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EVERY 3-MANIFOLD ADMITS A STRUCTURALLY STABLE NONSINGULAR FLOW WITH THREE BASIC SETS

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Abstract. This paper is devoted to proving that every closed orientable 3-manifold admits a simple Smale flow X_t. Here a simple Smale flow is a structurally stable nonsingular flow whose chain recurrent set is composed of a periodic orbit attractor, a periodic orbit repeller and a transitive saddle invariant set, i.e., a saddle basic set.

1. Introduction

1.1. **Background and main result.** A qualitative study of the dynamical behavior of a structurally stable system is a classical topic in dynamical systems. In the case of flows, people always hope to classify flows up to topological equivalence. But normally this goal is too complicated, even in dimension three. Partially motivated by this question, people are interested in some special flows; for instance, Morse-Smale flows ([Mo]), Smale flows ([Fr1], [Fr2]), Anosov flows ([Fe]), etc. Here, a Smale flow is a structurally stable flow such that the topological dimension of the chain recurrent set of the flow is no more than one and a Morse-Smale flow is a Smale flow such that the chain recurrent set is the union of finitely many periodic orbits and singular points.

For a fixed type of flows, it is natural to ask which 3-manifolds admit one of these kinds of flows.

- In the case of Morse-Smale flows, Morgan [Mo] nearly answered this question. In particular, an irreducible 3-manifold admits a nonsingular Smale flow if and only if it is a graph manifold.
- In the case of transitive Anosov flows, it is still an open question to decide which 3-manifolds admit Anosov flows. But there are some well-known obstructions. For instance, the manifold should admit foliations without Reeb components. See, for instance, [Fe] and [BBY1].
- Different from the two kinds of flows above, in the case of Smale flows, Proposition 6.1 in [Fr1] and the fundamental theorem of surgery on 3-manifolds by Lickorish and Wallace (see Chapter 9 of [Ro]) imply[1] that every closed orientable 3-manifold admits a nonsingular Smale flow.

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[1]Proposition 6.1 of [Fr1] says that for every link, there exists a nonsingular Smale flow on S^3 with this link as the set of attractors. Lickorish and Wallace's theorem says that for every closed orientable 3-manifold M, there exists a link L in S^3 so that M can be obtained by doing a Dehn surgery on L. From these two facts, one can easily get that every closed orientable 3-manifold admits a nonsingular Smale flow by some standard cut and paste arguments.

In this paper, we will discuss the realization question on 3-manifolds for a very special class of Smale flows, so-called *simple Smale flows*. A simple Smale flow is a smooth flow whose chain recurrent set is composed of a periodic orbit attractor, a periodic orbit repeller and a transitive saddle invariant set, i.e., a saddle basic set. This kind of flow is called a *simple Smale* flow after [Su]. Here, the adjective "simple" makes sense because of the following.

(1) The number of the basic sets[2] is small.
(2) The transitive saddle invariant set of the flow can be well described. On the one hand, by Bowen [Bo], the invariant set is abstractly topologically conjugate to a suspension of some subshift of finite type; on the other hand, by Birman and Williams [BW], the dynamical behavior of a neighborhood of the transitive saddle invariant set can be held very well by a compact branched surface with semiflow, which is called a template.

In the papers [Su], [Yu] and [HS], the authors analyze simple Smale flows on S^3 with some special subshift of finite types. Moreover, in a forthcoming paper [BBY2], the authors more systematically discuss the classification question of simple Smale flows on S^3. In this paper, we will discuss which 3-manifolds admit simple Smale flows. This discussion can be regarded as the first step in the classification of simple Smale flows on all 3-manifolds. We state the main result of this paper as follows.

Theorem 1.1. *Every closed orientable* 3*-manifold* M *admits a simple Smale flow.*

Notice that it is easy to show that a 3-manifold admits a nonsingular Smale flow with two basic sets if and only if the manifold is a lens space. Therefore, our result is sharpest in the category of Smale flows.

The proof of Theorem 1.1 strongly depends on the following key lemma, which may be valuable to the study of diffeomorphisms on surfaces.

