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RIGIDITY THEOREMS FOR COMPACT HYPERSURFACES IN LOCALLY SYMMETRIC RIEMANNIAN MANIFOLDS

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Abstract. In this paper, we prove some rigidity theorems for compact hypersurfaces without the constancy condition on the mean curvature or the scalar curvature in locally symmetric Riemannian manifolds.

1. Introduction

When the ambient manifolds possess very nice symmetry, for example the sphere, many results have been obtained in the study of the minimal hypersurface and hypersurface with constant mean curvature or constant scalar curvature in these ambient manifolds. (One can see [1]-[7]). Recently, Q.M. Cheng and H. Nakagawa [8] independently proved the optimal rigidity theorem for hypersurface of constant mean curvature in a sphere.

In order to study hypersurface with constant scalar curvature, Cheng and Yau [9] introduced a new self-adjoint differential operator $\square$ acting on C^2-functions defined on Riemannian manifolds. As a by-product of this approach they were able to classify closed hypersurface M^n with constant normalized scalar curvature R satisfying $R \geq c$ and non-negative sectional curvatures immersed in complete and simply connected $(n+1)$-dimensional Riemannian manifolds of constant sectional curvature c, which will be denoted by $Q^{n+1}(c)$ and are also known as space forms.

By using the Cheng-Yau technique, X. Liu and H. Li [7] also obtained some rigidity theorems for hypersurface with constant scalar curvature. Therefore, it is important and natural to extend the Riemannian space forms to the locally symmetric Riemannian manifolds.

Let the ambient manifold N^{n+1} be a locally symmetric Riemannian manifold with sectional curvature K_N and M^n be an n-dimensional complete hypersurface with constant mean curvature H in N^{n+1}. When $\frac{1}{2} < \delta \leq K_N \leq 1$ (δ is a constant) at all points $x \in M^n$ and the squared norm of the second fundamental form S satisfies $S < n + \frac{n^3H^2}{2(n-1)} - \frac{n(n-2)}{2(n-1)}\sqrt{n^2H^4 + 4(n-1)H^2}$, S. Shu [10] and S. Ding [11] have obtained that the hypersurface M^n is a totally umbilical hypersurface, respectively. H.W. Xu [12] has also obtained the same result when M^n is an n-dimensional closed minimal hypersurface with constant mean curvature H in N^{n+1} and sectional curvature K_N satisfying the condition $\delta \leq K_N \leq 1$ at all points $x \in M^n$ and the squared norm of the second fundamental form S satisfies $S \leq (2\delta - 1)n$.

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In [13], Q. Wang and C. Xia proved that M^n is isometric to a Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$ with $c^2 \leq (n-1)/n$ if the fundamental group $\pi_1(M^n)$ of M^n is infinite and that $S \leq n + n^3H^2/(2(n-1)) - n(n-2)|H|\sqrt{n^2H^2+4(n-1)}/(2(n-1))$. Here, motivated by the works described above, our aim is to study the closed hypersurfaces without the constancy condition on the mean curvature or the scalar curvature.

In this paper, let the ambient manifold N^{n+1} be an (n+1)-dimensional simply locally symmetric Riemannian manifold with δ pinched curvature, i.e. $\frac{1}{2} < \delta \leq K_N \leq 1$. From (2.4), we denote

$$n(n-1)P = n^2H^2 - S = n(n-1)r - \sum_{i,j=1}^{n} K_{ijij}, \tag{1.1}$$

where r is normalized scalar curvature of M^n. Here, our aim is to study the closed hypersurfaces in N^{n+1} without the constancy condition on the mean curvature or the scalar curvature. We prove some rigidity theorems for the Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$. Namely, we have the following theorems.

Theorem 1.1. *Let M^n be an n-dimensional compact hypersurface in a locally symmetric Riemannian manifold N^{n+1}. Assume that the fundamental group $\pi_1(M^n)$ of M^n is infinite and that the squared norm of the second fundamental form satisfies $S \leq \alpha(n,r)$. Then S is constant, $S = \alpha(n,r)$ and M^n is isometric to a Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$, where $\alpha(n,r) = (n-1)\frac{nP+2\delta}{n-2} + \frac{n-2}{nP+2\delta}$.*

Remark 1.1. If $\delta = 1$, i.e. the locally symmetric Riemannian manifold N^{n+1} is the unit sphere $S^{n+1}(1)$, our Theorem 1.1 reduces to Theorem 1.3 in [14].

