PROCEEDINGS OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 146, Number 5, May 2018, Pages 2217–2218
http://dx.doi.org/10.1090/proc/13949
Article electronically published on January 8, 2018

ADDENDUM TO "ON A GENERAL MACLAURIN'S INEQUALITY"

STEFANO FAVARO AND STEPHEN G. WALKER

(Communicated by Ken Ono)

Abstract. A short proof of a general Maclaurin inequality is presented.

1. Short proof

With $(x_1, \dots, x_n)$ as positive real numbers, define

$$E_k(\mathbf{x}) = \left[\frac{\sum_{1 \le i_1 < i_2 < \cdots < i_k \le n} x_{i_1} x_{i_2} \cdots x_{i_k}}{\binom{n}{k}} \right]^{\frac{1}{k}} \tag{1.1}$$

for any $k = 1, \dots, n$, where the numerator of (1.1) is the kth elementary symmetric polynomial in $\mathbf{x}$ and where the binomial coefficient in the denominator of (1.1) is the number of terms in the numerator. The Maclaurin's inequality is given by

$$E_1(\mathbf{x}) \ge E_2(\mathbf{x}) \ge \cdots \ge E_{n-1}(\mathbf{x}) \ge E_n(\mathbf{x}), \tag{1.2}$$

with the extreme terms $E_1(\mathbf{x})$ and $E_n(\mathbf{x})$ being the arithmetic mean and the geometric mean, respectively.

Suppose now we have $(y_1, \dots, y_m)$, which is comprised of r_i copies of x_i, for $i = 1, \dots, n$, and $\sum_{i=1}^n r_i = m$. Then the following equality can be proven:

$$\sum_{1 \le i_1 < i_2 < \cdots < i_l \le m} y_{i_1} \dots y_{i_l} = \sum_{\sum_{i=1}^n l_i = l} \prod_{i=1}^n \binom{r_i}{l_i} x_i^{l_i}, \tag{1.3}$$

and it is this which forms the basis of the short proof.

Then (1.3) combined with (1.2) implies

$$T_1(\mathbf{x}, \mathbf{r}) \ge T_2(\mathbf{x}, \mathbf{r}) \ge \cdots \ge T_{m-1}(\mathbf{x}, \mathbf{r}) \ge T_m(\mathbf{x}, \mathbf{r}),$$

where

$$T_l(\mathbf{x}, \mathbf{r}) = \left[\sum_{(l_1, \dots, l_n) \in \mathcal{P}_{n,l}} \frac{\prod_{i=1}^n \binom{r_i}{l_i} x_i^{l_i}}{\binom{m}{l}} \right]^{1/l},$$

and $\mathcal{P}_{n,l} = \{(l_1, \dots, l_n) : l_i \ge 0 \text{ and } \sum_{1 \le i \le n} l_i = l\}$. This result was proved in [1] using inequalities between Jacobi polynomials.

Acknowledgment

The authors acknowledge Richard Stong for pointing out the short proof.

Received by the editors August 15, 2017.
2010 *Mathematics Subject Classification.* Primary 26D15.

References

[1] S. Favaro and S. G. Walker, *On a General Maclaurin's inequality*. Proceedings of the American Mathematical Society, to appear.

Department of Economics and Statistics, University of Torino, Corso Unione Sovietica 218/bis, 10134 Torino, Italy
Email address: stefano.favaro@unito.it

Department of Mathematics, University of Texas at Austin, One University Station, C1200 Austin, Texas
Email address: s.g.walker@math.utexas.edu