Lemma 1.2. *Let* Σ *be a genus* g ($g \geq 1$) *once-punctured orientable surface and let* f *be a homeomorphism on* Σ*. Then there exists a diffeomorphism* h *on* Σ *which is isotopic to* f *and the chain recurrent set of* h *is composed of an isolated fixed point attractor* O *and a zero-dimensional transitive saddle invariant set* Λ_0*.*

1.2. **A further question.** If we consider all structurally stable flows, Theorem 1.1 naturally motivates us to ask a tough question: does every closed orientable 3-manifold admit a structurally stable flow so that the number of the basic sets is no more than two?

If there is exactly one basic set in a structurally stable flow X_t on a closed orientable 3-manifold M, then X_t is a transitive Anosov flow. As we have mentioned before, there are many topological obstructions for a closed orientable 3-manifold admitting an Anosov flow.

If X_t is a structurally stable flow on a closed orientable 3-manifold M whose chain recurrent set is composed of two basic sets, then one of them is a transitive repeller Λ_R and the other is a transitive attractor Λ_A. Moreover, there are two canonical neighborhoods (normally, they are called the filtrating neighborhoods) $N(\Lambda_R)$ and $N(\Lambda_A)$ of Λ_R and Λ_A respectively.

[2]Recall that in uniform hyperbolic theory, a basic set is a compact invariant set with a dense orbit and the chain recurrent set of a structurally stable flow on a closed manifold can always be decomposed to the union of finitely many pairwise disjoint basic sets

There are three cases for Λ_A and $N(\Lambda_A)$:

(1) Λ_A is an isolated singularity and $N(\Lambda_A)$ is a three-ball,
(2) Λ_A is a periodic orbit and $N(\Lambda_A)$ is a solid torus,
(3) Λ_A is an expanding attractor and $N(\Lambda_A)$ is a compact irreducible 3-manifold with finitely many tori boundaries (see [Br]).

It is similar for Λ_R and $N(\Lambda_R)$. Then by some standard combinatorial arguments, (M, X_t) should be one of following four cases:

(1) Each of Λ_R and Λ_A is an isolated singularity. In this case, M is homeomorphic to a three-sphere.
(2) Each of Λ_R and Λ_A is an isolated periodic orbit. In this case, M is homeomorphic to a lens space.
(3) Λ_R is an expanding repeller and Λ_A is an expanding attractor. In this case, M is a Haken manifold since there exists an incompressible torus (see [Br]).
(4) Λ_R is an isolated periodic orbit and Λ_A is an expanding attractor. In this case, there are no known topological obstructions for M.

So, now we can ask the following more concrete question.

Question 1.3. Does every closed orientable 3-manifold admit a nonsingular flow whose chain recurrent set is composed of an expanding attractor and a periodic orbit repeller?

Remark 1.4. The positive answer to Question 1.3 can imply Theorem 1.1. More precisely, if a closed orientable 3-manifold M admits a nonsingular flow with an expanding attractor Λ_A and a periodic orbit repeller Λ_R, then M admits a simple Smale flow Y_t. Y_t can be obtained by doing an unstable DA surgery along a periodic orbit of the expanding attractor. For more details about DA surgery, see [BBY1].

2. Preliminaries

A well-known theorem of Bowen ([Bo]) implies that a nonsingular transitive saddle invariant set Λ in a three-dimensional flow is abstractly topologically equivalent to the suspension of a subshift of finite type. But to recognize a basic set, such a description sometimes is not enough. One reason is that one can't read the hyperbolicity. Actually, people usually recognize a transitive saddle invariant set of a three-dimensional flow X_t by the suspension of a diffeomorphism f on some surface so that the maximal invariant set of f is a zero-dimensional transitive saddle invariant set. Moreover, a zero-dimensional transitive saddle invariant set can always be recognized as follows.

Proposition 2.1. *Let f be a diffeomorphism on a surface S. If we can choose finitely many pairwise disjoint disks $\Sigma = \bigsqcup_{i=1}^{n} D_i$ of S, so that they satisfy the following conditions, then the nontrivial maximal invariant set of f on Σ, $\bigcap_{n=-\infty}^{+\infty} f^n(\Sigma)$, is a zero-dimensional transitive saddle invariant set:*

(1) *Each D_i can be parameterized by $\Psi_i : [0,1] \times [0,1] \to D_i$. We call $\mathcal{F}^s = \{\Psi_i(\{x\} \times [0,1])\}$ and $\mathcal{F}^u = \{\Psi_i([0,1] \times \{y\})\}$ the stable and unstable foliations respectively. For every $p \in \Sigma$, there exist a unique stable leaf $W_s(p)$ and a unique unstable leaf $W_u(p)$ both of which contain p.*