Theorem 1.2. *Let M^n be an n-dimensional compact hypersurface in a locally symmetric Riemannian manifold N^{n+1}. Assume that the fundamental group $\pi_1(M^n)$ of M^n is infinite and that the squared norm of the second fundamental form and mean curvature satisfy $S \leq \alpha(n,H)$. Then S is constant, $S = \alpha(n,H)$ and M^n is isometric to a Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$, where $\alpha(n,H) = n\delta + \frac{n^3H^2}{2(n-1)} - \frac{n(n-2)}{2(n-1)}\sqrt{n^2H^4 + 4(n-1)H^2\delta}$.*

Remark 1.2. When the manifold N^{n+1} is the unit sphere $S^{n+1}(1)$, our Theorem 1.2 reduces to Theorem 1.1 in [13].

Theorem 1.3. *Let M^n be an n-dimensional compact hypersurface in a locally symmetric Riemannian manifold N^{n+1}. Assume that the fundamental group $\pi_1(M^n)$ of M^n is infinite and that the squared norm of the second fundamental form satisfies $S \leq 2\sqrt{n-1}\delta$. Then S is constant, $S = 2\sqrt{n-1}\delta$ and M^n is isometric to a Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$.*

Corollary 1.4. *Let M^n be an n-dimensional compact hypersurface in $S^{n+1}(1)$. Assume that the fundamental group $\pi_1(M^n)$ of M^n is infinite and that the squared norm of the second fundamental form satisfies $S \leq 2\sqrt{n-1}$. Then S is constant, $S = 2\sqrt{n-1}$ and M^n is isometric to a Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$.*

2. Preliminaries

If M^n is a hypersurface in N^{n+1}, let $\{e_1, e_2, \cdots, e_{n+1}\}$ be a local frame of orthonormal vector fields in N^{n+1} such that, restricted to M^n, the vectors $\{e_1, e_2, \cdots, e_n\}$ are tangent to M^n, and the vector e_{n+1} is normal to M^n.

Let $\{\omega^1,\ \omega^2,\ \cdots, \omega^{n+1}\}$ be its dual frame field. We use the following convention on the range of indices:

$$1 \le A, B, C, \cdots \le n+1, \quad 1 \le i, j, k, \cdots \le n.$$

Then the structure equations of N^{n+1} are given by

$$d\omega_A = \sum_{B=1}^{n+1} \omega_{AB} \wedge \omega_B,\ \omega_{AB} + \omega_{BA} = 0,$$

$$d\omega_{AB} = \sum_{C=1}^{n+1} \omega_{AC} \wedge \omega_{CB} - \frac{1}{2} \sum_{C,D=1}^{n+1} R_{ABCD} \omega_C \wedge \omega_D,$$

where R_{ABCD} denotes the components of the Riemannian curvature tensor of N^{n+1}. Let M be an arbitrary hypersurface of N^{n+1}. The structure equations of M^n are given by

$$d\omega_{ij} = -\sum_k \omega_{ik} \wedge \omega_{kj} + \frac{1}{2} \sum_{k,l} R_{ijkl} \omega^k \wedge \omega^l, \tag{2.1}$$

$$R_{ijkl} = K_{ijkl} + h_{ik}h_{jl} - h_{il}h_{jk}, \tag{2.2}$$

where R_{ijkl} denotes the components of the Riemannian curvature tensor of M^n;

$$R_{ij} = \sum_{k=1}^n K_{kikj} + nHh_{ij} - \sum_{k=1}^n h_{ik}h_{kj}, \tag{2.3}$$

$$n(n-1)r = \sum_{i,j=1}^n K_{ijji} + (nH)^2 - S, \tag{2.4}$$

where R_{ij} and $n(n-1)r$ are components of the Ricci curvature tensor and the scalar curvature of M^n, respectively, and $S = \sum_{ij=1}^n (h_{ij})^2$ is the squared norm of the second fundamental form of M^n.

The following lemmas are needed in the proof of Theorems 1.1, 1.2, and 1.3.

Lemma 2.1 ([15]). *If the Ricci curvature of a compact Riemannian manifold is non-negative and positive at a point, then the manifold carries a metric of positive Ricci curvature.*

Lemma 2.2 ([16]). *Let $A = (a_{ij})$ be a symmetric $n \times n$ matrix $(n \ge 2)$, and set $A_1 = \mathrm{tr} A$ and $A_2 = \sum_{i,j} (a_{ij})^2$. Then we have*

$$\sum_i (a_{in})^2 - A_1 a_{nn} \le \frac{1}{n^2}[n(n-1)A_2 + (n-2)\sqrt{n-1}|A_1|\sqrt{nA_2 - (A_1)^2} - 2(n-1)(A_1)^2]. \tag{2.5}$$

Equality holds if and only if either $n = 2$ or $n > 2$ and (a_{ij}) is of the form

$$\begin{pmatrix} a & & & 0 \\ & \ddots & & \\ & & a & \\ 0 & & & A_1 - (n-1)a \end{pmatrix}$$

with $(na - A_1)A_1 \ge 0$.

