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MINIMALLY ALMOST PERIODIC GROUP TOPOLOGIES ON COUNTABLY INFINITE ABELIAN GROUPS

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ABSTRACT. We obtain a positive answer to a question by Comfort: Every countable Abelian group G of infinite exponent admits a complete Hausdorff minimally almost periodic group topology.

1. INTRODUCTION

For an Abelian topological group G, $G^\wedge$ denotes the group of all continuous characters on G endowed with the compact-open topology. Denote by $\mathbf{n}(G) = \bigcap_{\chi \in G^\wedge} \ker\chi$ the von Neumann radical of G. Following von Neumann [21], the group G is called *minimally almost periodic* (MinAP) if $\mathbf{n}(G) = G$, and G is called *maximally almost periodic* (MAP) if $\mathbf{n}(G) = 0$.

Let G be an infinite Abelian group. Denote by $\mathcal{NR}(G)$ the set of all subgroups H of G for which there exists a non-discrete Hausdorff group topology τ on G such that the von Neumann radical of (G,τ) is H, i.e., $\mathbf{n}(G,\tau) = H$. Let $\mathcal{NRC}(G) \subseteq \mathcal{NR}(G)$ be the set of all subgroups H of G for which there exists a topology as above which is also complete.

Let G be an Abelian topological group. The richness of its dual group $G^\wedge$ is one of the most important properties of G which plays a crucial role in Harmonic Analysis (see [19, 20]). The von Neumann radical measures this richness, thus the following general question is important:

Question 1.1 ([12, Problem 2])**.** Describe the sets $\mathcal{NR}(G)$ and $\mathcal{NRC}(G)$.

One can show (see [14, Theorem 1]) that every infinite Abelian group admits a complete non-trivial Hausdorff group topology with trivial von Neumann radical. Thus, the trivial group $\{0\}$ belongs to $\mathcal{NRC}(G)$ for every infinite Abelian group G, i.e., $\mathcal{NRC}(G)$ is never empty. A much deeper question is whether any infinite Abelian group admits a Hausdorff group topology with *non-zero* von Neumann radical. The positive answer was given by Ajtai, Havas and Komlós [1]. Using the method of T-sequences, Protasov and Zelenyuk [26] strengthened their result as follows: every infinite Abelian group admits a *complete sequential* Hausdorff group topology for which characters do not separate points. So $\mathcal{NRC}(G) \neq \{\{0\}\}$ for every infinite Abelian group G.

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Question 1.1 has two interesting extreme cases. Denote by $\mathcal{S}(G)$ the set of all subgroups of an infinite Abelian group G.

Question 1.2 ([12, Problem 4])**.** Describe all infinite Abelian groups G such that $\mathcal{NR}(G) = \mathcal{S}(G)$ (respectively, $\mathcal{NRC}(G) = \mathcal{S}(G)$).

The second and, perhaps, the most interesting special case of Question 1.1 is the following:

Question 1.3 ([12, Problem 5])**.** Describe all infinite Abelian groups G such that $G \in \mathcal{NR}(G)$ (or $G \in \mathcal{NRC}(G)$).

Note that the existence of Abelian groups admitting no non-zero continuous characters has been known for a long time. The classic example of linear topological spaces that (when viewed as Abelian topological groups) have no non-trivial continuous characters was given by Day in [7]. There are other examples in [19, 23.32]. Nienhuys [22] showed the existence of a metric solenoidal monothetic MinAP group (see also [23]). Prodanov [24] gave an elementary example of a MinAP group.

Recall that an Abelian group G is *of finite exponent* or *bounded* if there exists a positive integer n such that $ng = 0$ for every $g \in G$. The minimal integer n with this property is called the *exponent* of G and is denoted by $\exp(G)$. When G is not bounded, we write $\exp(G) = \infty$ and say that G is *of infinte exponent* or *unbounded*.

Protasov [25] posed the question of whether every infinite Abelian group admits a minimally almost periodic group topology. Using a result of Graev [18], Remus [28] proved that for every natural number n there exists a connected MinAP group which is algebraically generated by elements of order n. On the other hand, he gave (see [6]) a simple example of a group G of finite exponent which does not admit any Hausdorff group topology τ such that (G, τ) is minimally almost periodic. So, for groups of finite exponent the answer to Protasov's question is negative. This justifies the following problems:

Question 1.4 ([6, Problem 521])**.** Does every Abelian group which is not of bounded exponent admit a minimally almost periodic topological group topology? What about the countable case?

Question 1.5 ([27, Question 2.6.1])**.** Let G be a torsion free countable Abelian group. Does there exist a Hausdorff group topology on G with only zero character?

The main goal of the article is to give the positive answer to Comfort's Question 1.4 for the countable case and, hence, to Protasov-Zelenyuk's Question 1.5.

