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A GENERALIZATION OF THE FORELLI-RUDIN CONSTRUCTION AND DEFLATION IDENTITIES

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Abstract. We establish a series representation formula of the Bergman kernel of a certain class of domains, which generalizes the Forelli-Rudin construction of the Hartogs domain. Our formula is applied to derive deflation type identities of the Bergman kernels for our domains.

1. Introduction

The main purpose of this paper is to generalize the Forelli-Rudin construction [14], which has a direct application to the *deflation identity* initiated by Boas-Fu-Straube [1]. We start this section with the motivational background.

1.1. **Forelli-Rudin construction.** Let D be a domain in $\mathbb{C}^n$ and φ a positive continuous function on D. Then the Bergman kernel of D is defined as the reproducing kernel of the space of square integrable holomorphic functions on D. As is known from [14] for the Hartogs domain $D_{m,\varphi} = \{(z,\zeta) \in D \times \mathbb{C}^m; \|\zeta\|^2 < \varphi(z)\}$, the Bergman kernel $K_{D_{m,\varphi}}$ has the following series representation formula (see also [18]):

$$K_{D_{m,\varphi}}((z,\zeta),(z',\zeta')) = \frac{1}{\pi^m}\sum_{k=0}^{\infty}(k+1)_m K_{D,\varphi^{k+m}}(z,z')\langle \zeta,\zeta'\rangle^k. \tag{1.1}$$

Here $K_{D,\varphi^{k+m}}$ is the weighted Bergman kernel of D with respect to φ^{k+m} and $(x)_m$ is the Pochhammer symbol. Since it was first proved by F. Forelli and W. Rudin [7] for the unit disk $D = \mathbb{D}$ and $\varphi(z) = 1 - |z|^2$, it is called the Forelli-Rudin construction. This formula plays a critical role in establishing explicit formulas of the Bergman kernels of some Hartogs domains (see [21], [22] and [23]). It is also effective in the study of the presence or absence of zeroes of the Bergman kernel, which is called the Lu Qi-Keng problem (see [3] and [24]). In [5], M. Engliš and G. Zhang obtained a generalization of the Forelli-Rudin construction for the following domain:

$$D^{\mathcal{F}}_{\varphi} = \{(z,\zeta) \in D \times \mathbb{C}^m; \varphi(z)^{-1/2}\zeta \in \mathcal{F}\},$$

where $\mathcal{F}$ is an irreducible bounded symmetric domain.

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In this paper we generalize this result. More precisely, the present paper generalizes the Forelli-Rudin construction to the domains (Theorem 3.1)

$$D^{\varphi}_{P,m} := \{(z,\zeta) \in D \times \mathbb{C}^m; P(|\zeta_1|^2, \cdots, |\zeta_m|^2) < \varphi(z)\},$$

for every quasi-homogeneous function P.

1.2. **Deflation identity.** A generalization of the Forelli-Rudin construction is interesting in its own right. Moreover, we emphasize that our formula enables us to generalize the deflation identity initiated by Boas-Fu-Straube [1], and it provides an additional motivation for our study.

Let Ω be a bounded domain in $\mathbb{C}^n$ which is given by an inequality $\psi(z) < 1$. Consider the following two domains:

$$\Omega_1 := \{(z,\zeta) \in \Omega \times \mathbb{C}; |\zeta|^{2/(q_1+q_2)} < 1 - \psi(z)\},$$
$$\Omega_2 := \{(z,\zeta) \in \Omega \times \mathbb{C}^2; |\zeta_1|^{2/q_1} + |\zeta_2|^{2/q_2} < 1 - \psi(z)\},$$

where q_1, q_2 are positive real numbers. Then the following identity was proved in [1]:

$$c_1 K_{\Omega_1}((z,0),(z',0)) = c_2 K_{\Omega_2}((z,0,0),(z',0,0)), \tag{1.2}$$

where c_1 and c_2 are constants. This identity is called the deflation identity. The proof is based on the fact that both sides of (1.2) represent the same weighted Bergman kernel $K_{\Omega,(1-\psi)^{q_1+q_2}}$. The deflation identity is an effective tool in the study of the Lu Qi-Keng problem (for details, see [1]).

In the case of the Hartogs domains, due to the Forelli-Rudin construction, the restriction of $K_{D_{m,\varphi}}$ to the subspace $\{\zeta, \zeta' = 0\}$ coincides with $K_{D,\varphi^m}(z,z')$ (up to a constant). This kind of property also holds for our domain $D^{\varphi}_{P,m}$ and it allows us to study deflation identities for $D^{\varphi}_{P,m}$.

The deflation identity (1.2) relates the Bergman kernels of domains in different dimensions, while our deflation identity also relates those of domains in the same dimension. For instance, the restrictions of the Bergman kernels $K_{\Omega_1}, K_{\Omega_2}$ to the subspace $\{\zeta, \zeta' = 0\}$ coincides, up to a constant multiple, with those of the following domains Ω_3, Ω_4:

$$\Omega_3 := \{(z,\zeta) \in \Omega \times \mathbb{C}^2; |\zeta_1|^2 + |\zeta_2|^{2/q_2} < (1-\psi(z))^{\frac{q_1+q_2}{1+q_2}}\},$$
$$\Omega_4 := \{(z,\zeta) \in \Omega \times \mathbb{C}^2; |\zeta_1|^{2/q_1} + |\zeta_2|^{2/q_1} < (1-\psi(z))^{\frac{q_1+q_2}{2q_1}}\}.$$

1.3. **Organization of the paper.** Subsequent to this introduction, Section 2 provides some basic properties of the Bergman kernel and the Reinhardt domain $D^m_P(r)$. Using the results given in Section 2, we prove our main result (Theorem 3.1) in Section 3. In Section 4, by using our Forelli-Rudin type formula, we study deflation identities for $D^{\varphi}_{P,m}$.

