions and generalizations. Noting that all abelian von Neumann algebras are of the form $L^\infty(X, \mu)$, Sakai [16] extended Grothendieck's result by showing that all von Neumann algebras have unique preduals.

In the non-selfadjoint realm, Ando [2] proved that $H^\infty(\mathbb{D})$, which is easily seen to be a dual space, has a unique (isometric) predual. Ando's result has been extended in two notable directions.

(1) The free semigroup algebras, the WOT closed operator algebras generated by the creation operators on the Full Fock space $\mathcal{F}(\mathcal{H})$ where $\mathcal{H}$ is a Hilbert space, were recently shown by Davidson and Wright [5] to have a unique predual.

(2) Finite maximal subdiagonal algebras, also called non-commutative H^∞ algebras, were introduced by Arveson [3] (see also Exel's [6] paper) and are defined thus: Let (M, τ) be a finite von Neumann algebra and let A be an abelian von Neumann subalgebra. A weak* closed subalgebra $\mathcal{A}$ of M is said to be maximal subdiagonal in M if
   (a) $\mathcal{A} + \mathcal{A}^*$ is weak* dense in M and
   (b) the unique normal trace preserving conditional expectation $E : M \to A$ is multiplicative on $\mathcal{A}$.

   In particular, $H^\infty(\mathbb{D}) \subset L^\infty(\mathbb{T})$ is a (finite) maximal subdiagonal algebra. Ueda [17] (see also [18]) recently proved that all maximal subdiagonal algebras have unique preduals.

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Ando's result, however, has not been generalized to other domains. It is currently unknown if H^∞ of a domain in $\mathbb{C}^n$, even, say, the ball in $\mathbb{C}^2$ or the two-torus $\mathbb{T}^2$, has a unique predual. In this paper we show that the situation in the plane is more tractable.

Theorem 4.1. *H^∞ for every finitely connected domain in the plane has a unique predual.*

We mention here that for operator algebras, having a unique predual is somewhat mysterious. The major result in this direction is Ruan's result [14] that local dual operator algebras (i.e. weak* closed operator algebras in which the compact operators are ultraweakly dense) have unique preduals.

Our proof is an adaptation of Ando's, whose proof hinges upon a result of Amar and Lederer [1] that says roughly that singular functionals on $H^\infty(\mathbb{D})$ are supported on peak sets of functions in H^∞. We extend this result of Amar and Lederer (Theorem 3.1), and our extension further allows us to extend another of Ando's results. There is a canonical decomposition of any element in $\phi \in L^\infty(\mathbb{D})^*$, $\phi = \phi_n + \phi_s$, into "normal" and "singular" functionals. The projection P given by $P(\phi) = \phi_n$ is an L-projection, i.e. $\|\phi\| = \|\phi_n\| + \|\phi_s\|$.

This projection is not weak* continuous, for that would imply that L^∞ is reflexive. However, Ando shows that the mapping is weak* sequentially continuous and that $H^\infty(\mathbb{D})^\perp$ is invariant under this projection. As a consequence, the L-projection P induces an L-projection from $H^\infty(\mathbb{D})^*$ onto $H^\infty(\mathbb{D})_*$. Our extension of Amar and Lederer's peak sets result allows us to prove analogous results in the case of finitely connected domains.

Theorem 4.2. *For any finitely connected domain A, the predual $(H^\infty(A))_*$ is the range of a weak* sequentially continuous L-projection.*

The proof of Theorem 4.2 uses Theorem 3.1 but not Theorem 4.1. A recent result of Pfitzner [13] says that if a predual is an L-summand in its double dual (i.e. the range of an L-projection), then it is a strongly unique predual (see Section 2 for a definition). Thus, the first part of Theorem 4.2 along with Pfitzner's result will imply Theorem 4.1. However, since the proof of Theorem 4.1 is simpler than that of Theorem 4.2, we include it.

Finally, we give an extension of yet another result of Ando by showing that the unit ball of the canonical predual has no extreme points, which in turn implies the following result:

Theorem 4.3. *H^∞ of a finitely connected domain has no second predual.*

We conclude with some consequences of these results.