Theorem 1.6. *Every countable Abelian group of infinite exponent admits a complete sequential Hausdorff minimally almost periodic group topology.*

A complete characterization of bounded Abelian groups which admit Hausdorff MinAP group topologies is given in [15]: An infinite bounded Abelian group G admits a MinAP group topology if and only if all its leading Ulm-Kaplansky invariants are infinite. This result with Theorem 1.6 gives a complete description of countably infinite Abelian groups admitting a Hausdorff MinAP group topology. This description has several applications (see [16, 17]).

2. Proof of Theorem 1.6

Let G be an Abelian group. If G is endowed with the discrete topology, we denote it by G_d. If κ is a cardinal number, we denote by $G^{(\kappa)}$ the direct sum of κ

copies of the group G. The subgroup of G generated by a subset A is denoted by $\langle A\rangle$.

Let X be an Abelian topological group. An element $x \in X$ is called a *topological generator* of X if $\langle x\rangle$ is a dense subgroup of X.

Following [9], for a sequence $\mathbf{d} = \{d_n\}_{n=0}^{\infty}$ of elements of an Abelian group G we set

$$s_{\mathbf{d}}((G_d)^\wedge) := \{x \in (G_d)^\wedge : (d_n, x) \to 1 \text{ in } \mathbb{T}\}.$$

Following Protasov and Zelenyuk (see [26, 27]), we say that a sequence $\mathbf{d} = \{d_n\}$ in an Abelian group G is a *T-sequence* if there is a Hausdorff group topology on G in which d_n converges to zero. The group G equipped with the finest group topology with this property is denoted by $(G, \tau_{\mathbf{d}})$.

Theorem 2.1 ([27, Theorem 2.1.5])**.** *Let G be a subgroup of an Abelian Hausdorff topological group S. Suppose a sequence $\{a_n\}_{n=0}^{\infty}$ converges to zero and a sequence $\{b_n\}_{n=0}^{\infty}$ converges to an element $b \in S$ satisfying the condition $\langle b\rangle \cap G = \{0\}$. For every $n \geq 0$, set*

$$d_{2n} = a_n \quad \textit{and} \quad d_{2n+1} = b_n.$$

Then $\mathbf{d} = \{d_k\}_{k=0}^{\infty}$ is a T-sequence in G.

Recall (see [3]) that a sequence $\mathbf{d} = \{d_n\}$ is called a *TB-sequence* in an Abelian group G if there is a precompact Hausdorff group topology on G in which $d_n \to 0$. The next lemma is a reformulation of Lemma 5.2 in [4]; for the sake of completeness we prove it.

Lemma 2.2. *Let $\pi : G \to X$ be a continuous monomorphism from a discrete countably infinite Abelian group G to a compact metrizable Abelian group X with dense image. For a sequence $\mathbf{d} = \{d_n\}$ in G the following assertions are equivalent:*

(i) $\pi^\wedge(X^\wedge) \subseteq s_{\mathbf{d}}(G^\wedge)$*, where $\pi^\wedge$ is the adjoint homomorphism of π.*

(ii) *$\pi(d_n)$ converges to zero in X.*

In particular, if (i) and (ii) hold, then $\mathbf{d}$ is a TB-sequence in G.

Proof. (i)⇒(ii) By definition and assumption, we have $(\pi(d_n), y) = (d_n, \pi^\wedge(y)) \to 1$ for every $y \in X^\wedge$. But this is possible only if $\pi(d_n) \to 0$, since the topology of X is determined by its continuous characters $y \in X^\wedge$.

(ii)⇒(i) Let $y \in X^\wedge$. Then $(\pi^\wedge(y), d_n) = (y, \pi(d_n)) \to 1$. Thus $\pi^\wedge(y) \in s_{\mathbf{d}}(G^\wedge)$, and hence $\pi^\wedge(X^\wedge) \subseteq s_{\mathbf{d}}(G^\wedge)$.

If (i) and (ii) hold, then $s_{\mathbf{d}}(G^\wedge)$ is dense in X because $\pi^\wedge$ has dense image by [19, 24.41]. Thus $\mathbf{d}$ is a TB-sequence in G (see [9]). □

We use this to prove the following theorem.

Theorem 2.3. *Let a countably infinite Abelian group G be isomorphic to a dense subgroup of a compact connected second countable Abelian group X. Then G admits a complete sequential Hausdorff minimally almost periodic group topology generated by a T-sequence.*

Proof. Let $\pi : G_d \to X$ be a continuous monomorphism with dense image. So $\pi^\wedge$ is injective and has dense image. Since $X^\wedge$ is countable [19, 24.15], Theorem 1.4 of [8] (or alternatively, Theorem 3.1 of [5]) implies that there exists a sequence $\mathbf{a} = \{a_n\}_{n=0}^{\infty}$ in G such that $s_{\mathbf{a}}((G_d)^\wedge) = \pi^\wedge(X^\wedge)$. By Lemma 2.2, $\pi(a_n) \to 0$ in X.