2. Preliminaries

2.1. **Bergman kernel.** In this section, we collect some basic properties of the Bergman kernel. Let D be a domain in $\mathbb{C}^n$, φ a positive continuous function on D and $L^2_a(D,\varphi)$ the Hilbert space of square integrable holomorphic functions with respect to the weight function φ on D with the inner product

$$\langle f, g\rangle = \int_D f(z)\overline{g(z)}\varphi(z)dV(z), \quad \text{for all } f,g \in \mathcal{O}(D).$$

The weighted Bergman kernel $K_{D,\varphi}$ of D with respect to the weight φ is the reproducing kernel of $L^2_a(D,\varphi)$. Namely, the weighted Bergman kernel $K_{D,\varphi}$ has the following reproducing property:

$$f(z) = \int_D K_{D,\varphi}(z,w) f(w)\varphi(w)dV(w), \quad \text{for all } f \in L^2_a(D,\varphi).$$

If $\varphi \equiv 1$, the reproducing kernel is called the (unweighted) Bergman kernel. Let $f : D_1 \to D_2$ be a biholomorphic map between two domains D_1 and D_2 in $\mathbb{C}^n$. Then the following transformation law is known:

$$K_{D_1}(z,w) = \det f'(z) K_{D_2}(f(z), f(w)) \overline{\det f'(w)}, \tag{2.1}$$

for all $z, w \in D_1$, where $f'(z)$ is the complex Jacobian of f. Let $\{f_\alpha\}$ be a complete orthonormal basis for $L^2_a(D,\varphi)$; then the kernel $K_{D,\varphi}$ is computed by

$$K_{D,\varphi}(z,w) = \sum_\alpha f_\alpha(z)\overline{f_\alpha(w)}, \tag{2.2}$$

which is independent of the choice of basis. Further information about the Bergman kernel can be found in [10, Chapter 3] and [13, Chapter 1].

2.2. **Reinhardt domain $D^m_P(r)$.** In this section we study a certain Reinhardt domain $D^m_P(r)$. We start this section with basic definitions.

Definition 2.1. A domain D in $\mathbb{C}^m$ is called a Reinhardt domain if it is invariant under the rotations:

$$(\zeta_1, \cdots, \zeta_m) \mapsto (e^{i\theta_1}\zeta_1, \cdots, e^{i\theta_m}\zeta_m), \quad (\theta_1, \cdots, \theta_m) \in \mathbb{R}^m.$$

Moreover a Reinhardt domain D is called complete if, whenever the point $(\zeta_1, \cdots, \zeta_m) \in D$, then $(c_1\zeta_1, \cdots, c_m\zeta_m) \in D$ for all complex constants c_j satisfying $|c_j| \leq 1$ for all $j = 1, \cdots, m$.

Definition 2.2. Let P be a real continuous function on $\mathbb{R}^m$. Assume that, for all $\lambda \geq 0$, there exist positive real numbers $\alpha_1, \cdots, \alpha_m$ such that

$$\lambda P(x_1, \cdots, x_m) = P(\lambda^{\alpha_1}x_1, \cdots, \lambda^{\alpha_m}x_m), \tag{2.3}$$

for all $(x_1, \cdots, x_m) \in \mathbb{R}^m$. A function P satisfying this condition is called quasi-homogeneous with weight α. In particular, a quasi-homogeneous function is called homogeneous if $\alpha_i = 1$ for all $1 \leq i \leq m$.

In this section, we consider the following Reinhardt domain $D^m_P(r)$:

$$D^m_P(r) := \{\zeta \in \mathbb{C}^m; P(|\zeta_1|^2, \cdots, |\zeta_m|^2) < r^2\}, \quad r > 0,$$

with the conditions

(i) P is quasi-homogeneous with weight α,
(ii) $D^m_P(r)$ is bounded and complete.

The aim of this section is to give a relation between $\|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^2_{L^2_a(D^m_P(r))}$ and $\|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^2_{L^2_a(D^m_P(1))}$ (Proposition 2.1).

By condition (i), we know that $(0, \cdots, 0) \in D^m_P(r)$. This, together with (ii) implies that the domain $D^m_P(r)$ is a complete Reinhardt domain with center at the origin. It is well known that for any complete Reinhardt domain D with center at the origin, the set of (normalized) monomials forms a complete orthonormal basis of $L_a^2(D)$ (see [17, Exercise 8.8]). Thus the Bergman kernel $K_{D_P^m(r)}$ of $D_P^m(r)$ is given by

$$K_{D_P^m(r)}(\zeta,\zeta') = \sum_{k\in\mathbb{N}^m} a_{k_1\cdots k_m}(r)(\zeta\overline{\zeta'})^k, \tag{2.4}$$

where $(\zeta\overline{\zeta'})^k = (\zeta_1\overline{\zeta'_1})^{k_1}\cdots(\zeta_m\overline{\zeta'_m})^{k_m}$ and the coefficient $a_{k_1\cdots k_m}(r)$ is given by

$$\begin{aligned} a_{k_1\cdots k_m}(r) &= \|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^{-2}_{L_a^2(D_P^m(r))} \\ &= \left(\int_{D_P^m(r)} |\zeta_1^{k_1}\cdots\zeta_m^{k_m}|^2 d\zeta\right)^{-1}. \end{aligned}$$

