PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 143, Number 1, January 2015, Pages 219–225
S 0002-9939(2014)12226-X
Article electronically published on September 15, 2014

ON PROPER HOLOMORPHIC MAPPINGS AMONG IRREDUCIBLE BOUNDED SYMMETRIC DOMAINS OF RANK AT LEAST 2

SUI-CHUNG NG

(Communicated by Franc Forstneric)

ABSTRACT. We give a characterization for totally geodesic embeddings between higher-rank irreducible bounded symmetric domains in terms of certain totally geodesic rank-1 symmetric subspaces. More explicitly, we prove that for two irreducible bounded symmetric domains Ω_1, Ω_2 of rank at least 2, a holomorphic map $F : \Omega_1 \to \Omega_2$ is a totally geodesic embedding with respect to the Bergman metrics if F maps the minimal disks of Ω_1 into rank-1 characteristic symmetric subspaces of Ω_2. As a simple corollary, we obtain a much simpler proof for a theorem of Tsai which says that F is totally geodesic if F is proper and $\text{rank}(\Omega_1) = \text{rank}(\Omega_2)$.

1. INTRODUCTION

Proper holomorphic mappings among domains on Euclidean spaces is a classical topic in Several Complex Variables. The literature dates back to the earliest results such as the theorem of H. Alexander [1], which says that any proper holomorphic self-map of the complex unit n-ball is a biholomorphism if $n \geq 2$. Since then, the study of the proper holomorphic mappings between complex unit balls of different dimensions has become a very popular topic in the field. Important input from various perspectives, such as Algebraic Geometry, Chern-Moser Theory, Segre variety and Bergman kernel, etc., has been made. It is apparent by now that the complexity of the problem grows with the codimension and one in general must impose certain regularity assumptions on the proper maps in order to give any satisfactory classification.

Comparing with those of the complex unit balls, the proper holomorphic mapping problems for irreducible bounded symmetric domains of higher rank are of a very different nature. On the one hand, the methods in the rank-1 case find limited applicability on the higher-rank cases due to the vast difference in their boundary structures. While the boundary of a complex unit ball is a smooth strictly pseudoconvex hypersurface defined by a simple real analytic equation in a Euclidean space, the boundary of a higher-rank irreducible bounded symmetric domain is non-smooth and contains complex analytic submanifolds. On the other hand, in the higher-rank cases, it appears that it is the rank difference which defines the difficulty of the problem rather than the codimension. This can be illustrated by, for example, the following statement which was originally conjectured by Mok and later proven by Tsai [7].

Received by the editors October 15, 2012 and, in revised form, March 20, 2013.
2010 *Mathematics Subject Classification.* Primary 32H35, 32M15.

Theorem 1.1 (Tsai). *Let Ω_1, Ω_2 be two irreducible bounded symmetric domains and $F : \Omega_1 \to \Omega_2$ be a proper holomorphic map. If* $\mathrm{rank}(\Omega_1) \geq \mathrm{rank}(\Omega_2) \geq 2$*, then F is a totally geodesic isometric embedding (up to a normalization constant) with respect to the Bergman metrics.*

The above theorem generalizes the classical result of Tumanov-Khenkin [10] in the special case where $\Omega_1 = \Omega_2$. The proof given in [7] is sophisticated and involves a very careful analysis of the infinitesimal behavior of the map F and a complete classification of the invariantly geodesic subspaces of irreducible bounded symmetric domains. While many of these intermediate results such as the classification of invariantly geodesic subspaces are of great significance to the theory of bounded symmetric domains, it is still worth the effort to look for a simpler and more direct proof of Theorem 1.1. To this end, we first remark that the major goal behind Tsai's proof is to show that F maps minimal disks of Ω_1 properly into minimal disks of Ω_2, and after this a simple argument will lead to the desired result. This approach has also been adopted in the subsequent works of Tu ([9],[8]) on the rigidity of proper holomorphic maps among higher-rank irreducible bounded symmetric domains. The purpose of the current article is to demonstrate that the total geodesy of a map between two higher-rank irreducible bounded symmetric domains can also be characterized by a weaker condition as follows.

