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A LOCALLY HYPERBOLIC 3-MANIFOLD THAT IS NOT HYPERBOLIC

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Abstract. We construct a locally hyperbolic 3-manifold M_∞ such that $\pi_1(M_\infty)$ has no divisible subgroup. We then show that M_∞ is not homeomorphic to any complete hyperbolic manifold. This answers a question of Agol.

Introduction

Throughout this paper, M is always an oriented, aspherical 3-manifold. A 3-manifold M is *hyperbolizable* if its interior is homeomorphic to $\mathbb{H}^3/\Gamma$ for $\Gamma \leqslant \text{Isom}(\mathbb{H}^3)$ a discrete, torsion free subgroup. An irreducible 3-manifold M is of *finite-type* if $\pi_1(M)$ is finitely generated, and we say it is of *infinite-type* otherwise. By Geometrization ([20–22]) and Tameness ([1, 6]) a finite type 3-manifold M is hyperbolizable if and only if M is the interior of a compact 3-manifold $\overline{M}$ that is atoroidal and with non-finite $\pi_1(\overline{M})$. On the other hand, if M is of infinite type not much is known, and we are very far from a complete topological characterization. Nevertheless, some interesting examples of these manifolds have been constructed in [5, 28]. What we do know are necessary conditions for a manifold of infinite type to be hyperbolizable. If M is hyperbolizable, then $M \cong \mathbb{H}^3/\Gamma$. Hence by discreteness of Γ and the classification of isometries of $\mathbb{H}^3$ we have that no element $\gamma \in \Gamma$ is *divisible* ([10, Lemma 3.2]). Here, $\gamma \in \Gamma$ is divisible if there are infinitely many $\alpha \in \pi_1(M)$ and $n \in \mathbb{N}$ such that: $\gamma = \alpha^n$. We say that a manifold M is *locally hyperbolic* if every cover $N \twoheadrightarrow M$ with $\pi_1(N)$ finitely generated is hyperbolizable. Thus, local hyperbolicity and having no divisible subgroups in π_1 are necessary conditions. In [9, 18] Agol asks whether these conditions could be sufficient for hyperbolization:

Question (Agol)**.** Is there a 3-dimensional manifold M with no divisible elements in $\pi_1(M)$ that is locally hyperbolic but not hyperbolic?

We give a positive answer:

Theorem 1. *There exists a locally hyperbolic 3-manifold with no divisible subgroups in its fundamental group that does not admit any complete hyperbolic metric.*

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Outline of the proof. The manifold M_∞ is a thickening of the 2-complex obtained by gluing to an infinite annulus A countably many copies of a genus two surface $\{\Sigma_i\}_{i\in\mathbb{Z}}$ along a fixed separating curve γ such that the i-th copy Σ_i is glued to $S^1\times\{i\}$. The manifold M_∞ covers a compact non-atoroidal manifold M containing an incompressible two-sided surface Σ. Since $\pi_1(M_\infty)\leqslant\pi_1(M)$ and M is Haken by [25] we have that $\pi_1(M_\infty)$ has no divisible elements. By construction M_∞ has countably many embedded genus two surfaces $\{\Sigma_i\}_{i\in\mathbb{Z}}$ that project down to Σ. By a surgery argument it can be shown that M_∞ is atoroidal. Moreover, if we consider the lifts Σ_{-i},Σ_i, they co-bound a submanifold M_i that is hyperbolizable, and we will use the M_i to show that M_∞ is locally hyperbolic (see Lemma 2.1). Thus, M_∞ satisfies the conditions of Agol's question.

The obstruction to hyperbolicity arises from the lift A of the essential torus T. The lift A is an open annulus such that the intersection with all M_i is an embedded essential annulus $A_i\doteq A\cap M_i$ with boundaries in $\Sigma_{\pm i}$. The surfaces $\Sigma_{\pm i}$ in the boundaries of the M_i have the important property that they have no homotopic essential subsurfaces except for the one induced by A. This gives us the property that both ends of A see an 'infinite' amount of topology. This is in sharp contrast with finite type hyperbolic manifolds in which, by Tameness, every such annulus only sees a finite amount of topology.

In future work we will give a complete topological characterization of hyperbolizable 3-manifolds for a class of infinite type 3-manifolds. This class contains M_∞ and the example of Souto-Stover [28] of a hyperbolizable Cantor set in S^3.