In the case of $r=1$, for simplicity of notation, we put $a_{k_1\cdots k_m} = a_{k_1\cdots k_m}(1)$. By condition (i), we know that

$$\begin{aligned} D_P^m(r) &= \{\zeta\in\mathbb{C}^m; P(|\zeta_1|^2,\cdots,|\zeta_m|^2) < r^2\} \\ &= \left\{\zeta\in\mathbb{C}^m; P\left(\frac{|\zeta_1|^2}{r^{2\alpha_1}},\cdots,\frac{|\zeta_m|^2}{r^{2\alpha_m}}\right) < 1\right\}. \end{aligned}$$

It implies that there is a biholomorphic mapping F from $D_P^m(r)$ to $D_P^m(1)$:

$$F: D_P^m(r)\to D_P^m(1), (\zeta_1,\cdots,\zeta_m)\mapsto\left(\frac{\zeta_1}{r^{\alpha_1}},\cdots,\frac{\zeta_m}{r^{\alpha_m}}\right).$$

By the transformation law of the Bergman kernel (2.1) and (2.4), it follows that

$$K_{D_P^m(r)}(\zeta,\zeta') = \frac{1}{r^{2|\alpha|}}\sum_{k\in\mathbb{N}^m} a_{k_1\cdots k_m}\left(\frac{\zeta_1\overline{\zeta'_1}}{r^{2\alpha_1}}\right)^{k_1}\cdots\left(\frac{\zeta_m\overline{\zeta'_m}}{r^{2\alpha_m}}\right)^{k_m}. \tag{2.5}$$

Comparing the above two expansions (2.4) and (2.5) of $K_{D_P^m(r)}$, we obtain a relation:

$$\|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^2_{L_a^2(D_P^m(r))} = r^{2|\alpha(1+k)|}\|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^2_{L_a^2(D_P^m(1))}, \tag{2.6}$$

where we put $|\alpha(1+k)| = \sum_{i=1}^m \alpha_i(1+k_i)$. In summary, we showed the following proposition:

Proposition 2.1. *For any positive real number r, we have the following relation:*

$$\|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^2_{L_a^2(D_P^m(r))} = r^{2|\alpha(1+k)|}\|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^2_{L_a^2(D_P^m(1))},$$

where $|\alpha(1+k)| = \sum_{i=1}^m \alpha_i(1+k_i)$.

Now we give some examples.

Example 2.3. Let us consider the case $P(x_1,\cdots,x_m) = x_1+\cdots+x_m$. In this case, $\alpha_1=\cdots=\alpha_m=1$ and the domain $D_P^m(r)$ coincides with the unit ball of radius r:

$$D_P^m(r) = \{\zeta\in\mathbb{C}^m; \|\zeta\|^2 < r^2\} =: \mathbb{B}^m(r).$$

It follows that

$$\|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^2_{L_a^2(\mathbb{B}^m(r))} = r^{2(m+|k|)}\|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^2_{L_a^2(\mathbb{B}^m(1))}.$$

The coefficient $a_{k_1\cdots k_m}$ has the following explicit form:

$$a_{k_1\cdots k_m} = \frac{(|k|+m)!}{\pi^m k_1!\cdots k_m!}.$$

Example 2.4. Let $q_1, \cdots, q_m$ be positive integers. We now consider the case $P(x_1, \cdots, x_m) = x_1^{q_1} + \cdots + x_m^{q_m}$. In this case, we know that $\alpha_i = q_i^{-1}$ and

$$D_P^m(r) = \{\zeta \in \mathbb{C}^m; |\zeta_1|^{2q_1} + \cdots + |\zeta_m|^{2q_m} < r^2\} =: D_q(r).$$

The domain $D_q(r)$ is called a complex ellipsoid. Then we have

$$\|\zeta_1^{k_1} \cdots \zeta_m^{k_m}\|^2_{L_a^2(D_q(r))} = r^{2|q^{-1}(1+k)|}\|\zeta_1^{k_1} \cdots \zeta_m^{k_m}\|^2_{L_a^2(D_q(1))}.$$

It is known that $a_{k_1 \cdots k_m}$ has the following expression (see [2]):

$$a_{k_1 \cdots k_m} = \frac{q_1 \cdots q_m \Gamma\left(1 + \sum_{j=1}^m (k_j+1)/q_j\right)}{\pi^m \prod_{j=1}^m \Gamma\left((k_j+1)/q_j\right)}. \tag{2.7}$$

Example 2.5. Let $q_1, \cdots, q_m$ be positive real numbers. We consider the case $P(x_1, \cdots, x_m) = \sum_{i=1}^m x_i^{1/q_i} + \prod_{i=1}^m x_i^{\frac{1}{mq_i}}$. Then we have

$$D_P^m(r) = \left\{\zeta \in \mathbb{C}^m; \sum_{i=1}^m |\zeta_i|^{2/q_i} + \prod_{i=1}^m |\zeta_i|^{\frac{2}{mq_i}} < r^2\right\}.$$

In this case, we know that $\alpha_i = q_i$ and

$$\|\zeta_1^{k_1} \cdots \zeta_m^{k_m}\|^2_{L_a^2(D_P^m(r))} = r^{2|q(1+k)|}\|\zeta_1^{k_1} \cdots \zeta_m^{k_m}\|^2_{L_a^2(D_P^m(1))}.$$