Proposition 1.2. *Let Ω_1, Ω_2 be irreducible bounded symmetric domains of rank at least* 2 *and let $F : \Omega_1 \to \Omega_2$ be a holomorphic map. If F maps the minimal disks of Ω_1 properly into the rank-*1 *characteristic symmetric subspaces of Ω_2, then F is a totally geodesic isometric embedding (up to a normalization constant) with respect to the Bergman metrics.*

When F is a proper and $\mathrm{rank}(\Omega_1) \geq \mathrm{rank}(\Omega_2) \geq 2$, it is much easier to verify the hypotheses of the above proposition than to show that F maps minimal disks into minimal disks. Thus, by proving the above proposition, we will obtain a much simpler proof for Theorem 1.1. The rest of the article will be devoted to this task.

2. Background and preliminaries

2.1. **Characteristic symmetric subspace.** To make this article more self-contained, in the following we gather some relevant background material on the geometry of bounded symmetric domains of rank at least 2. For more detail, we refer the readers to [5] (Section 1 and the references therein).

Let Ω be an irreducible bounded symmetric domain and $\mathrm{rank}(\Omega) = r$. It is well known that there are totally geodesic complex submanifolds of Ω holomorphically isometric to the r-disk (up to normalizing constants) and any such polydisk is maximal in dimension. If we write $\Omega = G/K$, where G is the identity component of the group of holomorphic isometries of Ω and K is the isotropy subgroup at the origin, then for a fix totally geodesic r-disk Δ^r passing through the origin we have $\Omega = \bigcup_{k \in K} k \cdot \Delta^r$. Now let $T_x^{(1,0)}(\Omega)$ be the holomorphic tangent space at an arbitrary point $x \in \Omega$. A vector in $T_x^{(1,0)}(\Omega)$ tangent to any direct factor of a totally geodesic r-disk is called a *characteristic vector* at x.

Let $\mathfrak{g}_o = \mathfrak{k} + \mathfrak{m}_o$ be the orthogonal symmetric Lie algebra associated to the Riemannian symmetric space $\Omega = G/K$. We write its complexification as $\mathfrak{g} = \mathfrak{k}^{\mathbb{C}} + \mathfrak{m}$ and let $\mathfrak{m} = \mathfrak{m}^+ + \mathfrak{m}^-$ be the eigenspace decomposition of $\mathrm{ad}(z)$, where z is in the center of $\mathfrak{k}$ representing the complex structure of Ω. As usual, we make the identification $\mathfrak{m}^+ = T_o^{(1,0)}(\Omega)$, where $o \in \Omega$ is the origin. For any proper subset $S \subset T_o^{(1,0)}(\Omega)$, we let $\mathcal{N}_S \subset T_o^{(1,0)}(\Omega)$ be the subset such that the holomorphic bisectional curvature $R_{\eta\bar{\eta}\xi\bar{\xi}} = 0$ for every $\eta \in S$ and $\xi \in \mathcal{N}_S$. Since Ω has non-positive holomorphic bisectional curvature, it follows that $\mathcal{N}_S$ is a complex vector subspace. Using the curvature formula for symmetric spaces and Jacobi identity, one further sees that, as a complex vector subspace of $\mathfrak{m}^+$, $\mathcal{N}_S$ forms a (complex) Lie triple system and hence is the holomorphic tangent space of a unique totally geodesic Hermitian symmetric subspace $\Omega_S \subset \Omega$.

Now take a totally geodesic r-disk $\Delta^r \subset \Omega$ passing through the origin and decompose it as $\Delta^k \times \Delta^{r-k}$, where $1 \le k \le r-1$. Then $\Omega_k := \Omega_{T_o^{(1,0)}(\Delta^k \times \{0\})}$ is known as a *characteristic symmetric subspace* passing through the origin. It turns out that it is also an irreducible bounded symmetric domain, and it is not difficult to see that there are totally geodesic embeddings $\Delta^k \times \Delta^{r-k} \hookrightarrow \Delta^k \times \Omega_k \hookrightarrow \Omega$. Thus, $\mathrm{rank}(\Omega_k) = r-k$. The characteristic symmetric subspaces of Ω_k are certainly also characteristic symmetric subspaces of Ω. In general, a characteristic symmetric subspace of Ω is just the image of Ω_k under any automorphism of Ω. When $k = 1$, we call such a subspace a *maximal characteristic symmetric subspace* of Ω.