We can assume without loss of generality that at any fixed point p, $e_1, \cdots, e_n$ is a local orthonormal frame field, and from Gauss' equation it follows that the Ricci curvature R_{ij} of M^n is

$$\mathrm{R}_{ij} = \sum_{k=1}^{n} K_{ikjk} + nHh_{ij} - \sum_{k=1}^{n} h_{ik}h_{kj},$$

then we have

$$\begin{aligned}\mathrm{R}_{ii} &= \sum_{k=1}^{n} K_{ikik} + nHh_{ii} - \sum_{i}(h_{ik})^2,\\ \mathrm{R}_{ij} &= 0 \qquad i \neq j.\end{aligned}$$

From Lemma 2.2, we have

$$\begin{aligned}\mathrm{R}_{ii} &\geq (n-1)\delta - \frac{1}{n^2}[n(n-1)S + n(n-2)\sqrt{n-1}|H|\sqrt{nS-n^2H^2} - 2n^2(n-1)H^2]\\ &= (n-1)\delta - \frac{n-1}{n}S - \frac{n-2}{\sqrt{n}}\sqrt{\frac{n-1}{n}}|H|\sqrt{S-nH^2} + 2n^2(n-1)H^2\\ &\geq \frac{n-1}{n}\left(n\delta + 2nH^2 - S - \frac{n(n-2)}{\sqrt{n(n-1)}}|H|\sqrt{S-nH^2}\right).\end{aligned}$$

3. Proof of the theorems

Proof of Theorem 1.1. It is a direct check that our assumption condition, i.e.

$$S \leq (n-1)\frac{nP+2\delta}{n-2} + \frac{n-2}{nP+2\delta},$$

is equivalent to

$$\frac{(n-2)^2}{n^2}\{(n(n-1)P+S)(S-nP)\} \leq \left(n\delta + 2(n-1)P - \frac{n-2}{n}S\right)^2.$$

From (2.4), then $S - nH^2 = \frac{n-1}{n}(S-nP)$, we have $S-nP \geq 0$ and $n(n-1)P+S \geq 0$. So, we get

$$\frac{n-2}{n}\sqrt{(n(n-1)P+S)(S-nP)} \leq n\delta + 2(n-1)P - \frac{n-2}{n}S.$$

Hence, we obtain

$$\begin{aligned}&n\delta - nH^2 + (S-nH^2) - \frac{n(n-2)}{\sqrt{n(n-1)}}|H|\sqrt{S-nH^2}\\ &= n\delta + 2(n-1)P + \frac{n-2}{n}S - \frac{n-2}{n}\sqrt{(n(n-1)P+S)(S-nP)}.\end{aligned}\tag{3.1}$$

This implies that

$$(S-nH^2)\left[n\delta - (S-nH^2) - n|H|\frac{n-2}{\sqrt{n(n-1)}}\sqrt{S-nH^2} + nH^2\right] \geq 0.\tag{3.2}$$

From (2.6), we have $R_{ii} \geq 0$. In particular, from the assertions above, we know that if $S < \alpha(n,r)$ holds, then $R_{ii} > 0$. This implies that, by Lemma 2.1, M^n carries a metric of positive Ricci curvature. According to Bonnet-Myer's theorem [17], we know that the fundamental group is finite. This is impossible because M^n has infinite fundamental group.

From Lemma 2.2, we can assume without loss of generality that at any fixed point $p \in M$, $R_{nn} = 0$. Therefore, from Lemma 2.2, all of the above inequalities should be equalities at p and $S = \alpha(n, r)$. That is, we have $K_{ijij} = \delta$, N^{n+1} is of constant sectional curvature δ, and $h_{ij} = 0$ if $i \neq j$, $h_{11} = \cdots = h_{n-1n-1}$ and $h_{nn} = nH - (n-1)h_{11}$. Hence, we conclude that M^n has two distinct principal curvatures, one of which is simple. Let $\{e_1, \cdots, e_n\}$ be a local orthonormal frame field such that $h_{ij} = \lambda_i \delta_{ij}$, where λ_i's are the principal curvature on M^n. Without loss of generality, we can assume that $\lambda_1 = \cdots = \lambda_{n-1} = \lambda$, $\lambda_n = \mu$. From