Notation. We use $\simeq$ for homotopic, and by $\pi_0(X)$ we intend the connected components of X. With $\Sigma_{g,k}$ we denote the genus g orientable surface with k boundary components. By $N\hookrightarrow M$ we denote embeddings, while $S\looparrowright M$ denotes immersions.

1. Background

We now recall some facts and definitions about the topology of 3-manifolds; more details can be found in [14–16].

An orientable 3-manifold M is said to be *irreducible* if every embedded sphere S^2 bounds a 3-ball. A map between manifolds is said to be *proper* if it sends boundaries to boundaries and pre-images of compact sets are compact. We say that a connected properly immersed surface $S\looparrowright M$ is π_1*-injective* if the induced map on the fundamental groups is injective. Furthermore, if $S\hookrightarrow M$ is embedded and π_1-injective we say that it is *incompressible*. If $S\hookrightarrow M$ is a non-π_1-injective two-sided surface by the Loop Theorem we have that there is a compressing disk $D\hookrightarrow M$ such that $\partial D=D\cap S$ and ∂D is non-trivial in $\pi_1(S)$.

An irreducible 3-manifold $(M,\partial M)$ is said to have *incompressible boundary* if every map $(D^2,\partial D^2)\hookrightarrow(M,\partial M)$ is homotopic via a map of pairs into ∂M. Therefore, $(M,\partial M)$ has incompressible boundary if and only if each component $S\in\pi_0(\partial M)$ is incompressible, that is, π_1-injective. An orientable, irreducible, and compact 3-manifold is called *Haken* if it contains a two-sided π_1-injective surface. A 3-manifold is said to be *acylindrical* if every map $(S^1\times I,\partial(S^1\times I))\to(M,\partial M)$ can be homotoped into the boundary via maps of pairs.

Definition 1.1. A 3-manifold M is said to be *tame* if it is homeomorphic to the interior of a compact 3-manifold $\overline{M}$.

Even 3-manifolds that are homotopy equivalent to compact manifolds need not be tame. For example, the Whitehead manifold [31] is homotopy equivalent to $\mathbb{R}^3$ but is not homeomorphic to it.

Definition 1.2. We say that a codimension zero submanifold $N \overset{\iota}{\hookrightarrow} M$ forms a *Scott core* if the inclusion map ι_* is a homotopy equivalence.

By [12, 23, 24] given an orientable irreducible 3-manifold M with finitely generated fundamental group, a Scott core exists and is unique up to homeomorphism.

Let M be a tame 3-manifold. Then given a Scott core $C \hookrightarrow M \subseteq \overline{M}$ with incompressible boundary we have that, by Waldhausen's cobordism theorem [30], every component of $\overline{\overline{M} \setminus C}$ is a product submanifold homeomorphic to $S \times I$ for $S \in \pi_0(\partial C)$.

Definition 1.3. Given a core $C \hookrightarrow M$ we say that an end $E \subseteq \overline{M \setminus C}$ is *tame* if it is homeomorphic to $S \times [0, \infty)$ for $S = \partial E$.

A core $C \subseteq M$ gives us a bijective correspondence between the ends of M and the components of ∂C. We say that a surface $S \in \pi_0(\partial C)$ *faces the end* E if E is the component of $\overline{M \setminus C}$ with boundary S. It is a simple observation that if an end E facing S is exhausted by submanifolds homeomorphic to $S \times I$, then it is a tame end.

2. Proof of Theorem 1

Consider a surface of genus two Σ and denote by α a separating curve that splits it into two punctured tori. To $\Sigma \times I$ we glue a thickened annulus $C \doteq (S^1 \times I) \times I$ so that $S^1 \times I \times \{i\}$ is glued to a regular neighbourhood of $\alpha \times i$, for $i = 0, 1$. We call the resulting manifold M (see Figure 1).

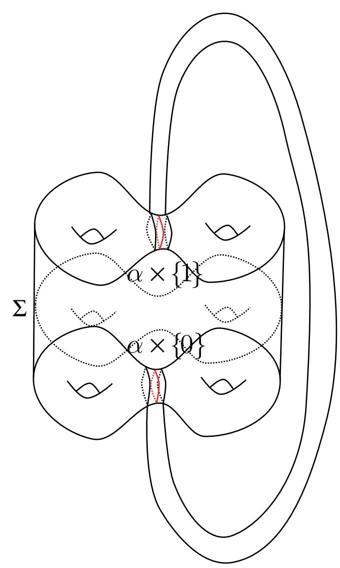

Figure 1. The manifold M.