(2) *f restricted on* Σ *satisfies:*
   (a) *for every* $p \in \Sigma$*, if* $f(W_s(p)) \cap \Sigma \neq \emptyset$*, then* $f(W_s(p))$ *is a subset of* $W_s(q)$ *for some* $q \in \Sigma$*;*
   (b) *for every* $p \in \Sigma$*, if* $q \in f(W_u(p))$*, then* $W_u(q) \subset f(W_u(p))$*;*
   (c) *f is uniformly attractive on* $\mathcal{F}^s$ *and uniformly expansive on* $\mathcal{F}^u$*;*
   (d) *f satisfies an irreducible property: for every two disks* $D_i, D_j \in \Sigma$*, there exist disks* $D^0, D^1, \ldots, D^m \in \{D_k\}$ *so that* $D^0 = D_i, D^m = D_j$*,* $\Phi(D^m) \cap \Phi(D^0) \neq \emptyset$ *and* $\Phi(D^s) \cap \Phi(D^{s+1}) \neq \emptyset$ $(s \in \{0, \ldots, m-1\})$*.*

Now we introduce a fibered knot in three-dimensional topology which will be the cornerstone of our constructive proof of Theorem 1.1.

Definition 2.2. Let M be a closed orientable 3-manifold. Then a simple closed curve $\gamma \subset M$ is called a *fibered knot* if γ satisfies the following conditions:

- $M - \gamma$ is homeomorphic to a once-punctured surface bundle over a circle, i.e., $\Sigma \times [0,1]/(x,0) \sim (f(x),1)$; here Σ is a once-punctured surface.
- $\partial\Sigma \times \{0\}$ is isotopic to γ in M.

The following theorem is a classical result in three-dimensional topology; see Chapter 10 of Rolfsen's book [Ro].

Theorem 2.3. *For every closed orientable* 3*-manifold* M*, there exists a fibered knot* $\gamma \subset M$ *so that* $M - \gamma$ *is homeomorphic to* $\Sigma \times [0,1]/(x,0) \sim (f(x),1)$*. Here* Σ *is a genus* g $(g \geq 1)$ *once-punctured orientable surface.*

Remark 2.4. The genus of Σ usually is called the genus of the fibered knot γ. In the case where the Heegaard genus of M is no more than one, the genus of γ constructed in [Ro] is zero. But if one does a standard surgery in low-dimensional topology, i.e., stabilization, then we can obtain a new fibered knot γ' whose genus is more than zero. See [Et] for more details.

3. Proof of the main theorem

Under the key lemma, i.e. Lemma 1.2, we can easily finish the proof as follows.

Proof of Theorem 1.1. By Theorem 2.3, there exists a simple closed curve $\gamma \subset M$ such that $M - \gamma$ is homeomorphic to a once-punctured surface bundle over a circle, i.e., $\Sigma \times [0,1]/(x,0) \sim (f(x),1)$; here Σ is a once-punctured surface. By Lemma 1.2, we can build a flow Z_t on $M_0 = \Sigma \times [0,1]/(x,0) \sim (f(x),1) \subset M$ which is the suspension of h on Σ. There exists a torus $T \subset M_0$ which is closed to γ and transverse to Z_t so that T bounds a solid torus V so that V is a tubular neighborhood of γ in M. Then we can find a smooth flow X_t on M such that $X_t|(M-V) = Z_t|(M-V)$ and V is a filtrating neighborhood of an isolated repeller γ_r which is isotopic to γ in V. Let Λ be the suspension of Λ_0 and let γ_a be the suspension of O. Then γ_a and Λ are an isolated attractor and a one-dimensional transitive saddle invariant set of X_t correspondingly. By the construction, the chain recurrent set of X_t is the union of γ_a, γ_r and Λ. Therefore, X_t is a simple Smale flow on M. $\square$

Now we are left to prove the key lemma, Lemma 1.2.

Let $E = [0,1] \times [0,1]$. Assume that $(a^1, a^2) = (\{0\} \times [0,1], \{1\} \times [0,1]) \subset E$ is the *attaching edges pair* of E. We call E with attaching edges pair (a^1, a^2) a 1*-handle* and call $a = [0,1] \times \{\frac{1}{2}\}$ the *core* of the 1-handle. We call $\mathcal{F}_s = \{\{x\} \times [0,1]\}, x \in [0,1]$, and $\mathcal{F}_u = \{[0,1] \times \{y\}\}, y \in [0,1]$, the stable and unstable foliation of E correspondingly.