$$\mathrm{R}_{nn} = (n-1)\delta + (\lambda_1 + \cdots + \lambda_{n-1} + \lambda_n)\lambda_n - \lambda_n^2 = (n-1)(\delta + \lambda\mu) = 0,$$

we have $\delta + \lambda\mu = 0$. From (2.4), we have

$$\mu = \frac{nP}{2\lambda} - \frac{n-2}{2}\lambda.$$

Hence, we get

$$\lambda^2 = \frac{nP + 2\delta}{n-2} \ \textit{and}\ \mu^2 = \frac{n-2}{nP + 2\delta}.$$

Similarly as in the proof of the Theorem [14], we consider the integral submanifold of the corresponding distribution of the space of principal vectors corresponding to the principal curvature λ. Since the multiplicity of the principal curvature λ is greater than 1, we know that the principal curvature λ is constant on this integral submanifold. From $\lambda^2 = \frac{nP+2\delta}{n-2}$ and $\mu^2 = \frac{n-2}{nP+2\delta}$, we have that the scalar curvature $n(n-1)r$ and the principal curvature μ are constant. Thus, we obtain that M^n is the isoparametric hypersurface of the sphere N^{n+1} with two distinct principal curvatures. Therefore, $S = (n-1)\frac{nP+2\delta}{n-2} + \frac{n-2}{nP+2\delta}$, and we conclude that M^n is isometric to the Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$. This completes the proof of Theorem 1.1. □

Proof of Theorem 1.2. Let

$$A = \sqrt{S - nH^2} + \frac{n(n-2)|H|}{2\sqrt{n(n-1)}} + \left[\frac{n^3H^2}{4(n-1)} + n\delta\right]^{\frac{1}{2}},$$

$$B = -\sqrt{S - nH^2} - \frac{n(n-2)|H|}{2\sqrt{n(n-1)}} + \left[\frac{n^3H^2}{4(n-1)} + n\delta\right]^{\frac{1}{2}}.$$

By hypothesis

$$S \leq n\delta + \frac{n^3H^2}{2(n-1)} - \frac{n(n-2)|H|}{2(n-1)}\sqrt{n^2H^4 + 4(n-1)\delta}.$$

This implies that

$$S - nH^2 \leq \left(\left[\frac{n^3H^2}{4(n-1)} + n\delta\right]^{\frac{1}{2}} - \frac{n(n-2)|H|}{2\sqrt{n(n-1)}}\right)^2,$$

which jointly with $\sqrt{S - nH^2} \geq 0$ and $[\frac{n^3H^2}{4(n-1)} + n\delta]^{\frac{1}{2}} > \frac{n(n-2)|H|}{2\sqrt{n(n-1)}}$ implies that

$$\sqrt{S - nH^2} \leq \left[\frac{n^3H^2}{4(n-1)} + n\delta\right]^{\frac{1}{2}} - \frac{n(n-2)|H|}{2\sqrt{n(n-1)}},$$

which implies that

$$B = -\sqrt{S - nH^2} - \frac{n(n-2)|H|}{2\sqrt{n(n-1)}} + \left[\frac{n^3H^2}{4(n-1)} + n\delta\right]^{\frac{1}{2}} \geq 0.$$

We get

$$n\delta - (S - nH^2) - n|H|\frac{n-2}{\sqrt{n(n-1)}}\sqrt{S - nH^2} + nH^2 = A \cdot B \geq 0. \tag{3.3}$$

From (2.6) and (3.3), we have $R_{ii} \geq 0$. In particular, from the assertions above, we know that if $S < \alpha(n, H)$ holds, then $R_{ii} > 0$. This implies that, by Lemma 2.1, M^n carries a metric of positive Ricci curvature. According to Bonnet-Myer's theorem [17], we know that the fundamental group is finite. This is impossible because M^n has infinite fundamental group.