The manifold M is not hyperbolic since it contains a non-boundary parallel essential torus T induced by the cylinder C. Moreover, M has a surjection p onto S^1 obtained by projecting the surfaces in $\Sigma \times I$ onto I and also mapping the cylinder onto an interval. We denote by H the kernel of the surjection map $p_* : \pi_1(M) \twoheadrightarrow \pi_1(S^1)$.

Consider an infinite cyclic cover M_∞ of M corresponding to the subgroup H. The manifold M_∞ is an infinite collection of $\{\Sigma \times I\}_{i\in\mathbb{Z}}$ glued to each other via annuli along the separating curves $\alpha \times \{0, 1\}$. Therefore, we have the covering in Figure 2:

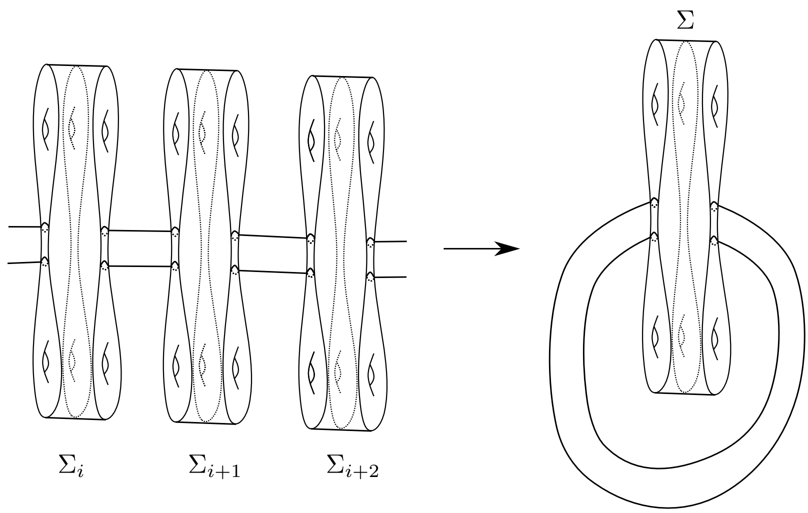

FIGURE 2. The infinite cyclic cover.

where the Σ_i are distinct lifts of Σ and so are incompressible in M_∞. Since $\pi_1(M_\infty)$ is a subgroup of $\pi_1(M)$ and M is Haken (M contains the incompressible surface Σ) by [25] we have that $\pi_1(M)$ has no divisible elements; thus $\pi_1(M_\infty)$ has no divisible subgroups as well.

Lemma 2.1. *The manifold M_∞ is locally hyperbolic.*

Proof. We claim that M_∞ is atoroidal and exhausted by hyperbolizable manifolds. Let $T^2 \hookrightarrow M_\infty$ be an essential torus with image T. Between the surfaces Σ_i and Σ_{i+1} we have incompressible annuli C_i that separate them; see Figure 3. Since T is compact it intersects at most finitely many $\{C_i\}$. Moreover, up to isotopy we can assume that T is transverse to all C_i and it minimizes $|\pi_0(T \cap \bigcup C_i)|$. If T does not intersect any C_i we have that it is contained in a submanifold homeomorphic to $\Sigma \times I$ which is atoroidal, and so T wasn't essential.

Since both C_i and T are incompressible we can isotope T so that the components of the intersection $T \cap C_i$ are essential simple closed curves. Thus, T is divided by $\bigcup_i T \cap C_i$ into finitely many parallel annuli, and $T \cap C_i$ are disjoint core curves for C_i. Consider C_k such that $T \cap C_k \neq \varnothing$ and $\forall n \geq k : T \cap C_n = \varnothing$. Then T cannot