2. Preliminaries

Given a Banach space X, we say that Y is a predual for X if $X \cong Y^*$, where $\cong$ means an isometric isomorphism. Thus, Y is a norm closed subspace of X^* that norms X and also the unit ball $(X)_1 := \{x \in X \mid ||x|| \leq 1\}$ of X is $\sigma(X,Y)$ compact. The converse is also true. Suppose Y is a norm closed subspace of X^* so that $(X)_1$ is $\sigma(X,Y)$ compact. In fact, the bipolar theorem gives us that Y norms X, i.e.,

$$\sup_{\phi\in(Y)_1} |\phi(x)| = ||x|| \quad \text{for all} \quad x \in X. \tag{1}$$

Thus, X sits in Y^* isometrically. By the compactness condition, the unit ball $(X)_1$ is $\sigma(Y^*, Y)$ closed in $(Y^*)_1$, and therefore, by the Krein-Smulian theorem, X is $\sigma(Y^*, Y)$ closed in Y^*. As $Y \subset X^*$, the annihilator of X in Y is $\{0\}$, which implies that $X = Y^*$.

If a Banach space X is known to have a predual $X_* \subset X^*$, we will say that it has a strongly unique predual if X_* is the unique norm closed subspace of X^* which makes $(X)_1$ compact in the induced topology. No examples are known of dual Banach spaces that have unique but not strongly unique preduals. We will show that H^∞ of a finitely connected domain in the plane indeed has a strongly unique predual.

We start with a finitely connected domain on the sphere S^2, whose complement consists of $n+1$ closed components relative to the sphere. By applying the Riemann mapping theorem $n+1$ times, we map the domain biholomorphically onto a bounded domain in the complex plane whose boundary consists of $n+1$ disjoint analytic simple curves. Biholomorphic maps induce isometric isomorphisms of H^p spaces for $1 \leq p \leq \infty$. Thus, the latter avatar will be the setting for our results.

Let A be a domain in $\mathbb{D}$ bounded by simple analytic curves $\Gamma_0, \Gamma_1, \cdots, \Gamma_n$, so that Γ_0 is the unit circle. Set $\Gamma = \Gamma_0 \cup \Gamma_1 \cup \cdots \cup \Gamma_n$. By m, denote the Lebesgue measure on Γ, normalized so that $m(\Gamma_i) = 1$ for $0 \leq i \leq n$. Let us denote by U_i the unbounded component of $\mathbb{C} \setminus \Gamma_i$ for $i > 0$.

It is well known that every $f \in H^\infty(A)$ can be represented in the form

$$f(z) = f_0(z) + f_1(z) + \cdots + f_n(z)$$

where $f_0 \in H^\infty(\mathbb{D})$ and $f_j \in H^\infty(U_j)$. As a consequence of this result the boundary values of f exists almost everywhere and the map $f \to f^*$, where $f^* \in H^\infty(\Gamma)$ is the corresponding boundary function, is an isometry. (See [7] or [4] for details.) From now on H^∞ will refer to $H^\infty(A)$ or $H^\infty(\Gamma)$ depending on the context.

Denote the maximal ideal space of $L^\infty(A)$ by Ω, which is a totally disconnected compact Hausdorff space. In particular, its topology is given by a basis of clopen sets. The set Ω can be identified with the Shilov boundary of $H^\infty(A)$, though this is not something we will use. We will write L^∞ for $L^\infty(\Gamma)$.

Let $f \to \hat{f}$ denote the Gelfand transform on L^∞. Any bounded linear functional on L^∞ is represented by a regular Borel measure on Ω, and, in particular, we have a measure $\hat{m}$ so that

$$\int f dm = \int \hat{f} d\hat{m}. \tag{2}$$

By the Hahn-Banach theorem, any bounded linear functional on H^∞ can be extended to a bounded linear functional on L^∞ and thus has a (non-unique) representing measure on Ω.

Suppose now that ϕ is some element in $(L^\infty)^*$. Write $\phi(f) = \int_\Omega \hat{f} d\nu$ and take the Lebesgue decomposition of ν with respect to $d\hat{m}$, that is, $\nu = \nu_a + \nu_s$ with $\nu_a \ll \hat{m}$ and $\nu_s \perp \hat{m}$. Letting $\phi_a(f) = \int_\Omega \hat{f} d\nu_a$ and $\phi_s(f) = \int_\Omega \hat{f} d\nu_s$, we have a decomposition $\phi = \phi_a + \phi_s$. The map P which takes ϕ to ϕ_a is an L-projection; i.e. $||\phi|| = ||\phi_a|| + ||\phi_s||$ by the Radon Nikodym Theorem.