Let Σ_g be a genus g ($g \geq 1$) orientable compact surface with one boundary. Then Σ_g can be parameterized by attaching $2g$ 1-handles $A_1, B_1, A_2, B_2, \dots, A_g, B_g$ to a 0-handle D so that they satisfy the following gluing rule. Let (c_i^1, c_i^2) and (d_i^1, d_i^2) be the attaching edges of A_i and B_i correspondingly. The gluing rule is that the gluing order of these attaching edges along ∂D is as follows: $c_1^1, d_1^1, c_1^2, d_1^2, c_2^1, d_2^1, \dots, c_g^2, d_g^2$. We denote the cores of $A_1, B_1, A_2, B_2, \dots, A_g, B_g$ by $a_1, b_1, a_2, b_2, \dots, a_g, b_g$ correspondingly. We can choose a point O as the center of D. For each $i \in \{1, \dots, g\}$, we can connect two segments starting at O and ending at the two ends of a_i (or b_i). The union of all these segments, all the edges a_i and b_i ($i \in \{1, \dots, g\}$) and the point O form a graph $G \subset \Sigma_g$ which is $2g$ circles over one point O. We define these circles α_i and β_i ($i \in \{1, \dots, g\}$) which contain a_i and b_i as sub-arcs correspondingly. We call G a *core* of Σ_g. Figure 1 illustrates this notation in Σ_2.

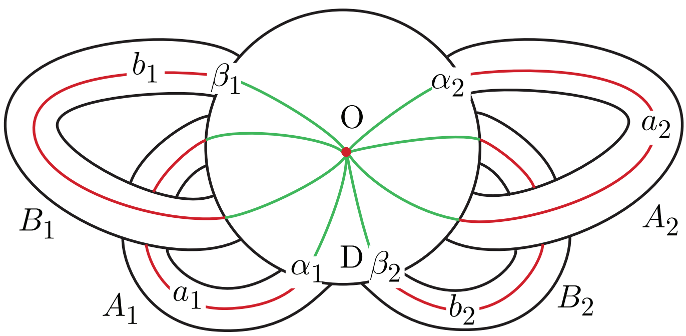

FIGURE 1. Σ_g with this notation

Lemma 3.1. *Let Σ be a genus g once-punctured orientable surface and let f be a homeomorphism on Σ. Then there exists a diffeomorphism f_0 on Σ and a simple closed curve $c \subset \Sigma$ which satisfy the following conditions:*

(1) *c cuts Σ to an annulus A and a genus g compact surface Σ_g so that $\partial\Sigma$ is a connected component of ∂A. We parameterize Σ_g as above.*
(2) *f_0 is isotopic to f.*
(3) *Σ_g and D are two attracting regions of f_0, i.e., $f_0(\Sigma_g)$ and $f_0(D)$ are in the interiors of Σ_g and D respectively.*
(4) *The maximal invariant set of f_0 restricted to D is a sink O, i.e., $\bigcap_{n=1}^{\infty}(f_0^n(D)) = \{O\} \in D$.*
(5) *If $x \in A_i \cap f_0^{-1}(B_j)$, then $f_0(W^s(x)) \subset W^s(f_0(x))$ and $W^u(f_0(x)) \subset f_0(W^u(x))$. Moreover, f_0 is exponentially attractive on $W^s(x)$ and exponentially expansive on $W^u(x)$. It is similar for the points in $A_i \cap f_0^{-1}(A_j)$, $B_i \cap f_0^{-1}(A_j)$ and $B_i \cap f_0^{-1}(B_j)$.*
(6) *$\Sigma - \Sigma_g$ is a wandering set of f_0. More precisely, for every $x \in \Sigma - \Sigma_g$, there exists $n \in \mathbb{N}$ such that $f_0^n(x) \in \Sigma_g$.*

Proof. First we choose a simple closed curve c which cuts Σ to an annulus A and a genus g compact surface Σ_g. Σ_g can be parameterized as above.

G is a core of Σ_g and $f(G)$ is an embedded graph which is homeomorphic to $2g$ circles with one vertex. We can push $f(G)$ up to isotopy to G^1 which satisfies the following two conditions:

- The vertex of G^1 is O.
- If an edge e of G^1 satisfies $e \cap A_i \neq \emptyset$ (or $e \cap B_i \neq \emptyset$), then e is transverse to the stable foliation of A_i (or B_i).