Remark 2.6. It is known that the Bergman kernel of $D_q(r)$ is expressed in terms of hypergeometric functions (see [8]). In particular, J.-D. Park proved that the Bergman kernel of the 2-dimensional complex ellipsoid

$$D_{(q_1,q_2)} = \{(z_1, z_2) \in \mathbb{C}^2; |z_1|^{2q_1} + |z_2|^{2q_2} < 1\}$$

is represented by means of elementary functions if and only if $q = (1, q_2), (q_1, 1), (2, 2)$ and obtained the Bergman kernel in explicit form when $q = (2, 2)$ (see [15]). Recently, J.-D. Park also obtained a closed form of the Bergman kernel of the 3-dimensional complex ellipsoid [16]:

$$D_{(4,4,4)} = \{(z_1, z_2, z_3) \in \mathbb{C}^3; |z_1|^4 + |z_2|^4 + |z_3|^4 < 1\}.$$

We note that condition (i) is closely related to bounded Reinhardt domains with non-compact automorphism group and C^k-smooth boundary, $k \geq 1$. Indeed, the following result is known (cf. [11] and [12]):

Theorem 2.7. *Let $D \subset \mathbb{C}^n$ be a bounded Reinhardt domain with C^k-smooth boundary, and if $Aut(D)$ is not compact, then, up to dilations and permutations of coordinates, D is a domain of the form*

$$\{(z_1, \cdots, z_n) \in \mathbb{C}^n; |z_1|^2 + \psi(|z_2|, ..., |z_n|) < 1\}, \tag{2.8}$$

where ψ is a non-negative C^k-smooth function in $\mathbb{R}^{n-1}$ that is strictly positive in $\mathbb{R}^{n-1}\backslash\{0\}$ and such that $\psi(|z_2|, \cdots, |z_n|)$ is C^k-smooth in $\mathbb{C}^{n-1}$, and

$$\psi(t^{\frac{1}{\alpha_1}} x_2, \cdots, t^{\frac{1}{\alpha_n}} x_n) = t\psi(x_2, \cdots, x_n),$$

in $\mathbb{R}^{n-1}$ for all $t \geq 0$. Here $\alpha_j > 0$, $j = 2, \cdots, n$, and each α_j is either an even integer or $\alpha_j > 2k$.

We also remark that a classification of Reinhardt domains with a description of their automorphism groups was obtained by Shimizu [19] and Sunada [20] (see also [10, Chapter 8]). We close this section with one more remark on condition (i).

Remark 2.8. Consider a function P on $\mathbb{R}^2$ with the following condition:

$$h(\lambda)P(x_1, x_2) = P(\lambda^{\alpha_1}x_1, \lambda^{\alpha_2}x_2),$$

for all $x \in \mathbb{R}^2$ and $\lambda > 0$. Here h is a given continuous function. Then h can be written in the form $h(\lambda) = \lambda^r$ for some $r \in \mathbb{R}$. Indeed, it can be shown by checking $h(\lambda_1\lambda_2) = h(\lambda_1)h(\lambda_2)$ for all $\lambda_1, \lambda_2 > 0$.

3. Generalized Forelli-Rudin construction

Let D be a domain $\in \mathbb{C}^n$ and φ be a positive continuous function on D. In this section, we consider the following domain:

$$D^{\varphi}_{P,m} := \{(z,\zeta) \in D \times \mathbb{C}^m; P(|\zeta_1|^2, \cdots, |\zeta_m|^2) < \varphi(z)\}$$

with the same assumptions (i), (ii). In the previous section, using condition (i), we showed Proposition 2.1 for $D^m_P(r)$. Proposition 2.1 gives us a relation between the Bergman kernel of $D^{\varphi}_{P,m}$ and the weighted Bergman kernels of the base domain D.

3.1. **Main result.**

As is shown in [18, Proposition 1.1], for the Hartogs domain

$$\widehat{D} = \{(z,\zeta) \in D \times \mathbb{C}; |\zeta|^2 < \varphi(z)\},$$

the set $S = \{(k+1)^{1/2} f_{j,k+1}(z)\zeta^k\}_{k\in\mathbb{N}, j\in J_{k+1}}$ is a complete orthonormal basis of $L^2_a(\widehat{D})$. Here $\{f_{j,k}\}_{j\in J_k}$ is a complete orthonormal basis of $L^2_a(D, \varphi^k)$. This fact, together with (2.2), implies the Forelli-Rudin construction for $\widehat{D}$. Our proof of the main theorem is based on the idea mentioned above.

Now we state the main theorem.

Theorem 3.1. *The Bergman kernel $K_{D^{\varphi}_{P,m}}$ of $D^{\varphi}_{P,m}$ has the following series representation:*

$$K_{D^{\varphi}_{P,m}}((z,\zeta),(z',\zeta')) = \sum_{k\in\mathbb{N}^m} a_{k_1\cdots k_m} K_{D,\varphi^{|\alpha(1+k)|}}(z,z')(\zeta\overline{\zeta'})^k.$$

Here $a_{k_1\cdots k_m} = \|\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^{-2}_{L^2_a(D^m_P(1))}$.

