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CORRIGENDUM AND IMPROVEMENT TO "CHAOTIC SOLUTION FOR THE BLACK-SCHOLES EQUATION"

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Abstract. We correct an error and improve the main result in our paper *Chaotic solution for the Black-Scholes equation*, Proc. Amer. Math. Soc. **140** (2012), no. 6, 2043–2052.

In the proof of Lemma 3.5 in our paper *Chaotic solution for the Black-Scholes equation*, Proc. Amer. Math. Soc. **140** (2012), no. 6, 2043–2052, the function we denoted by $g(z) = \nu^2 z^2 + (r - \nu^2)z - r$ should have been, according to (3.2), $z^2 + (r/\nu - \nu)z - r$. Thus

$$\operatorname{Re} g(z) = x^2 - y_0^2 + (\frac{r}{\nu} - \nu)x - r = 0$$

with $z = x + \mathrm{i}y_0$. We must find (x, y_0) with $0 < x < \nu s$, $y_0 \in \mathbb{R}$ such that

$$x^2 + (\frac{r}{\nu} - \nu)x - r = y_0^2. \tag{3.3}$$

Call $\mathscr{C}$ the curve represented by the graph of the quadratic function $y = x^2 + (\frac{r}{\nu} - \nu)x - r$. As Figure 1′ shows, for $\nu < x < \nu s$, there are uncountably many points (x, y) on the dashed portion of $\mathscr{C}$ with $y > 0$. For each such point let $y_0 = \sqrt{y}$. This gives uncountably many solutions of (3.3).

With this correction in the proof, our main results, Theorems 3.6 and 3.7, have the same conclusions under weaker hypotheses. The following is a precise statement of this.

Theorem 3.6′. *Let $s > 1$, $\tau \geq 0$ and define the complex Banach space*

$$Y^{s,\tau} := \{u \in C(0,\infty) : \lim_{x\to 0} \left|u(x)/(1 + x^{-\tau})\right| = \lim_{x\to 0} |u(x)/(1 + x^s)| = 0\}$$

with norm

$$\|u\|_{s,\tau} = \sup_{x>0} \left|u(x)/[(1 + x^{-\tau})(1 + x^s)]\right|.$$

The Black-Scholes equation

$$\partial v/\partial t = (\sigma^2/2)x^2\partial^2 v/\partial x^2 + rx\partial v/\partial x - rv,$$

for $\sigma > 0$, $r > 0$, is governed by a (C_0) semigroup $T = \{T(t) : t \geq 0\}$ on $Y^{s,\tau}$. This semigroup is chaotic. If $Y^{s,\tau}_{\mathbb{R}}$ consists of the real functions in $Y^{s,\tau}$, then S_T, the restriction of T to $Y^{s,\tau}_{\mathbb{R}}$, is a chaotic (C_0) semigroup.

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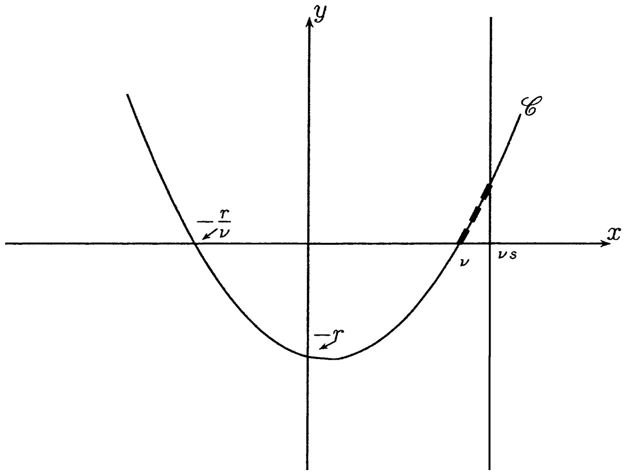

FIGURE 1′.

Thus we do not need the assumption $\sigma s > \sqrt{2}$, which was made in the original Theorems 3.6 and 3.7. Consequently the choice of spaces for the chaosity of the Black-Scholes semigroup is independent of the volatility σ, which gives a conceptually cleaner and improved result.

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