The above geometric objects canonically associated to Ω have good realizations under the Harish-Chandra coordinates for $\Omega \Subset \mathbb{C}^n$. In particular, for every $k \in \{1, \ldots, r-1\}$, we can choose Harish-Chandra coordinates $(z_1, \ldots, z_n)$ such that

$$\{(z_1, \ldots, z_k, z_{k+1}, \ldots, z_{k+m}, 0, \ldots, 0) \in \Omega\}$$

is the image of the totally geodesic embedding $\Delta^k \times \Omega_k \hookrightarrow \Omega$ described above, where $m = \dim(\Omega_k)$. One can obtain these realizations from the construction of characteristic symmetric subspaces using root systems as in [11]. Finally, let $\mathbb{B}^m \subset \Omega$ be a rank-1 characteristic symmetric subspace passing through the origin. Then we can choose Harish-Chandra coordinates such that $\mathbb{B}^m = \{(z_1, \ldots, z_m, 0, \ldots, 0) \in \Omega\}$ and for the canonical projection onto the first m coordinates $\pi_m : \mathbb{C}^n \to \mathbb{C}^m$, we have $\pi_m(\Omega) = \mathbb{B}^m$. (The proof of this is parallel to that of Lemma 2.2.2. in [4] using Hermann Convexity Theorem [2].)

We now use Type-I domains to illustrate the above notions. Let $M(p, q; \mathbb{C})$ be the set of p-by-q complex matrices and use $(\cdot)^H$ to denote Hermitian conjugation. The Type-I irreducible bounded symmetric domains can be defined as $D_{p,q} = \{Z \in M(p, q; \mathbb{C}) : ZZ^H < I\}$. Since $D_{p,q} \cong D_{q,p}$, we may assume that $p \le q$. Then $\mathrm{rank}(D_{p,q}) = p$ and a totally geodesic p-disk through the origin is

$$\begin{bmatrix} z_1 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & z_2 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & z_p & 0 & \cdots & 0 \end{bmatrix}.$$

For $k \in \{1, \ldots, p-1\}$, the totally geodesic embedding $\Delta^k \times \Omega_k \hookrightarrow D_{p,q}$ has the image

$$\begin{bmatrix} z_1 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & \cdots & z_k & 0 & \cdots & 0 \\ 0 & \cdots & 0 & z_{1,1} & \cdots & z_{1,q-k} \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & 0 & z_{p-k,1} & \cdots & z_{p-k,q-k} \end{bmatrix}.$$

Here we have $\Omega_k \cong D_{p-k,q-k}$.

2.2. **Bounded holomorphic functions.** In what follows, for every $a \in \mathbb{C}^n$ and $r > 0$, we write

$$B(a, r) := \{z \in \mathbb{C}^n : \|z - a\| < r\}.$$

We also denote the set of bounded holomorphic functions on a complex manifold X by $H^\infty(X)$.

Let $D \subset \mathbb{C}^n$ be a domain. A point $b \in \partial D$ is said to be a *local peak point* (Rudin [6]) of D if there is an $r > 0$ and a function h, holomorphic in $D \cap B(b, r)$, continuous on $\overline{D \cap B(b, r)}$ such that $h(b) = 1$ but $|h(z)| < 1$ for every $z \in \overline{D \cap B(b, r)} - \{b\}$.

We have the following lemma by the maximal principle [6].

Lemma 2.1. *Let $D \Subset \mathbb{C}^n$ be a bounded domain and $b \in \partial D$ be a local peak point of D. Suppose $\{F_i\}$ is a sequence of holomorphic maps from a domain $U \subset \mathbb{C}^k$ into D and there is a point $z_0 \in U$ such that $F_i(z_0) \to b$ as $i \to \infty$. Then $F_i(z) \to b$ uniformly on every compact subset of U.*

We need another lemma which is proven in [5] using Fatou's theorem on the existence of radial limits for bounded holomorphic functions.

Lemma 2.2. *Let Δ be the unit disk and $W \Subset \mathbb{C}^n$ be a bounded domain. Write the coordinates for a point $p \in \Delta \times W$ as $p = (z, w)$. Suppose $f \in H^\infty(\Delta \times W)$, then for almost every $b \in \partial\Delta$, we have that*

$$f_b(w) = \lim_{r \to 1^-} f(rb, w)$$

exists for every $w \in W$. Moreover, $f_b \in H^\infty(W)$. If f_b is constant for $b \in E \subset \partial\Delta$, where E is of positive measure with respect to the measure of $\partial\Delta$, then f is independent on w.