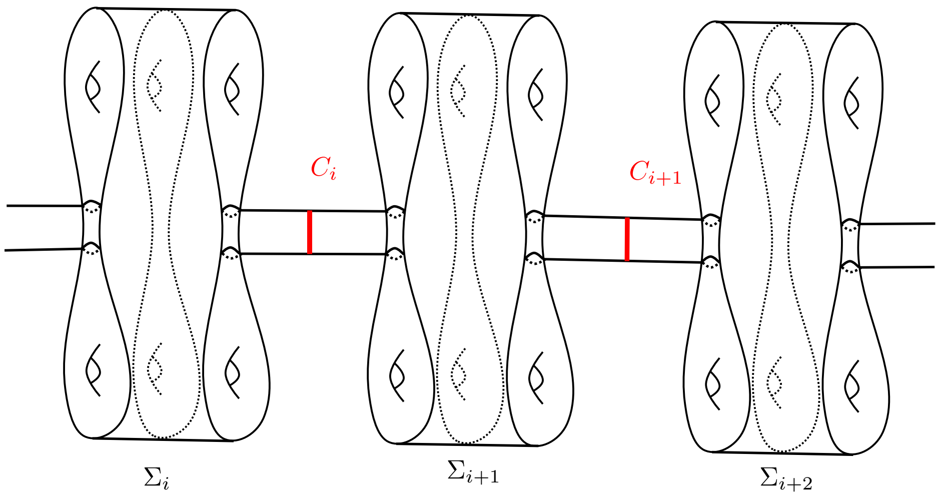

FIGURE 3

intersect C_k in only one component, so it has to come back through C_k. Thus, we have an annulus $A \subset T$ that has both boundaries in C_k and is contained in a submanifold of M_∞ homeomorphic to $\Sigma_{k+1} \times I$. The annulus A gives an isotopy between isotopic curves in $\partial(\Sigma_{k+1} \times I)$ and is therefore boundary parallel. Hence, by an isotopy of T we can reduce $|\pi_0(T \cap \bigcup C_i)|$, contradicting the fact that it was minimal and non-zero.

We define the submanifold of M_∞ co-bounded by Σ_k and Σ_{-k} by M_k. Since M_∞ is atoroidal so are the M_k. Moreover, since the M_k are compact manifolds with infinite π_1 they are hyperbolizable by Thurston's Hyperbolization Theorem [17].

We now want to prove that M_∞ is locally hyperbolic. To do so it suffices to show that given any finitely generated $H \leqslant \pi_1(M_\infty)$ the cover $M_\infty(H)$ corresponding to H factors through a cover $N \twoheadrightarrow M_\infty$ that is hyperbolizable. Let $\gamma_1, \dots, \gamma_n \subset M_\infty$ be loops generating H. Since the M_k exhaust M_∞ we can find some $k \in \mathbb{N}$ such that $\{\gamma_i\}_{i \leq n} \subset M_k$; hence the cover corresponding to H factors through the cover induced by $\pi_1(M_k)$. We now want to show that the cover $M_\infty(k)$ of M_∞ corresponding to $\pi_1(M_k)$ is hyperbolizable.

Since $\pi : M_\infty \twoheadrightarrow M$ is the infinite cyclic cover of M we have that $M_\infty(k)$ is the same as the cover of M corresponding to $\pi_*(\pi_1(M_k))$. The resolution of the Tameness [1,6] and the Geometrization conjecture [20–22] imply Simon's conjecture, that is: covers of compact irreducible 3-manifolds with finitely generated fundamental groups are tame [8,26]. Therefore, since M is compact by Simon's conjecture we have that $M_\infty(k)$ is tame. The submanifold $M_k \hookrightarrow M_\infty$ lifts homeomorphically to $\widetilde{M}_k \hookrightarrow M_\infty(k)$. By Whitehead's theorem [13] the inclusion is a homotopy equivalence; hence $\widetilde{M}_k$ forms a Scott core for $M_\infty(k)$. Thus, since $\partial\widetilde{M}_k$ is incompressible and $M_\infty(k)$ is tame we have that $M_\infty(k) \cong \operatorname{int}(M_k)$, and so it is hyperbolizable. $\square$

In the infinite cyclic cover M_∞ the essential torus T lifts to a π_1-injective annulus A that is properly embedded: $A = \gamma \times \mathbb{R} \hookrightarrow M_\infty$ for γ the lift of the curve $\alpha \hookrightarrow \Sigma \subseteq M$.

Remark 2.2. Consider two distinct lifts Σ_i, Σ_j of the embedded surface $\Sigma \hookrightarrow M$. Then we have that the only essential subsurface of Σ_i homotopic to a subsurface of Σ_j is a neighbourhood of γ. This is because by construction the only curve of Σ_i homotopic into Σ_j is γ.

Given a hyperbolic 3-manifold M, a *useful simplicial hyperbolic surface* is a surface S with a 1-vertex triangulation $\mathcal{T}$, a preferred edge e, and a map $f : S \to M$, such that:

(1) $f(e)$ is a geodesic in M;
(2) every edge of $\mathcal{T}$ is mapped to a geodesic segment in M;
(3) the restriction of f to every face of $\mathcal{T}$ is a totally geodesic immersion.