Proof. Let $f \in L^2_a(D^{\varphi}_{P,m})$. By the Taylor expansion, with respect to the variable ζ, we have

$$f(z,\zeta) = \sum_{k\in\mathbb{N}^m} b_k(z)\zeta_1^{k_1}\cdots\zeta_m^{k_m}. \tag{3.1}$$

Here the function $b_k(z)$ is holomorphic in z and the series converges uniformly on compact subsets of $D^{\varphi}_{p,m}$. Let us compute $\|f\|^2_{L^2_a(D^{\varphi}_{P,m})}$:

$$\begin{aligned}
\infty > \|f\|^2_{L^2_a(D^{\varphi}_{P,m})} &= \lim_{\epsilon\to 0}\int_{D^{\varphi-\epsilon}_{P,m}} |f(z,\zeta)|^2 dz d\zeta \\
&= \lim_{\epsilon\to 0}\sum_{k\in\mathbb{N}^m}\int_{z\in D}|b_k(z)|^2\left(\int_{D^m_P(\sqrt{(\varphi-\epsilon)})}|\zeta_1^{k_1}\cdots\zeta_m^{k_m}|^2 d\zeta\right)dz \\
&= \lim_{\epsilon\to 0}\sum_{k\in\mathbb{N}^m} a^{-1}_{k_1\cdots k_m}\int_{z\in D}|b_k(z)|^2(\varphi(z)-\epsilon)^{|\alpha(1+k)|}dz \\
&= \sum_{k\in\mathbb{N}^m} a^{-1}_{k_1\cdots k_m}\|b_k(z)\|^2_{L^2_a(D,\varphi^{|\alpha(1+k)|})}.
\end{aligned}$$

Here the third equality follows from Proposition 2.1. Thus if $f \in L^2_a(D^{\varphi}_{P,m})$, then $b_k(z) \in L^2_a(D, \varphi^{|\alpha(1+k)|})$. For all $h \in L^2_a(D, \varphi^{|\alpha(1+k)|})$, we have

$$\begin{aligned}\|h(z)\zeta_1^{k_1}\cdots\zeta_m^{k_m}\|^2_{L^2_a(D^{\varphi}_{P,m})} &= \int_{z\in D}|h(z)|^2\left(\int_{D^m_P(\sqrt{\varphi(z)})}|\zeta_1^{k_1}\cdots\zeta_m^{k_m}|^2 d\zeta\right)dz\\ &= a^{-1}_{k_1\cdots k_m}\|h(z)\|^2_{L^2_a(D,\varphi^{|\alpha(1+k)|})}\\ &< \infty.\end{aligned}$$

Let $\{e_{jk}\}_{j\in J_k}$ be a complete orthonormal basis of $L^2_a(D, \varphi^{|\alpha(1+k)|})$. Put $\zeta^k = \zeta_1^{k_1}\cdots\zeta_m^{k_m}$. Then $e_{jk}\zeta^k$ is an element of $L^2_a(D^{\varphi}_{P,m})$ for all $k \in \mathbb{N}^m$, $j \in J_k$. Now we show that $S = \{a^{1/2}_{k_1\cdots k_m}e_{jk}\zeta^k\}_{j\in J_k, k\in\mathbb{N}^m}$ is a complete orthonormal basis of $L^2_a(D^{\varphi}_{P,m})$. Put $g_{jk} = a^{1/2}_{k_1\cdots k_m}e_{jk}\zeta^k$ (resp. $g_{j'k'} = a^{1/2}_{k'_1\cdots k'_m}e_{j'k'}\zeta^{k'}$). Then it is easy to see that

$$\begin{aligned}\langle g_{jk}, g_{j'k'}\rangle_{D^{\varphi}_{P,m}} &= \int_{D^{\varphi}_{P,m}} g_{jk}(z,\zeta)\overline{g_{j'k'}(z,\zeta)}dzd\zeta\\ &= \int_{z\in D}\langle g_{jk}(z,\cdot), g_{j'k'}(z,\cdot)\rangle_{D^m_P(\sqrt{\varphi(z)})}dz.\end{aligned}$$

Thus g_{jk} and $g_{j'k'}$ are orthogonal in $L^2_a(D^{\varphi}_{P,m})$ if $k \neq k'$. Now assume that $k = k'$ and $j \neq j'$. Then we have

$$\begin{aligned}\langle g_{jk}, g_{j'k}\rangle_{D^{\varphi}_{P,m}} &= \int_{D^{\varphi}_{P,m}} g_{jk}(z,\zeta)\overline{g_{j'k}(z,\zeta)}dzd\zeta\\ &= \langle e_{jk}, e_{j'k}\rangle_{L^2_a(D,\varphi^{|\alpha(1+k)|})} = 0.\end{aligned}$$

Since the normality is easily verified, we know the set S is an orthonormal set. Assume that $\langle f, g_{kj}\rangle = 0$ for all $j \in J_k$ and $k \in \mathbb{N}^m$. Then we have

$$\begin{aligned}\langle f, g_{jk}\rangle_{D^{\varphi}_{P,m}} &= \int_{D^{\varphi}_{P,m}} f(z,\zeta)\overline{g_{jk}(z,\zeta)}dzd\zeta\\ &= a^{-1/2}_{k_1\cdots k_m}\langle b_k(z), e_{jk}\rangle_{L^2_a(D,\varphi^{|\alpha(1+k)|})} = 0,\end{aligned}$$

for all $j \in J_k$ and $k \in \mathbb{N}^m$. Since $b_k(z)$ is an element of $L^2_a(D, \varphi^{|\alpha(1+k)|})$ and $\{e_{jk}\}_{j\in J_k}$ is a complete orthonormal basis of $L^2_a(D, \varphi^{|\alpha(1+k)|})$, the condition $\langle f, g_{kj}\rangle = 0$ implies that $b_k = 0$ for all $k \in \mathbb{N}^m$. Therefore the set S is a complete orthonormal basis of $L^2_a(D^{\varphi}_{P,m})$. Hence we finally obtain

$$\begin{aligned}K_{D,\varphi^{|\alpha(1+k)|}}(z,z') &= \sum_{j\in J_k} e_{jk}(z)\overline{e_{jk}(z')},\\ K_{D^{\varphi}_{P,m}}((z,\zeta),(z',\zeta')) &= \sum_{j\in J_k, k\in\mathbb{N}^m} a_{k_1\cdots k_m}e_{jk}(z)\overline{e_{jk}(z')}(\zeta\overline{\zeta'})^k\\ &= \sum_{k\in\mathbb{N}^m} a_{k_1\cdots k_m}K_{D,\varphi^{|\alpha(1+k)|}}(z,z')(\zeta\overline{\zeta'})^k.\end{aligned}$$