It is well known that a polydisk cannot be properly mapped into any complex unit ball. We follow the idea of the proof of this fact and obtain the following:

Proposition 2.3. *Let $W \Subset \mathbb{C}^n$ be a bounded domain and $F : \Delta \times W \longrightarrow \mathbb{B}^k$ be a holomorphic map such that $F(z, 0) : \Delta \longrightarrow \mathbb{B}^k$ is proper. Then $F(z, w) = F(z, 0)$.*

Proof. As all the component functions of F are in $H^\infty(\Delta \times W)$, by Lemma 2.2, for almost every $b \in \partial\Delta$,

$$F_b(w) = \lim_{r \to 1^-} F(rb, w)$$

exists for every $w \in W$ and $F_b(w)$ is a holomorphic map from W into $\mathbb{C}^k$. Fix one $b \in \partial\Delta$ such that F_b exists. As $F(z, 0)$ is proper, we have $F(rb, 0) \to \zeta(b)$ for some $\zeta(b) \in \partial\mathbb{B}^n$ as $r \to 1^-$. Since every boundary point of $\mathbb{B}^k$ is a local peak point, by Lemma 2.1, $F_b(w) \equiv \zeta(b)$. Hence we have $F(z, w) = F(z, 0)$ by Lemma 2.2. $\square$

3. Proof of Proposition 1.2

Let Ω_1, Ω_2 be irreducible bounded symmetric domains of rank at least 2.

Lemma 3.1. *Let $F : \Omega_1 \to \Omega_2$ be a holomorphic map such that F maps minimal disks of Ω_1 properly into rank-1 characteristic symmetric subspaces of Ω_2. Let $\mu, \nu \in T_p^{(1,0)}(\Omega_1)$ be two characteristic vectors at $p \in \Omega_1$ such that $R^{(1)}_{\mu\bar{\mu}\nu\bar{\nu}} = 0$, where $R^{(1)}_{\alpha\bar{\beta}\gamma\bar{\delta}}$ is the curvature tensor of Ω_1. Then $g_2(dF(\mu), \overline{dF(\nu)}) = 0$, where g_2 is the Bergman metric of Ω_2.*

Proof. Without loss of generality, we may take p to be the origin and assume that $F(0) = 0$. Since μ, ν are characteristic and $R^{(1)}_{\mu\bar{\mu}\nu\bar{\nu}} = 0$, we can find a totally geodesic two-disk $\Delta^2 \subset \Omega_1$ and choose a coordinate system (z, w) of Δ^2 which is the restriction of some choice of Harish-Chandra coordinates of Ω_1 such that, $\Delta_\mu := \{(z, 0) \in \Delta^2\}$, $\Delta_\nu := \{(0, w) \in \Delta^2\}$ are minimal disks of Ω_1 and μ, ν are tangent to Δ_μ and Δ_ν respectively. We may simply take $\mu = \frac{\partial}{\partial z}(0)$ and $\nu = \frac{\partial}{\partial w}(0)$.

By hypotheses we have $F(\Delta_\mu) \subset B \subset \Omega_2$, where $B \cong \mathbb{B}^k$, $k \in \mathbb{N}^+$, is a rank-1 characteristic symmetric subspace of Ω_2. By applying an automorphism in Ω_2, we may assume that $B = \{(z_1, \ldots, z_k, 0, \ldots, 0) \in \Omega_2 : |z_1|^2 + \cdots + |z_k|^2 < 1\}$, where $(z_1, \ldots, z_k, \ldots, z_m)$ are Harish-Chandra coordinates of $\Omega_2 \Subset \mathbb{C}^m$. We also have $\pi(\Omega_2) = \mathbb{B}^k$, where $\pi : \mathbb{C}^m \to \mathbb{C}^k$ is the canonical projection to the first k direct factors (see Section 2.1). Therefore if we write $F = (f_1, \ldots, f_k, \ldots, f_m)$, and $F_k := (f_1, \ldots, f_k)$, then the restriction $F_k|_{\Delta_\mu} : \Delta_\mu \to \mathbb{B}^k$ is a proper holomorphic map.