We denote the edge of G^1 corresponding to α_i (β_i) by α_i^1 (β_i^1) ($i \in \{1, \dots, g\}$).

We choose a small neighborhood of G^1, $N(G^1)$, which admits a handle decomposition with one 0-handle D^1 and $2g$ 1-handles A_i^1 and B_i^1 ($i \in \{1, \dots, g\}$). They satisfy the following conditions:

- $N(G^1) \subset \Sigma_g$.
- The center of D^1 is O and D^1 is in the interior of D.
- The cores of A_i^1 and B_i^1 are a_i^1 and b_i^1 ($i \in \{1, \dots, g\}$) correspondingly.
- For any $x \in A_i^1 \cap B_j$, the stable leaf of x in A_i^1 is in the interior of a stable leaf of x in B_j and the unstable leaf of x in A_i^1 is transverse to the stable foliation of B_j. It is similar to (A_i^1, A_j), (B_i^1, B_j) and (B_i^1, A_j) ($i, j \in \{1, \dots, g\}$).

Since G^1 is isotopic to $f(G)$ in Σ, there exists a homeomorphism f_1 on Σ such that:

(1) f_1 is isotopic to f.
(2) $f_1(\Sigma_g) = N(G^1)$.
(3) $f_1|D$ is exponentially contractive and $f_1(O) = O$.
(4) f_1 preserves the stable and unstable foliations of 1-handles from the handle decomposition of Σ_g to the handle decomposition of $N(G^1)$. Moreover, f_1 is exponentially attractive and exponentially expansive on the stable and unstable foliations respectively.

Moveover, since $\Sigma_g - N(G^1)$ is homeomorphic to an annulus and $\Sigma - \Sigma_g$ is homeomorphic to a once-punctured disk, it is easy to build a homeomorphism f_0 on Σ which is an extension of f_1 so that, for every $x \in \Sigma - \Sigma_g$, there exists $n \in \mathbb{N}$ so that $f_0^n(x) \in \Sigma_g$.

Now it is easy to check that f_0 satisfies all the conditions in the lemma. □

Remark 3.2. Generally, the chain recurrent set of f_0 is the union of the isolated fixed point attractor O, finitely many saddle periodic orbits and finitely many nontrivial saddle basic sets. Here a trivial saddle basic set means a saddle periodic orbit. The maximal invariant set of a Smale horseshoe map is a canonical example of nontrivial saddle basic sets.

Proof of Lemma 1.2. We begin our construction from G^1 in the proof of Lemma 3.1. Let $c = \partial\Sigma_g$ and let w be a small arc in c which doesn't intersect the 1-handles of Σ_g. We can push the closure of $c - w$ a little into Σ_g to a new arc u_1 with two ends P_1 and Q_1 so that:

(1) u_1 transversely intersects each 1-handle of Σ_g with stable foliation,
(2) u_1 is disjoint to G^1.

We choose one point R_1 in the intersection of α_1^1 and the interior of D. Since $\Sigma_g - (G^1 \cup u_1)$ is connected and for every $i \in \{1, \dots, g\}$, either $A_i - (G^1 \cup u_1)$ or

$B_i - (G^1 \cup u_1)$ is the union of finitely many rectangles foliated by sub-leaves of the stable leaves, there exists an arc v_1 in Σ_g so that:

(1) $\partial v_1 = v_1 \cap (G^1 \cup u_1) = \{Q_1, R_1\}$.

(2) v_1 is transverse to the stable foliations of the 1-handles of Σ_g.

Let l_1 be the union of u_1 and v_1 which is an interval with two ends P_1 and R_1. Obviously l_1 transversely intersects to each 1-handle of Σ_g. Figure 2 illustrates an example of u_1 and v_1 in Σ_1 so that f can be represented by $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ under a suitable coordinate.