This completes the proof of the theorem. □

Let us describe how to recover the Forelli-Rudin construction (1.1) of a Hartogs domain. Recall that $D^{\varphi}_{P,m}$ is a Hartogs domain if $P(x_1, \cdots x_m) = x_1 + \cdots + x_m$. By

Example 2.3, we know that $\varphi^{|\alpha(1+k)|} = \varphi^{m+|k|}$. Using Theorem 3.1, we have the following series representation of the Bergman kernel K of the Hartogs domain:

$$K((z,\zeta),(z',\zeta')) = \sum_{k_1,\cdots,k_m\geq 0} \frac{(|k|+m)!}{\pi^m k_1!\cdots k_m!} K_{D,\varphi^{|k|+m}}(z,z')(\zeta_1\overline{\zeta'_1})^{k_1}\cdots(\zeta_m\overline{\zeta'_m})^{k_m}$$
$$= \frac{1}{\pi^m}\sum_{k\geq 0}(k+1)_m K_{D,\varphi^{k+m}}(z,z')\langle\zeta,\zeta'\rangle^k.$$

Here the second equality follows from the following simple identity:

$$\frac{(x_1+\cdots+x_m)^k}{k!} = \sum_{|\alpha|=k}\frac{x^\alpha}{\alpha!}.$$

Thus the Forelli-Rudin construction (1.1) for the Hartogs domain is recovered.

We next consider the following case:

$$D^\varphi_{q,m} = \{(z,\zeta)\in D\times\mathbb{C}^m; |\zeta_1|^{2q_1}+\cdots+|\zeta_m|^{2q_m} < \varphi(z)\}.$$

By Theorem 3.1 and Example 2.4, we have the following corollary:

Corollary 3.2. *The Bergman kernel of $D^\varphi_{q,m}$ has the following series representation:*

$$K_{D^\varphi_{q,m}}((z,\zeta),(z',\zeta')) = \sum_{k\in\mathbb{N}^m}^{\infty} a_{k_1\cdots k_m} K_{D,\varphi^{|q^{-1}(1+k)|}}(z,z')(\zeta\overline{\zeta'})^k,$$

where $a_{k_1\cdots k_m}$ is given by (2.7).

3.2. **Roos' problem.** Here we discuss further generalizations of Corollary 3.2 and related results. Let q_i be a positive integer and φ_i a positive continuous function on D for $1\leq i\leq m$. Consider the following domain:

$$\widetilde{D}^\varphi_{q,m} := \left\{(z,\zeta)\in D\times\mathbb{C}^m; \sum_{i=1}^m \frac{|\zeta_i|^{2q_i}}{\varphi_i(z)} < 1\right\}.$$

The domain $\widetilde{D}^\varphi_{q,m}$ includes as a special case the domain $D^\varphi_{q,m}$. In a completely analogous way to the proof of Theorem 3.1, we can prove a Forelli-Rudin type formula for $\widetilde{D}^\varphi_{q,m}$. To avoid the repetition, we only formulate the theorem omitting its proof.

Theorem 3.3. *The Bergman kernel $K_{\widetilde{D}^\varphi_{q,m}}$ of $\widetilde{D}^\varphi_{q,m}$ has the following series representation:*

$$K_{\widetilde{D}^\varphi_{q,m}}((z,\zeta),(z',\zeta')) = \sum_{k\in\mathbb{N}^m} a_{k_1\cdots k_m} K_{m,q,k}(z,z')(\zeta\overline{\zeta'})^k,$$

where $a_{k_1\cdots k_m}$ is given by (2.7) *and the kernel function $K_{m,q,k}$ is the weighted Bergman kernel of D with respect to a weight $\prod_{i=1}^m \varphi_i(z)^{\frac{1+k_i}{q_i}}$.*

Recall that the Hartogs domain can be rewritten as

$$D_{m,\varphi} = \{(z,\zeta)\in D\times\mathbb{C}^m; \varphi(z)^{-1/2}\zeta\in\mathbb{B}^m\}. \tag{3.2}$$

Moreover, the Forelli-Rudin construction (1.1) can be rewritten as

$$K_{D_{m,\varphi}}((z,\zeta),(z',\zeta')) = \frac{1}{\pi^m}\frac{\partial^m}{\partial r^m}L_{D,\varphi}(z,z';r)\bigg|_{r=\langle\zeta,\zeta'\rangle}. \tag{3.3}$$

Here the function $L_{D,\varphi}(z,z';r)$ is given by

$$L_{D,\varphi}(z,z';r)=\sum_{k=0}^{\infty}K_{D,\varphi^k}(z,z')r^k,$$

and it is called the virtual Bergman kernel of (D,φ). In [18], G. Roos introduced the following domain D_φ^Ω, which is a generalization of the right hand side of the above expression (3.2):

$$D_\varphi^\Omega:=\{(z,\zeta)\in D\times\mathbb{C}^m;\varphi(z)^{-1/2}\zeta\in\Omega\},$$

where $\Omega\subset\mathbb{C}^m$ is a domain. Then he put forth the following problem [18, Problem 4.2]:

Problem 3.4. (1) If Ω is any circled domain in $\mathbb{C}^m$, what can be said about the Bergman kernel of D_φ^Ω? (2) For some families $\{F\}$ other than the family $\{\mathbb{B}^m\}$ of Hermitian balls, is it possible to define an analogue of the virtual Bergman kernel $L_{\Omega,\varphi}$ and obtain an analogue of (3.3)?