Now, by Proposition 2.3, we have $F_k(z, w) = F_k(z, 0)$, i.e. $f_j(z, w) = f_j(z, 0)$ for $1 \le j \le k$ and this implies that $f_j(0, w) \equiv 0$ for $1 \le j \le k$. Hence, we have $F(z, 0) = (f_1(z, 0), \ldots, f_k(z, 0), 0, \ldots, 0)$ and $F(0, w) = (0, \ldots, 0, f_{k+1}(0, w), \ldots, f_m(0, w))$. It then follows that $g_{\mathbb{C}^m}(dF(\mu), \overline{dF(\nu)}) = 0$, where $g_{\mathbb{C}^m}$ is the Euclidean metric. But the Bergman metric g_2 of Ω_2 agrees with $g_{\mathbb{C}^m}$ at the origin and thus the lemma follows. □

We are now ready to prove our main proposition.

Proof of Proposition 1.2. Let g_1 and g_2 be the Bergman metrics of Ω_1 and Ω_2 respectively. We are going to show that $F^*g_2 = cg_1$ for some $c > 0$.

Take two distinct unit-length characteristic vectors $\mu, \nu \in T_0^{(1,0)}(\Omega_1)$ at the origin such that $R^{(1)}_{\mu\bar{\mu}\nu\bar{\nu}} = 0$. Choose Harish-Chandra coordinates $(z_1, \ldots, z_n)$ on $\Omega_1 \Subset \mathbb{C}^n$ such that $\mu = \frac{\partial}{\partial z_1}(0)$, $\nu = \frac{\partial}{\partial z_2}(0)$ and $(z_1, z_2, 0, \ldots, 0)$ is a totally geodesic two-disk $\Delta^2 \subset \Omega_1$ as in Lemma 3.1.

Let $h = F^*g_2$ and write $h_{i\bar{j}} = h(\frac{\partial}{\partial z_i}, \frac{\partial}{\partial \bar{z}_j})$. Note that as for μ and ν, the vector fields $\frac{\partial}{\partial z_1}$ and $\frac{\partial}{\partial z_2}$ give a zero bisectional curvature everywhere in Δ^2 and thus, by Lemma 3.1, we have $h_{1\bar{2}} = h_{2\bar{1}} \equiv 0$ on Δ^2. Since h is Kähler, it follows that $\frac{\partial h_{1\bar{1}}}{\partial z_2} = \frac{\partial h_{2\bar{1}}}{\partial z_1} \equiv 0$ on Δ^2. Let R^h be the curvature tensor of h. By direct computation, we have

$$R^h_{1\bar{1}2\bar{2}} = -\frac{\partial^2 h_{1\bar{1}}}{\partial z_2 \partial \bar{z}_2} + \sum_{1\le i,j\le n} h^{i\bar{j}} \frac{\partial h_{1\bar{j}}}{\partial z_2} \frac{\partial h_{i\bar{1}}}{\partial \bar{z}_2}.$$

Since h has non-positive bisectional curvature, it follows that

$$\begin{aligned} 0 \ge R^h_{1\bar{1}2\bar{2}}(0) &= -\frac{\partial^2 h_{1\bar{1}}}{\partial z_2 \partial \bar{z}_2}(0) + \sum_{1\le i,j\le n} h^{i\bar{j}} \frac{\partial h_{1\bar{j}}}{\partial z_2} \frac{\partial h_{i\bar{1}}}{\partial \bar{z}_2}\Bigg|_{z=0} \\ &= \sum_{1\le i,j\le n} h^{i\bar{j}} \frac{\partial h_{1\bar{j}}}{\partial z_2} \frac{\partial h_{i\bar{1}}}{\partial \bar{z}_2}\Bigg|_{z=0} \ge 0. \end{aligned}$$

Hence, $\dfrac{\partial h_{1\bar{j}}}{\partial z_2}(0) = 0$ for every j. As Harish-Chandra coordinates are complex geodesic coordinates at the origin, we have for every $\eta \in T_0^{(1,0)}(\Omega_1)$,

$$\nabla_\mu h_{\nu\bar{\eta}}(0) = 0.$$

By a polarization argument as given by Mok ([3], Chapter 6), the complex vector space $T_0^{(1,0)}(\Omega_1) \otimes T_0^{(1,0)}(\Omega_1)$ is spanned by its elements of the form $\xi \otimes \zeta$, where ξ, ζ are characteristic and $R_{\xi\bar{\xi}\zeta\bar{\zeta}} = 0$. We therefore can conclude that $\nabla h = 0$ at the origin. By exploiting the homogeneity, we deduce furthermore that $\nabla h \equiv 0$, i.e. h is parallel and hence $h = cg_1$ for some $c \ge 0$ as Ω_1 is irreducible. Since F is non-constant, we have $c \ne 0$ and hence F is an isometric embedding up to a normalizing constant. By the preservation of the zeros of the bisectional curvature by a holomorphic isometric embedding between bounded symmetric domains, it follows easily that the associated second fundamental form is identically zero and hence the embedding is totally geodesic. The proof is complete. □

Theorem 1.1 is now a simple corollary.