By [3, 7] every π_1-injective map $f : S \to M$ with a 1-vertex triangulation with a preferred edge can be homotoped so that it becomes a useful simplicial surface. Moreover, with the path metric induced by M a useful simplicial surface is negatively curved and the map becomes 1-Lipschitz.

Proposition 2.3. *The manifold M_∞ is not hyperbolic.*

Proof. The manifold M_∞ has two non-tame ends $E^\pm$, and the connected components of the complement of a region co-bounded by distinct lifts of Σ give neighbourhoods of these ends. Let A be the annulus obtained by the lift of the essential torus $T \hookrightarrow M$. The ends $E^\pm$ of M_∞ are in bijection with the ends $A^\pm$ of the annulus A. Let γ be a simple closed curve generating $\pi_1(A)$. Denote by $\{\Sigma_i\}_{i\in\mathbb{Z}} \subset M_\infty$ the lifts of $\Sigma \subset M$ and let $\{\Sigma_i^\pm\}_{i\in\mathbb{Z}}$ be the lifts of the punctured tori that form the complement of α in $\Sigma \subseteq M$. The proof is by contradiction, and it will follow by showing that γ is neither homotopic to a geodesic in M_∞ nor out of a cusp.

Step 1. We want to show that the curve γ cannot be represented by a hyperbolic element.

By contradiction assume that γ is represented by a hyperbolic element and let $\overline{\gamma}$ be the unique geodesic representative of γ in M_∞. Consider the incompressible embeddings $f_i : \Sigma_2 \hookrightarrow M_\infty$ with $f_i(\Sigma_2) = \Sigma_i$ and let $\gamma_i \subset \Sigma_i$ be the simple closed curve homotopic to γ. By picking a 1-vertex triangulation of Σ_i where γ_i is represented by a preferred edge we can realize each (f_i, Σ_i) by a useful simplicial hyperbolic surface $g_i : S_i \to M_\infty$ with $g_i(S_i) \simeq \Sigma_i$ (see [3, 7]). By an abuse of notation we will also use S_i to denote $g_i(S_i)$. Since all the S_i realize $\overline{\gamma}$ as a geodesic we see the configuration in M_∞; see Figure 4.

On the simplicial hyperbolic surfaces S_i a maximal one-sided collar neighbourhood of $\overline{\gamma}$ has area bounded by the total area of S_i. Since the simplicial hyperbolic surfaces are all genus two by Gauss-Bonnet we have that $A(S_i) \leq 2\pi|\chi(S_i)| = 4\pi$. Therefore, the radius of a one-sided collar neighbourhood is uniformly bounded by some constant $K = K(\chi(\Sigma_2), \ell(\gamma)) < \infty$. Then, for any $\xi > 0$ the $K + \xi$ bi-collar neighbourhood of $\overline{\gamma}$ in the simplicial hyperbolic surface S_i is not embedded and contains a 4-punctured sphere. Since simplicial hyperbolic surfaces are 1-Lipschitz the 4-punctured sphere is contained in a $K + \xi$ neighbourhood C of $\overline{\gamma}$; thus it

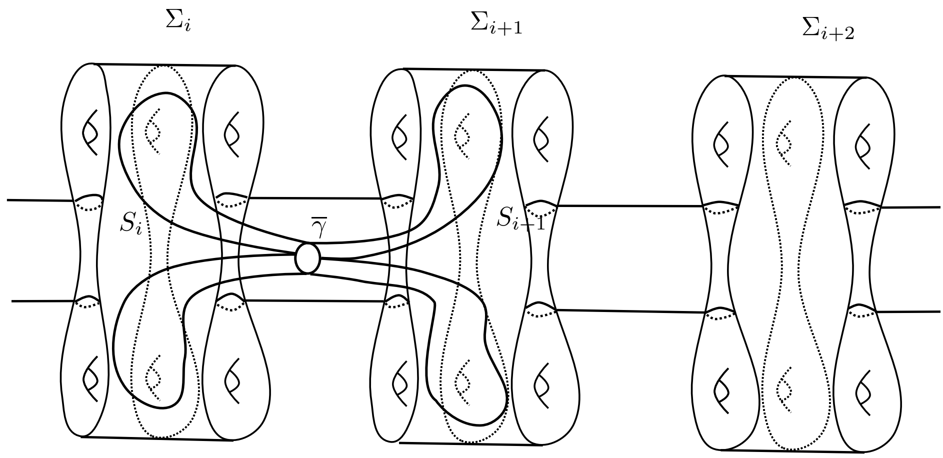

FIGURE 4. The simplicial hyperbolic surfaces S_i exiting the ends.

lies in some fixed set M_h. Therefore for every $|n| > h$ we have that $\Sigma_{\pm n}$ has an essential subsurface, homeomorphic to a 4-punctured sphere, homotopic into $\Sigma_{\pm h}$ respectively. But this contradicts Remark 2.2.