M. Engliš and G. Zhang [5] answered this problem when F is a bounded symmetric domain. Namely, they obtained a Forelli-Rudin type formula of D_φ^Ω when Ω is an irreducible bounded symmetric domain and also obtained an analogue of (3.3) when Ω is an irreducible bounded symmetric domain of Type I or Type IV. Recently, the domain D_φ^Ω was also investigated by Z. Feng [6] in a different context. Feng introduced invariant Hilbert spaces of holomorphic functions on D_φ^Ω and computed their reproducing kernels explicitly.

As an application of Theorem 3.3, we derive a Forelli-Rudin type formula for the domain D_φ^Ω when Ω is the complex ellipsoid D_q of radius 1, which is not bounded symmetric in general:

Theorem 3.5. *The Bergman kernel K of D_φ^Ω has the following series representation when Ω is the complex ellipsoid D_q of radius* 1*:*

$$K((z,\zeta),(z',\zeta'))=\sum_{k\in\mathbb{N}^m}a_{k_1\cdots k_m}K_{D,\varphi^{m+|k|}}(z,z')(\zeta\overline{\zeta'})^k,$$

where $a_{k_1\cdots k_m}$ is given by (2.7)*.*

Proof. Put $\varphi_i=\varphi^{q_i}$ for any $1\le i\le m$. Then we know that

$$\begin{aligned}\widetilde{D}_{q,m}^{\varphi}&=\left\{(z,\zeta)\in D\times\mathbb{C}^m;\sum_{i=1}^{m}\frac{|\zeta_i|^{2q_i}}{\varphi(z)^{q_i}}<1\right\}\\&=\left\{(z,\zeta)\in D\times\mathbb{C}^m;\varphi(z)^{-\frac{1}{2}}\zeta\in D_q\right\}.\end{aligned}$$

Thus the domain $\widetilde{D}_{q,m}^{\varphi}$ coincides with D_φ^Ω. In this case, the weight function $\prod_{i=1}^m\varphi_i(z)^{\frac{k_i+1}{q_i}}$ is given by

$$\prod_{i=1}^{m}\varphi_i(z)^{\frac{k_i+1}{q_i}}=\varphi^{\sum_{i=1}^m q_i\left(\frac{k_i+1}{q_i}\right)}=\varphi(z)^{m+|k|}.$$

Thus the proof is complete. ☐

This answers Problem 3.4(1) when Ω is the complex ellipsoid D_q. However we do not succeed in obtaining an analogue of (3.3).

4. Deflation identity

Let Ω_1, Ω_2 be domains in $\mathbb{C}^{m_1}$ and in $\mathbb{C}^{m_2}$ respectively. In general, there is no relation between the Bergman kernels K_{Ω_1} and K_{Ω_2}. However, as explained in Section 1, if we consider two domain Ω_1 and Ω_2 as in Section 1.2, then the restrictions of the Bergman kernels of Ω_1 and Ω_2 to the subspace $\{\zeta, \zeta' = 0\}$ coincide with each other (up to a constant multiple).

In this section, as an application of our results, we study deflation identities for $D^{\varphi}_{P,m}$. We start our study with the observation that the restriction of the Bergman kernel of $D^{\varphi}_{P,m}$ to the subspace $\{\zeta, \zeta' = 0\}$ represents the weighted Bergman kernel $K_{D,\varphi^{|\alpha|}}$. Indeed, as a consequence of Theorem 3.1, we have the following lemma.

Lemma 4.1. *The restriction of the Bergman kernel of $D^{\varphi}_{P,m}$ to the subspace $\{\zeta, \zeta' = 0\}$ is given by*

$$K_{D^{\varphi}_{P,m}}((z,0),(z',0)) = a_0 K_{D,\varphi^{|\alpha|}}(z,z').$$

Before stating our result, it is convenient to introduce the following equivalence relation:

Definition 4.2. Let r_i be a positive real number for $i = 1, 2$. We say that the domains $D^{\varphi^{r_1}}_{P,m}$ and $D^{\varphi^{r_2}}_{P',m'}$ are equivalent if $r_1|\alpha| = r_2|\alpha'|$.

Let K_1 (resp. K_2) be the Bergman kernel of $D^{\varphi^{r_1}}_{P,m_1}$ (resp. $D^{\varphi^{r_2}}_{P',m_2}$). We can now prove the following deflation identity:

Theorem 4.3. *Assume that the domains $D^{\varphi^{r_1}}_{P,m_1}$ and $D^{\varphi^{r_2}}_{P',m_2}$ are equivalent. Then the restrictions of the Bergman kernels K_1, K_2 to the subspace $\{\zeta, \zeta' = 0\}$ coincide with each other (up to a constant multiple):*

$$c_1 K_1((z,0),(z',0)) = c_2 K_2((z,0),(z',0)),$$

where c_1 and c_2 are constants.