Denote by Ω the set of all topological generators of X. The set Ω is a dense G_δ-subset of X as it coincides with the intersection $\bigcap_{\chi\in X^\wedge\setminus\{0\}} \chi^{-1}(\mathbb{T}\setminus(\mathbb{Q}/\mathbb{Z}))$. So Ω has size continuum. Hence there exists $b\in\Omega$ such that $\pi(G)\cap\langle b\rangle=\{0\}$.

Let a sequence $\mathbf{b}=\{b_n\}_{n=0}^\infty$ in G be such that $\pi(b_n)$ converges to b in X. Then, by Theorem 2.1, the sequence $\mathbf{d}=\{d_n\}_{n=0}^\infty$, where for every $n\geq 0$

$$d_{2n}=a_n \quad \text{and} \quad d_{2n+1}=b_n$$

is a T-sequence in G.

By [13, Theorem 4], to show that $(G,\tau_{\mathbf{d}})$ is MinAP, it is enough to prove that $s_{\mathbf{d}}\left((G_d)^\wedge\right)=\{0\}$. Note that

$$s_{\mathbf{d}}\left((G_d)^\wedge\right)=s_{\mathbf{a}}\left((G_d)^\wedge\right)\cap s_{\mathbf{b}}\left((G_d)^\wedge\right)=\pi^\wedge\left(X^\wedge\right)\cap s_{\mathbf{b}}\left((G_d)^\wedge\right)$$

by the choice of $\mathbf{a}$. Now if $x\in s_{\mathbf{d}}\left((G_d)^\wedge\right)$, then $x=\pi^\wedge(y)$ for some $y\in X^\wedge$ and $(b_n,x)\to 1$. Since

$$(b_n,x)=(b_n,\pi^\wedge(y))=(\pi(b_n),y)\to(b,y),$$

we obtain $(b,y)=1$. But since b is a topological generator of X, this immediately yields $y=0$. Thus $x=0$ and $s_{\mathbf{d}}\left((G_d)^\wedge\right)$ is trivial.

Finally, Theorems 2.3.1 and 2.3.11 of [27] implies that the group $(G,\tau_{\mathbf{d}})$ is sequential and complete. □

We are now ready to prove Theorem 1.6.

Proof of Theorem 1.6. Corollary 9.4 of [10] implies that G is isomorphic to a dense subgroup of $\mathbb{T}^{\mathbb{N}}$. Now the assertion immediately follows from Theorem 2.3. □

For the sake of completeness we prove the next folklore proposition:

Proposition 2.4. *If an Abelian topological group G has a family of minimally almost periodic subgroups $\{G_i : i\in I\}$, such that their (not necessarily direct) sum $\sum_{i\in I} G_i$ is dense in G, then G is MinAP.*

Proof. Set $H:=\sum_{i\in I} G_i$. Let $\chi\in G^\wedge$. By assumption, $\chi|_{G_i}=0$ for every $i\in I$. Hence $\chi|_H=0$. Since H is dense in G, we obtain $\chi=0$. Thus G is MinAP. □

Proposition 2.4 immediately implies the next corollary:

Corollary 2.5. *Let $\{G_i\}_{i\in I}$ be a family of Hausdorff MinAP Abelian groups. Then the direct sum $\bigoplus_{i\in I} G_i$ endowed with the product topology is also MinAP.*

Corollary 2.6. *For every infinite cardinal κ and each natural number m, the group $G:=\mathbb{Z}(m)^{(\kappa)}$ admits a Hausdorff MinAP group topology.*

Proof. By [1,26], the group $H:=\mathbb{Z}(m)^{(\omega)}$ admits a Hausdorff MinAP group topology τ. Then $(H,\tau)^{(\kappa)}$ endowed with the product topology is MinAP by Corollary 2.5. Note that $G\cong H^{(\kappa)}$ algebraically. Now applying Corollary 2.5 we obtain that the group G admits a Hausdorff MinAP group topology. □

The main open problem still remaining is:

Question 2.7. Let G be an uncountable Abelian group of infinite exponent (for example, G is an uncountable torsion free Abelian group). Does G admit a Hausdorff MinAP group topology?

However, we prove the following special case. The set of all prime numbers we denote by $\mathbb{P}$.