Any element f in L^1 gives rise to a functional on H^∞ by the formula

$$h \longrightarrow \int_\Gamma hf dm \quad \text{for all } f \in H^\infty.$$

Throughout this paper, we will denote the pre-annihilator of H^∞ in L^1 by Z. We will also use the notation $(H^\infty)_*$ to refer to the canonical predual. In this notation, the canonical predual of H^∞ is $(H^\infty)_* = L^1/Z$.

It is easy to see that a functional ψ on H^∞ is in the canonical predual L^1/Z iff there is some representing measure for it on Ω that is absolutely continuous with respect to $\hat{m}$; i.e. there is a measure ν on Ω such that

$$\psi(f) = \int_\Omega \hat{f} d\nu \quad \text{for every } f \in H^\infty \text{ and } \nu \ll \hat{m}.$$

The dual $(H^\infty)^*$ is isometric isomorphic to $(L^\infty)^*/(H^\infty)^\perp$. So if one can show that the annihilator $(H^\infty)^\perp$ is invariant under the projection P, the L-projection P will induce an L-projection from $(H^\infty)^*$ to $(H^\infty)_*$.

We also need the notion of unconditional weak summability. Let X be a Banach space. A sequence $(x_n)_n$ in X is said to be *unconditionally weakly summable* if for any $\phi \in X^*$, we have $\sum_{n=1}^\infty |\phi(x_n)| < \infty$. This is equivalent to saying that there exists C so that for all finite sequences $(\epsilon_1, \cdots, \epsilon_n)$ with $\max_n |\epsilon_n| \leq 1$, we have that $||\epsilon_1 x_1 + \cdots + \epsilon_n x_n|| < C$. (See [11] [Page 34].) The following proposition is trivial.

Proposition 2.1. *A sequence $\{f_n\}_n$ in $H^\infty(A)$ is unconditionally weakly summable iff there is a constant C so that for every $z \in A$, $\sum_n |f_n(z)| < C$.*

3. Peak sets for $H^\infty(A)$

The main ingredient in the proof of our main result is the following extension of Amar and Lederer's well known peak sets result. In this paper, the term "peak set" will refer to:

Definition 3.1. A compact subset K of Ω is called a peak set for H^∞ if there exists an element $f \in H^\infty$ such that $\hat{f}(x) = 1$ for every $x \in K$ and $|\hat{f}(x)| < 1$ for every $x \in X \setminus K$.

It is well known (see for instance [15], page 249) that the boundary values of an H^∞ function on a simply connected domain cannot vanish on a set of positive Lebesgue measure. The following lemma is an easy adaptation of this result to the case of finitely connected domains.

Lemma 3.1. *If the boundary values of a function $f \in H^\infty$ is zero on a set of positive measure in Γ, then $f \equiv 0$ on A.*

Proof. Let us denote the boundary function with f^*. Assume first that $f^* = 0$ almost everywhere. Then $\|f^*\|_\infty = 0$. Thus, $\|f\|_\infty$ is also zero, as the mapping $f \mapsto f^*$ is an isometry. Assume now that f^* vanishes on a set of positive measure in Γ. Then in particular it vanishes on a set of positive measure in one boundary component. We may assume without loss of generality that boundary part is the unit circle. The rest of the proof is the same as the one for the simply connected case. □

Theorem 3.1 (Amar-Lederer Redux)**.** *Let K be a compact set in Ω so that $\hat{m}(K) = 0$. Then, K is contained in a compact set F with $\hat{m}(F) = 0$, which is a peak set for H^∞.*

Proof. Since $\hat{m}$ is a regular measure on a totally disconnected set Ω, we can find clopen sets K_n so that $K_1 = \Omega$, $K_n \supset K_{n+1}$, $K_n \supset K$, and $\dfrac{1}{(n+1)^3} < \hat{m}(K_n) < \dfrac{1}{n^3}$ for $n \geq 2$. Since the sets $\{K_n\}_n$ are clopen, the characteristic functions χ_{K_n} are continuous on Ω. Via the Gelfand transformation, we can find measurable subsets E_n of Γ such that $E_1 = \Gamma$, $E_n \supset E_{n+1}$, $\hat{\chi_{E_n}} = \chi_{K_n}$, and $m(E_n) < \frac{1}{n^3}$. Actually, note that by (2), we have $m(E_n) = \hat{m}(K_n)$. Let $f = \sum_{n=1}^{\infty} n\chi_{E_n}$. By the statement above, $f \in L^1$. Let $P_z(\zeta)$, for $z \in A$ and $\zeta \in \Gamma$, be the Poisson kernel on A and define u_n as the Poisson transform of $n\chi_{E_n}$:

$$u_n(z) = \int_\Gamma P_z(\zeta) n\chi_{E_n} ds(\zeta).$$

Let $u = \sum_n u_n$. By the the monotone convergence theorem, we have

$$u(z) = \int_\Gamma P_z(\zeta) f(\zeta) ds(\zeta).$$

It is a standard fact (see for instance [8, Theorem II.2.5]) that $P_z(\zeta) = -\dfrac{\partial g(\zeta, z)}{\partial n_\zeta}$, where $g(\zeta, z)$ is the Green's function for A. Further, since Green's function $g(\zeta, z)$ is harmonic on a neighbourhood of Γ, thus real analytic, we conclude that for any z in A, the function $P_z(\zeta)$ is continuous on Γ. In particular, $u(z)$ is everywhere finite on A. So by Harnack's principle u is a harmonic function.

Let $a_1, \cdots, a_n$ be points, fixed in the sequel, in the interior of the bounded domains, bounded by $\Gamma_1, \cdots, \Gamma_n$ respectively. By [7, proof of Theorem 4.2.3, page 80], there are real constants $c_1, \cdots, c_n$ so that $u + \sum_{i=1}^n c_i \log|z - a_i|$ has a (single valued) harmonic conjugate v. By adding a real positive constant C we can assume that the function $\sum_{i=1}^n c_i \log|z - a_i| + C > 0$ for all points in $\overline{A}$. This is clearly possible since the domain is bounded. This gives rise to the analytic function

$$g = \tilde{u} + iv = u + \sum_{i=1}^n c_i \log|z - a_i| + C + iv$$

with positive real part.

Let $k = \dfrac{g}{1+g}$. Since g has positive real part, k is a bounded analytic function and thus has non-tangential limits almost everywhere. We will show that for $m \geq 1$, we have $|1 - k| \leq \dfrac{1}{1 + \frac{m}{3}}$ almost everywhere on E_m.

Pick a point ζ so that both $k = \dfrac{g}{1+g}$ and u_m (which is the Poisson transform of a bounded function) have boundary values at ζ. We will show that $|1 - k(\zeta)| \leq \dfrac{1}{1 + \frac{m}{3}}$. Suppose not, i.e. $|1 - k(\zeta)| > \dfrac{1}{1 + \frac{m}{3}}$. We can pick a non-tangential neighbourhood B of ζ so that

(1) $|1 - k| > \dfrac{1}{1 + \frac{m}{3}}$ on B and

(2) $u_m > \dfrac{2m}{3}$ on B. This is possible since $\lim_{z \in B, z \to \zeta} u_n(z) = m$.

(1) implies the following: Since $|\frac{1}{1+g}| = |1 - \frac{g}{1+g}| = |1-k|$, we see that $|1+g| < 1 + \frac{m}{3}$ on B. Since $Re(g) > 0$, we have that $Re(g) < \frac{m}{3}$. Since, $u = Re(g) - \sum_{i=1}^{n} c_i \log|z - a_i| - C$, we conclude that $u < \frac{m}{3}$ on B.

From (2): Since $u \geq u_m$, we conclude that $u > \frac{2m}{3}$ on B. We have a contradiction.

Thus, $|1-k| \leq \frac{1}{1+\frac{m}{3}}$ almost everywhere on E_m for $m \geq 1$, and hence $|1-\hat{k}| \leq \frac{1}{1+\frac{m}{3}}$ on K_m. This implies that $\hat{k}$ takes value 1 on $\bigcap_n K_n$. Further, since k is not identically 1, it follows from the above lemma that k is 1 on at most a set of m measure zero, and hence $\hat{k}$ is 1 on a set of $\hat{m}$ measure at most 0. Thus, the set of points $F = \{x \in \Omega : \hat{k}(x) = 1\}$ is a set of $\hat{m}$ measure 0 that contains K. Further, it is the peak set for $\frac{1+\hat{k}}{2}$. □

We will use a consequence of this result, Proposition 3.1, in the proof of our main theorem. The proof of this proposition is analogous to a theorem used by Ando in the case of $H^\infty(\mathbb{D})$ to prove uniqueness of the predual, and so we omit the proof.