Proof. By Lemma 4.1, we have

$$K_i((z,0),(z',0)) = a_0^{(i)} K_{D,\varphi^{r_i|\alpha|}}(z,z').$$

Since $D^{\varphi^{r_1}}_{P,m_1}$ and $D^{\varphi^{r_2}}_{P',m_2}$ are equivalent, we conclude that

$$\begin{aligned} K_1((z,0),(z',0)) &= a_0^{(1)} K_{D,\varphi^{r_1|\alpha|}}(z,z') \\ &= a_0^{(1)} K_{D,\varphi^{r_2|\alpha'|}}(z,z') = \frac{a_0^{(1)}}{a_0^{(2)}} K_2((z,0),(z',0)). \end{aligned}$$

Thus the proof is complete. □

Now let us turn back to the original deflation identity (1.2) of the following domains Ω_1, Ω_2:

$$\begin{aligned} \Omega_1 &= \{(z,\zeta) \in \Omega \times \mathbb{C}; |\zeta|^2 < (1-\psi(z))^{q_1+q_2}\}, \\ \Omega_2 &= \{(z,\zeta) \in \Omega \times \mathbb{C}^2; |\zeta_1|^{2/q_1} + |\zeta_2|^{2/q_2} < 1-\psi(z)\}. \end{aligned}$$

To recover the deflation identity (1.2) from Theorem 4.3, it is enough to check that Ω_1 and Ω_2 are equivalent. Actually the equivalence of Ω_1 and Ω_2 is obvious from the definitions. Moreover, the following domains are also equivalent to Ω_1 and Ω_2:

$$\Omega_3 = \{(z,\zeta) \in \Omega \times \mathbb{C}^2; |\zeta_1|^2 + |\zeta_2|^{2/q_2} < (1-\psi(z))^{\frac{q_1+q_2}{1+q_2}}\},$$
$$\Omega_4 = \{(z,\zeta) \in \Omega \times \mathbb{C}^2; |\zeta_1|^{2/q_1} + |\zeta_2|^{2/q_1} < (1-\psi(z))^{\frac{q_1+q_2}{2q_1}}\}.$$

In summary, we have

Corollary 4.4. *The restrictions of the Bergman kernels $K_{\Omega_i}, K_{\Omega_j}$ to the subspace $\{\zeta, \zeta' = 0\}$ coincide with each other (up to a constant multiple) for any $1 \le i, j \le 4$.*

Thus we have recovered the deflation identity (1.2). Next, we turn to studying deflation identities in a more general setting. Consider the following domains:

$$D^{\varphi^{r_1}}_{p,m_1} := \{(z,\zeta) \in D \times \mathbb{C}^{m_1}; |\zeta_1|^{2/p_1} + \cdots + |\zeta_{m_1}|^{2/p_{m_1}} < \varphi(z)^{r_1}\},$$
$$D^{\varphi^{r_2}}_{q,m_2} := \{(z,\zeta) \in D \times \mathbb{C}^{m_2}; |\zeta_1|^{2/q_1} + \cdots + |\zeta_{m_2}|^{2/q_{m_2}} < \varphi(z)^{r_2}\}.$$

Then these domains $D^{\varphi^{r_1}}_{p,m_1}, D^{\varphi^{r_2}}_{q,m_1}$ are equivalent if and only if $r_1|p| = r_2|q|$. Let K_1 (resp. K_2) denote the Bergman kernel of $D^{\varphi^{r_1}}_{p,m_1}$ (resp. $D^{\varphi^{r_2}}_{q,m_1}$). Consequently, we obtain the following identity between K_1 and K_2.

Corollary 4.5. *Assume that $r_1|p| = r_2|q|$. Then the restrictions of the Bergman kernels K_1, K_2 to the subspace $\{\zeta, \zeta' = 0\}$ coincide with each other (up to a constant multiple).*

Remark 4.6. We remark that in the case of the Hartogs domain, the condition $r_1|p| = r_2|q|$ is more restrictive. Consider two Hartogs domains:

$$D_{m_i,\varphi^{r_i}} = \{(z,\zeta) \in D \times \mathbb{C}^{m_i}; \|\zeta\|^2 < \varphi(z)^{r_i}\}, \quad i = 1, 2.$$

It is easily verified that $D_{m_1,\varphi^{r_1}}$ and $D_{m_2,\varphi^{r_2}}$ are equivalent if and only if $r_1 m_1 = r_2 m_2$. Therefore if $r_1 = r_2$ and $m_1 \neq m_2$, then $D_{m_1,\varphi^{r_1}}$ and $D_{m_2,\varphi^{r_2}}$ are not equivalent. On the contrary, in the case of $D^{\varphi^{r_1}}_{p,m_1}$ and $D^{\varphi^{r_2}}_{q,m_2}$, there is the possibility $D^{\varphi^{r_1}}_{p,m_1} \sim D^{\varphi^{r_2}}_{q,m_2}$ even if we assume that $r_1 = r_2$ and $m_1 \neq m_2$.

Using Theorem 3.1 and Theorem 3.5, we also obtain deflation identities between $D^{\varphi}_{P,m}$ and D^{Ω}_{φ} in an analogous way. For instance, we have a deflation identity between the Hartogs domain $D_{m,\varphi}$ and D^{Ω}_{φ}.

Theorem 4.7. *Let Ω be a complex ellipsoid of radius 1. Then the restrictions of the Bergman kernels $K_{D_{m,\varphi}}, K_{D^{\Omega}_{\varphi}}$ to the subspace $\{\zeta, \zeta' = 0\}$ coincides with each other (up to a constant multiple).*

Since the proof proceeds along the same lines as that of Theorem 4.3, we omit it.

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