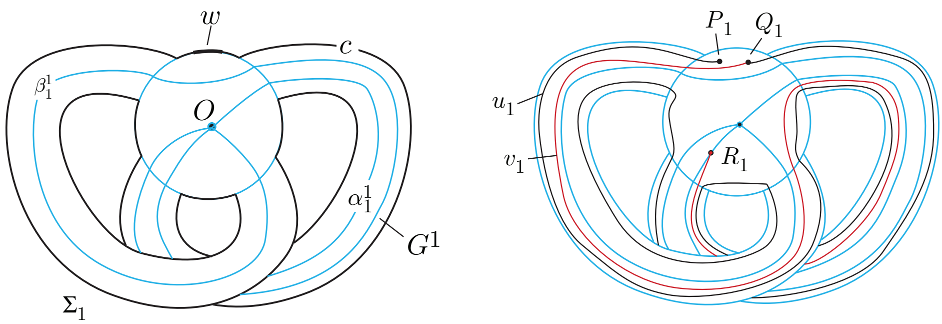

FIGURE 2. An example of u_1 and v_1 in Σ_1

We choose a small interval neighborhood of R_1 in α_1^1, say $\overline{R_1^1R_1^2}$, so that it is an arc in the interior of D with two ends R_1^1 and R_1^2. Then we can push the short interval $\overline{R_1^1R_1^2}$ along a small neighborhood of l_1 to a long interval $\widehat{R_1^1R_1^2}$ relative to the ends R_1^1 and R_1^2 so that $\widehat{R_1^1R_1^2}$ satisfies the following conditions (see Figure 3 as an illustration):

(1) $\widehat{R_1^1R_1^2} \cap l_1 = \emptyset$.

(2) $\widehat{R_1^1R_1^2}$ transversely intersects to each 1-handle of Σ_g with stable foliation.

We obtain a_1^2 from a_1^1 by replacing $\overline{R_1^1R_1^2}$ by $\widehat{R_1^1R_1^2}$. Globally, G^1 is replaced by G^2 after this surgery. Obviously, the new graph G^2 is isotopic to G^1 and a_1^2 transversely intersects to each 1-handle of Σ_g with stable foliation. These two facts are important in the following.

Next, we will do the same surgeries one by one along $b_1^1, a_2^1, b_2^1, \ldots, a_g^1, b_g^1$. First, we obtain G^3 by a surgery on G^2 along b_1^1. Then we obtain G^4 by a surgery on G^3 along a_2^1. Repeating this procedure, finally we obtain G^{2g+1} by a surgery on G^{2g} along b_g^1. One can easily check that G^{2g+1} satisfies the following conditions:

(1) G^{2g+1} is isotopic to G^1 and the vertex of G^{2g+1} is O.

(2) Every edge of G^{2g+1} transversely intersects each 1-handle of Σ_g with stable foliation.

Now, similar to the construction of f_0 by G^1 in the proof of Lemma 3.1, we can construct homeomorphism h on Σ from G^{2g+1} which satisfies all the conclusions in Lemma 3.1. Moveover, since every edge of G^{2g+1} transversely intersects each 1-handle of Σ_g with stable foliation, h restricted to $\overline{\Sigma_g - D}$ satisfies the irreducible

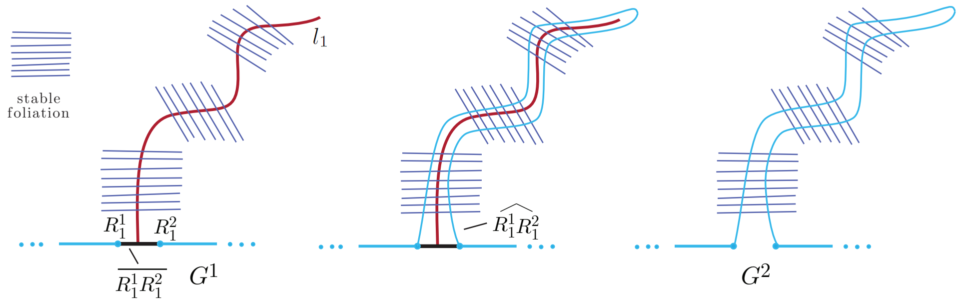

FIGURE 3. The pushing surgery

property in Proposition 2.1. All these facts imply that h restricted to $\overline{\Sigma_g - D}$ satisfies all conditions in Proposition 2.1. Therefore, the maximal invariant set of h restricted to $\overline{\Sigma_g - D}$ is a zero-dimensional transitive saddle invariant set Λ_0.

On the other hand, by the construction of h, the chain recurrent set of h on Σ is composed of an isolated attractor $\{O\}$ and the maximal invariant set of h restricted to $\overline{\Sigma_g - D}$, i.e., Λ_0. □

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