CORRIGENDUM AND IMPROVEMENT TO “CHAOTIC SOLUTION FOR THE BLACK-SCHOLES EQUATION”

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Abstract. We correct an error and improve the main result in our paper Chaotic solution for the Black-Scholes equation, Proc. Amer. Math. Soc. 140 (2012), no. 6, 2043–2052.

In the proof of Lemma 3.5 in our paper Chaotic solution for the Black-Scholes equation, Proc. Amer. Math. Soc. 140 (2012), no. 6, 2043–2052, the function we denoted by $g(z) = \nu^2 z^2 + (r - \nu^2)z - r$ should have been, according to (3.2), $z^2 + (r/\nu - \nu)z - r$. Thus

$$\text{Re} \ g(z) = x^2 - y_0^2 + \left(\frac{r}{\nu} - \nu\right)x - r = 0$$

with $z = x + iy_0$. We must find $(x, y_0)$ with $0 < x < \nu s$, $y_0 \in \mathbb{R}$ such that

$$x^2 + \left(\frac{r}{\nu} - \nu\right)x - r = y_0^2. \tag{3.3}$$

Call $C$ the curve represented by the graph of the quadratic function $y = x^2 + \left(\frac{r}{\nu} - \nu\right)x - r$. As Figure 1 shows, for $\nu < x < \nu s$, there are uncountably many points $(x, y)$ on the dashed portion of $C$ with $y > 0$. For each such point let $y_0 = \sqrt{y}$. This gives uncountably many solutions of (3.3).

With this correction in the proof, our main results, Theorems 3.6 and 3.7, have the same conclusions under weaker hypotheses. The following is a precise statement of this.

Theorem 3.6'. Let $s > 1$, $\tau \geq 0$ and define the complex Banach space

$$Y^{s,\tau} := \{u \in C(0, \infty) : \lim_{x \to 0} \frac{|u(x)/(1 + x^{-\tau})|}{u(x)/(1 + x^s)} = 0\}$$

with norm

$$\|u\|_{s,\tau} = \sup_{x > 0} \frac{|u(x)/(1 + x^{-\tau})(1 + x^s)|}{u(x)/(1 + x^{-\tau})}.$$

The Black-Scholes equation

$$\frac{\partial v}{\partial t} = \left(\sigma^2/2\right)x^2 \frac{\partial^2 v}{\partial x^2} + rx \frac{\partial v}{\partial x} - rv,$$

for $\sigma > 0$, $r > 0$, is governed by a $(C_0)$ semigroup $T = \{T(t) : t \geq 0\}$ on $Y^{s,\tau}$. This semigroup is chaotic. If $Y^{s,\tau}_{\mathbb{R}}$ consists of the real functions in $Y^{s,\tau}$, then $S_T$, the restriction of $T$ to $Y^{s,\tau}_{\mathbb{R}}$, is a chaotic $(C_0)$ semigroup.

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Thus we do not need the assumption $\sigma s > \sqrt{2}$, which was made in the original Theorems 3.6 and 3.7. Consequently the choice of spaces for the chaosy of the Black-Scholes semigroup is independent of the volatility $\sigma$, which gives a conceptually cleaner and improved result.

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