Step 2. We now show that γ cannot be represented by a parabolic element.

Let $\epsilon > 0$ be less than the 3-dimensional Margulis constant μ_3 [2] and let P be a cusp neighbourhood of γ such that the horocycle representing γ in ∂P has length ϵ. The cusp neighbourhood P is contained in one end E of M_∞. Without loss of generality we can assume that P is not contained in the end E^+ of M_∞.

Let $\{\Sigma_i^+\}_{i\geq 0} \subset \{\Sigma_i\}_{i\geq 0}$ be the collection of subsurfaces of the Σ_i formed by the punctured tori with boundary γ_i that are exiting E^+. By picking an ideal triangulation of Σ_i where the cusp γ_i is the only vertex we can realize the embeddings $f_i : \Sigma_i^+ \hookrightarrow M_\infty$ by simplicial hyperbolic surfaces (g_i, S_i^+) in which γ_i is sent to the cusp [3, 7]. The $\{S_i^+\}_{i\geq 0}$ are all punctured tori with cusp represented by γ. See Figure 5.

All simplicial hyperbolic surfaces S_i^+ intersect ∂P in a horocycle $f_i(c_i)$ of length $\ell(f_i(c_i)) = \epsilon$. Therefore, in each S_i^+ the horocycle c_i has a maximally embedded one-sided collar whose radius is bounded by some constant $K = K(\epsilon, 2\pi)$. Then for any $\xi > 0$, a $K+\xi$ neighbourhood of c_i in S_i^+ contains a pair of pants $P_i \subset S_i^+$ that has c_i as a boundary component. Since simplicial hyperbolic surfaces are 1-Lipschitz the pair of pants P_i are contained in a $K+\xi$ neighbourhood of $f_i(c_i)$ in M_∞. Thus, the Σ_i have pairs of pants that are homotopic a uniformly bounded distance from ∂P. Let $k \in \mathbb{N}$ be minimal such that Σ_k lies outside a $K + \xi$ neighbourhood of ∂P. Then for any $j > k$ we have that Σ_j has a pair of pants homotopic into Σ_k, contradicting Remark 2.2.

$\square$

This concludes the proof of Theorem 1.

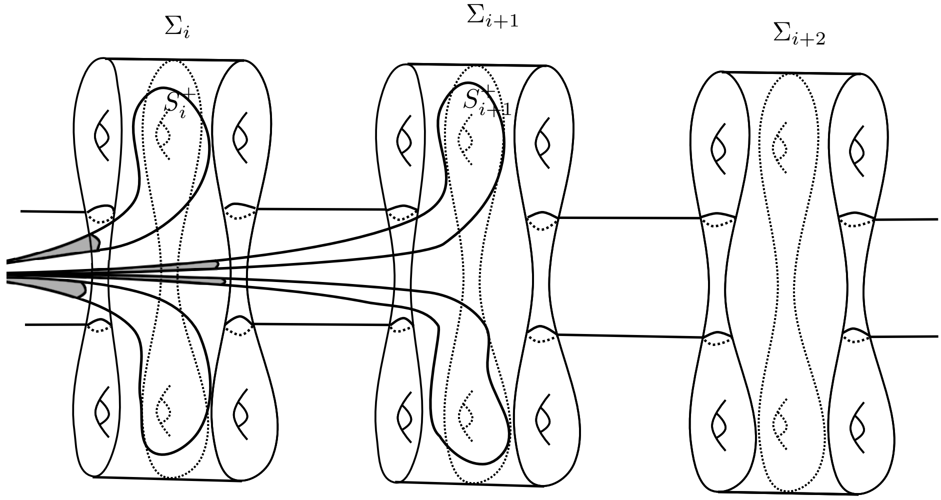

FIGURE 5. The ϵ-thin part is in grey.

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