Proposition 2.8. *Let $G=\bigoplus_{i\in I} G_i$, where I is a non-empty set of indices and G_i is a nonzero countable Abelian group for every $i \in I$. If G is of infinite exponent, then G admits a Hausdorff MinAP group topology.*

Proof. The idea of our proof is the following. Using purely combinatorial arguments we show first that $G=G'\oplus G''$, where G' is a direct sum of unbounded countable groups, while G'' is a direct sum of homogeneous bounded groups H_j (i.e., each H_j is a direct sum of infinitely many copies of some fixed cyclic p-group, where p may depend on j). Then we apply Corollary 2.5 (via Theorem 1.6) and Corollary 2.6.

Let I_1 be the set of all indices i such that G_i is of finite exponent. Set $I_2=I\setminus I_1$. We distinguish between two cases.

Case 1. $I_1=\emptyset$. By Theorem 1.6, for every $i\in I$ the group G_i admits a Hausdorff MinAP group topology. Thus G also admits a Hausdorff MinAP group topology by Corollary 2.5.

Case 2. $I_1\neq\emptyset$. For every $i\in I_1$, the group G_i is a direct sum of finite cyclic groups [11, 11.2]:

$$G_i=\bigoplus_{(p,n)\in A_i}\mathbb{Z}(p^n)^{(k^i_{(p,n)})},$$

where $A_i\subset\mathbb{P}\times\mathbb{N}$ is finite and $k^i_{(p,n)}$ are non-zero cardinal numbers.

Set $C:=\bigcup_{i\in I_1}A_i$ and $k_{(p,n)}:=\sum_{i\in I_1}k^i_{(p,n)}$. Put $C_1:=\{(p,n)\in C:\ k_{(p,n)}<\infty\}$ and $C_2=C\setminus C_1$. Then we can represent the group G in the following form:

$$G=\left(\bigoplus_{(p,n)\in C_1}\mathbb{Z}(p^n)^{(k_{(p,n)})}\right)\oplus\bigoplus_{(p,n)\in C_2}\mathbb{Z}(p^n)^{(k_{(p,n)})}\oplus\bigoplus_{i\in I_2}G_i. \tag{2.1}$$

Set $H:=\bigoplus_{(p,n)\in C_1}\mathbb{Z}(p^n)^{(k_{(p,n)})}$ and $H_{p,n}:=\mathbb{Z}(p^n)^{(k_{(p,n)})}$ for $(p,n)\in C_2$. By Theorem 1.6 and Corollary 2.6, for each $(p,n)\in C_2$ and each $i\in I_2$, the groups $H_{p,n}$ and G_i admit a Hausdorff MinAP group topology.

Subcase 2(*a*). Assume that H is either countably infinite (and hence $\exp(H)=\infty$) or trivial. By Theorem 1.6, the group H in (2.1) admits a Hausdorff MinAP group topology. Thus G also admits a Hausdorff MinAP group topology by Corollary 2.5.

Subcase 2(*b*). Let H be non-zero and finite. If $I_2\neq\emptyset$, take arbitrarily $i_0\in I_2$. Then we have

$$G=(H\oplus G_{i_0})\oplus\bigoplus_{(p,n)\in C_2}H_{p,n}\oplus\bigoplus_{i\in I_2,i\neq i_0}G_i. \tag{2.2}$$

By Theorem 1.6, the group $H\oplus G_{i_0}$ in (2.2) admits a Hausdorff MinAP group topology. Hence G also admits a Hausdorff MinAP group topology by Corollary 2.5.

Assume that $I_2=\emptyset$. Then C_2 is infinite since $\exp(G)=\infty$. Since $H_{p,n}\cong\mathbb{Z}(p^n)\oplus H_{p,n}$ we have

$$G\cong\left(H\oplus\bigoplus_{(p,n)\in C_2}\mathbb{Z}(p^n)\right)\oplus\bigoplus_{(p,n)\in C_2}H_{p,n}. \tag{2.3}$$

Also in this case, by Theorem 1.6, the group $H \oplus \bigoplus_{(p,n)\in C_2} \mathbb{Z}(p^n)$ in (2.3) admits a Hausdorff MinAP group topology. Therefore G admits a Hausdorff MinAP group topology by Corollary 2.5. $\Box$

Since a divisible group is a direct sum of full rational groups $\mathbb{Q}$ and groups of the form $\mathbb{Z}(p^\infty)$ [11, Theorem 19.1], we obtain the following:

Corollary 2.9. (i) [28] *Every free Abelian group admits a Hausdorff MinAP group topology.*

(ii) [28] *Each divisible Abelian group admits a Hausdorff MinAP group topology.*

(iii) *Every linear space over $\mathbb{R}$ or $\mathbb{C}$ admits a Hausdorff MinAP group topology.*

(iv) *The circle group $\mathbb{T}$ admits a Hausdorff MinAP group topology.*

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