From Lemma 2.2, we can assume without loss of generality that at any fixed point $p \in M$, $R_{nn} = 0$. Therefore, from Lemma 2.2, all of the above inequalities should be equalities at p and $S = \alpha(n, r)$. That is, we have $K_{ijij} = \delta$, N^{n+1} is of constant sectional curvature δ, and $h_{ij} = 0$ if $i \neq j$, $h_{11} = \cdots = h_{n-1n-1}$ and $h_{nn} = nH - (n-1)h_{11}$. Hence, we conclude that M^n has two distinct principal curvatures, one of which is simple. Let $\{e_1, \cdots, e_n\}$ be a local orthonormal frame field such that $h_{ij} = \lambda_i\delta_{ij}$, where the λ_i's are principal curvature on M^n. Without loss of generality, we can assume $\lambda_1 = \cdots = \lambda_{n-1} = \lambda$, $\lambda_n = \mu$. From

$$\mathrm{R}_{nn} = (n-1)\delta + (\lambda_1 + \cdots + \lambda_{n-1} + \lambda_n)\lambda_n - \lambda_n^2 = (n-1)(\delta + \lambda\mu) = 0,$$

we have $\delta + \lambda\mu = 0$. On the other hand, $nH = (n-1)\lambda + \mu$.

Similarly as in the proof of Theorem 1.1, we consider the integral submanifold of the corresponding distribution of the space of principal vectors corresponding to the principal curvature λ. Since the multiplicity of the principal curvature λ is greater than 1, we know that the principal curvature λ is constant on this integral submanifold. Thus, we obtain that M^n is the isoparametric hypersurface of the sphere N^{n+1} with two distinct principal curvatures. Therefore, we conclude that M^n is isometric to the Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$. This completes the proof of Theorem 1.2. □

Proof of Theorem 1.3. For a real number $d = \frac{n+2\sqrt{n-1}}{n-2}\sqrt{n} > 0$, we have

$$2|H||\phi| \leq dH^2 + \frac{1}{d}|\phi|^2. \tag{3.4}$$

From (2.6) and (3.4), we obtain

$$\begin{aligned} \mathrm{R}_{ii} &\geq \frac{n-1}{n}\left(n\delta + 2nH^2 - S - \frac{n(n-2)}{\sqrt{n(n-1)}}|H|\sqrt{S - nH^2}\right) \\ &= \frac{n-1}{n}\Bigg[n\delta + nH^2\left(2 - \frac{(n-2)d}{2\sqrt{n(n-1)}} + \frac{n(n-2)}{2\sqrt{n(n-1)}d}\right) \\ &\qquad\qquad - S\left(1 + \frac{n(n-2)}{2\sqrt{n(n-1)}d}\right)\Bigg] \\ &\geq \frac{n-1}{n}(n\delta - \frac{n}{2\sqrt{n-1}}S). \end{aligned} \tag{3.5}$$

From the assumption $S \leq 2\sqrt{n-1}\delta$ and (3.5), we have $R_{ii} \geq 0$. In particular, from the assertions above, we know that if $S < 2\sqrt{n-1}\delta$ holds, then $R_{ii} > 0$. This implies that, by Lemma 2.1, M^n carries a metric of positive Ricci curvature. According to Bonnet-Myer's theorem [17], we know that the fundamental group is finite. This is impossible because M^n has infinite fundamental group.

From Lemma 2.2, similarly as in the proof of Theorem 1.1, we get $S = 2\sqrt{n-1}\delta$. That is, we have $K_{ijij} = \delta$, N^{n+1} is of constant sectional curvature δ, and $h_{ij} = 0$ if $i \neq j$, $h_{11} = \cdots = h_{n-1n-1}$ and $h_{nn} = nH-(n-1)h_{11}$. Hence, we conclude that M^n has two distinct principal curvatures, one of which is simple. Let $\{e_1, \cdots, e_n\}$ be a local orthonormal frame field such that $h_{ij} = \lambda_i\delta_{ij}$, where the λ_i's are principal curvature on M^n. Without loss of generality, we can assume that $\lambda_1 = \cdots = \lambda_{n-1} = \lambda$, $\lambda_n = \mu$. From

$$\mathrm{R}_{nn} = (n-1)\delta + (\lambda_1 + \cdots + \lambda_{n-1} + \lambda_n)\lambda_n - \lambda_n^2 = (n-1)(\delta + \lambda\mu) = 0,$$

we have $\delta + \lambda\mu = 0$. On the other hand, $S = 2\sqrt{n-1}\delta = (n-1)\lambda^2 + \mu^2$. Hence, we get

$$\lambda^2 = \frac{\delta}{\sqrt{n-1}} \ \textit{and} \ \mu^2 = \sqrt{n-1}\delta.$$

Thus, we obtain that M^n is the isoparametric hypersurface of the sphere N^{n+1} with two distinct principal curvatures. Therefore, we conclude that M^n is isometric to the Riemannian product $S^1(\sqrt{1-c^2}) \times S^{n-1}(c)$. This completes the proof of Theorem 1.3. $\square$

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