Proposition 3.1. *If a measure ν on Ω is singular with respect to $\hat{m}$, then there is a compact subset F of Ω and a weakly summable sequence $\{g_n\}$ in H^∞ so that*

$$|\nu|(\Omega \setminus F) = 0, \text{ and}$$

$$\sum_{n=1}^{\infty} \hat{g}_n(\omega) = \chi_F(\omega) \quad (\omega \in \Omega) \quad \text{and} \quad \sum_{n=1}^{\infty} g_n(\zeta) = 0 \text{ a.e. on } \Gamma.$$

We will also use a result by Godefroy and Talagrand (Theorem 4 in [9]).

Theorem 3.2 (Godefroy-Talagrand)**.** *Let $X = Y^*$ be a Banach space. Let $C(Y)$ be the set of all elements $\phi \in X^*$ such that*

$$\sum_n \phi(x_n) = \phi \text{ (weak}^* \text{ limit} \sum_{n=1}^{N} x_n)$$

for every unconditionally summable sequence $(x_n)_n$ in X. Then every predual of X belongs to $C(Y)$. Here, we identify a predual of X with its canonical embedding in $X^{**} = Y^*$

4. The predual of H^∞

Once we have Theorem 3.1, the proof of the following theorem is analogous to that of Ando's. We provide it for the sake of completeness.

Theorem 4.1. *H^∞ has a unique predual.*

Proof. Let Y be a closed subspace of the dual $H^{\infty *}$ so that the unit ball of H^∞ is compact with respect to $\sigma(H^\infty, Y)$. Let ϕ be an arbitrary element in Y and ν be its representing measure. Then if $\nu = \nu_a + \nu_s$ is the Lebesgue decomposition of ν, we write $\phi = \phi_a + \phi_s$ for the corresponding decomposition. Let h be an arbitrary element in H^∞. The proof is complete if we can show that $\phi_s(h) = 0$.

Let E be the support of ν_s in Ω. Since ν_s is singular with respect to $\hat{m}$, we have $\hat{m}(E) = 0$. By Proposition 3.1, there exist a compact set $F \supset E$ with $\hat{m}(F) = 0$ and a weakly summable sequence $\{g_n\}$ in H^∞ so that

$$\mathrm{Supp}(\nu_s) \subset F, \quad \sum_{n=1}^{\infty} \hat{g}_n(\omega) = \chi_F(\omega) \ \text{ for } \omega \in \Omega \quad \text{and} \quad \sum_{n=1}^{\infty} g_n(\zeta) = 0 \text{ a.e. on } \Gamma.$$

Thus,

$$\phi_s(h) = \int_\Omega \hat{h} d\nu_s = \int_\Omega \hat{h}\chi_F d\nu = \int_\Omega \hat{h} \sum_{n=1}^{\infty} \hat{g}_n d\nu.$$

By the dominated convergence theorem, we have

$$\phi_s(h) = \sum_{n=1}^{\infty} \int_\Omega \hat{h}\hat{g}_n d\nu = \sum_{n=1}^{\infty} \phi(hg_n).$$

Since $\sum_1^\infty g_n = 0$, with the limit taken weak*, we have that $\sum_{n=1}^\infty hg_n$ has weak* limit 0 as well. Further, since $\{g_n\}_n$ is unconditionally weakly summable, so is $\{hg_n\}_n$. By Theorem 3.2, the subspace Y, being a predual, belongs to $C({H^\infty}_*)$. Hence, $\sum_{n=1}^\infty \phi(hg_n) = 0$. Then $\phi_s(h) = 0$, which implies that $Y \subset (H^\infty)_*$. □

4.1. **Two theorems on the space $(H^\infty)_*$.** With Theorem 3.1 in hand, the proof of Theorem 2 of Ando's paper [2] applies without a change to give us that the projection P is weak* sequentially continuous and induces an L-projection on $(H^\infty)^*$.

Theorem 4.2. *The predual $(H^\infty)_*$ is the range of a weak* sequentially continuous L-projection.*

Ando proved in [2] that H^∞ does not have a second predual. The same holds in the case of finitely connected domains. Using the version of Amar-Lederer's result in this paper, Theorem 3.1, Ando's proof applies without any other change. We omit the proof.

Theorem 4.3. *H^∞ of a finitely connected domain has no second predual.*

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