Proof of Theorem 1.1. If $F : \Omega_1 \to \Omega_2$ is a proper holomorphic map, then F maps maximal characteristic symmetric subspaces of Ω_1 into maximal characteristic symmetric subspaces of Ω_2. (This fact has been essentially proven in [5] and later explicitly stated in [7].) Since every maximal characteristic symmetric subspace is of rank one less than that of the ambient space, it follows by induction that F maps rank-1 characteristic symmetric subspaces of Ω_1 into rank-1 characteristic symmetric subspaces of Ω_2. But every minimal disk of Ω_1 is contained in some rank-1 characteristic symmetric subspace and therefore F satisfies the hypotheses of Proposition 1.2 and hence is a totally geodesic isometric embedding up to a normalization constant. □

Acknowledgements

The essential part of this research was done when the author was a postdoctoral fellow at The University of Hong Kong. He would like to thank Professor Ngaiming Mok for many helpful discussions, especially for pointing out how to use Kählerness in the last part of the proof for Proposition 1.2.

References

[1] H. Alexander, *Holomorphic mappings from the ball and polydisc*, Math. Ann. **209** (1974), 249–256. MR0352531 (50 #5018)

[2] Robert Hermann, *Geometric aspects of potential theory in the symmetric bounded domains. II*, Math. Ann. **151** (1963), 143–149. MR0155342 (27 #5276)

[3] Ngaiming Mok, *Metric rigidity theorems on Hermitian locally symmetric manifolds*, Series in Pure Mathematics, vol. 6, World Scientific Publishing Co. Inc., Teaneck, NJ, 1989. MR1081948 (92d:32046)

[4] Ngaiming Mok and Sui Chung Ng, *Germs of measure-preserving holomorphic maps from bounded symmetric domains to their Cartesian products*, J. Reine Angew. Math. **669** (2012), 47–73. MR2980451

[5] Ngaiming Mok and I Hsun Tsai, *Rigidity of convex realizations of irreducible bounded symmetric domains of rank* ≥ 2, J. Reine Angew. Math. **431** (1992), 91–122. MR1179334 (93h:32046)

[6] Walter Rudin, *Function theory in the unit ball of* $\mathbf{C}^n$, Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Science], vol. 241, Springer-Verlag, New York, 1980. MR601594 (82i:32002)

[7] I Hsun Tsai, *Rigidity of proper holomorphic maps between symmetric domains*, J. Differential Geom. **37** (1993), no. 1, 123–160. MR1198602 (93m:32038)

[8] Zhen-Han Tu, *Rigidity of proper holomorphic mappings between nonequidimensional bounded symmetric domains*, Math. Z. **240** (2002), no. 1, 13–35, DOI 10.1007/s002090100353. MR1906705 (2003g:32034)

[9] Zhen-Han Tu, *Rigidity of proper holomorphic mappings between equidimensional bounded symmetric domains*, Proc. Amer. Math. Soc. **130** (2002), no. 4, 1035–1042 (electronic), DOI 10.1090/S0002-9939-01-06383-3. MR1873777 (2003a:32027)

[10] A. E. Tumanov and G. M. Khenkin, *Local characterization of analytic automorphisms of classical domains* (Russian), Dokl. Akad. Nauk SSSR **267** (1982), no. 4, 796–799. MR681032 (85b:32048)

[11] Joseph A. Wolf, *Fine structure of Hermitian symmetric spaces*, Symmetric spaces (Short Courses, Washington Univ., St. Louis, Mo., 1969–1970), Pure and App. Math., Vol. 8, Dekker, New York, 1972, pp. 271–357. MR0404716 (53 #8516)

Department of Mathematics, Temple University, 1805 N. Broad Street, Philadelphia, Pennsylvania 19122

E-mail address: `scng@temple.edu`