due to H. A. Webb[3] covers the effect of a concentrated bending couple applied at an intermediate point of the span. As an application, R. V. Southwell suggests (Example 14, l.c.) the derivation of the theorem of three moments for a continuous beam by the same method. The use of the method for a beam with a stepwise variation of bending rigidity seems however to be Mr. Brown's original contribution.

It is hoped that the discussed method, whatever the symbols used, will get more attention from engineers on this side of the Atlantic, as it carries with it a very substantial shortening of the computations.

FORMULAS FOR COMPLEX INTERPOLATION*

By A. N. LOWAN and H. E. SALZER, (*Math. Tables Project, Nat. Bureau of Standards*)

An analytic function of $z=x+iy$ may be approximated by a complex polynomial of degree n passing through $n+1$ points in accordance with the Lagrange-Hermite formula of interpolation. For the important special case when the given $n+1$ points are equidistantly spaced along any straight line in the z-plane, the following tables give the real and imaginary parts of the coefficients $A_k(P)$ of the interpolation polynomial $f(z)=A_k(P)f(z_k)$, where $P=(z-z_0)/h=p+iq$ and h is the complex tabular interval. The formulas cover the cases ranging from complex quadratic (3 points) to complex quintic interpolation (6 points).

Quadratic interpolation (3 points)

$$\Re e A_{-1}(P) = \tfrac{1}{2}[p(p-1)-q^2], \qquad \Im m A_{-1}(P) = q(p-.5),$$

$$\Re e A_0(P) = 1-p^2+q^2, \qquad \Im m A_0(P) = -2pq,$$

$$\Re e A_1(P) = \tfrac{1}{2}[p(p+1)-q^2], \qquad \Im m A_1(P) = q(p+.5).$$

Cubic interpolation (4 points)

$$\Re e A_{-1}(P) = \frac{(1-p)}{6}[p(p-2)-3q^2], \qquad \Im m A_{-1}(P) = \frac{q}{6}[q^2-2+3p(2-p)],$$

$$\Re e A_0(P) = 1+\tfrac{1}{2}[p(p^2-2p-1)+q^2(2-3p)], \qquad \Im m A_0(P) = \frac{q}{2}[p(3p-4)-q^2-1],$$

$$\Re e A_1(P) = -\tfrac{1}{2}[p(p-2)(p+1)+q^2(1-3p)], \qquad \Im m A_1(P) = \frac{q}{2}[p(2-3p)+q^2+2],$$

$$\Re e A_2(P) = \frac{p}{6}(p^2-3q^2-1), \qquad \Im m A_2(P) = \frac{q}{6}[3p^2-q^2-1].$$

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Quartic interpolation (5 points)

$$\mathfrak{Re}A_{-2}(P) = \frac{1}{24}\left[p(p^2-1)(p-2) + q^2(q^2+1) + 6pq^2(1-p)\right],$$

$$\mathfrak{Im}A_{-2}(P) = \frac{q}{12}\left[q^2(1-2p) + 2p^3 - 3p^2 - p + 1\right],$$

$$\mathfrak{Re}A_{-1}(P) = -\frac{1}{6}\left[p(p-1)(p^2-4) + q^2(q^2+4) + 3pq^2(1-2p)\right],$$

$$\mathfrak{Im}A_{-1}(P) = -\frac{q}{6}\left[4p^3 - 3p^2 - 8p + 4 - q^2(4p-1)\right],$$

$$\mathfrak{Re}A_0(P) = \frac{1}{4}\left[(p^2-1)(p^2-4) + q^2(q^2-6p^2+5)\right],$$

$$\mathfrak{Im}A_0(P) = \frac{pq}{2}(2p^2 - 2q^2 - 5),$$

$$\mathfrak{Re}A_1(P) = -\frac{1}{6}\left[p(p+1)(p^2-4) + q^2(q^2+4) - 3pq^2(1+2p)\right],$$

$$\mathfrak{Im}A_1(P) = -\frac{q}{6}\left[4p^3 + 3p^2 - 8p - 4 - q^2(1+4p)\right],$$

$$\mathfrak{Re}A_2(P) = \frac{1}{24}\left[p(p+2)(p^2-1) + q^2(q^2+1) - 6pq^2(1+p)\right],$$

$$\mathfrak{Im}A_2(P) = \frac{q}{12}\left[2p^3 + 3p^2 - p - 1 - q^2(2p+1)\right].$$

Quintic interpolation (6 points)

$$\mathfrak{Re}A_{-2}(P) = \frac{1}{120}\left[-p(p^2-1)(p-3)(p-2) + 5q^2(p-1)(2p^2-4p-1) + 5q^4(1-p)\right],$$

$$\mathfrak{Im}A_{-2}(P) = \frac{q}{120}\left[-5p(p-2)(p^2-2p-1) - (q^2-6)(q^2+1) + 10pq^2(p-2)\right],$$

$$\mathfrak{Re}A_{-1}(P) = \frac{p}{24}\left[(p-1)(p^2-4)(p-3) - q^2(10p^2-24p-5q^2-3)\right] - \frac{q^2}{6}(q^2+4),$$

$$\mathfrak{Im}A_{-1}(P) = \frac{q}{24}\left[(q^2+4)(q^2-3) + p(5p^3-16p^2-3p+32) + pq^2(16-10p)\right],$$

$$\mathfrak{Re}A_0(P) = \frac{1}{12}\left[(p^2-1)(p^2-4)(3-p) + q^2(10p^3-18p^2-15p+15) + q^4(3-5p)\right],$$

$$\mathfrak{Im}A_0(P) = -\frac{q}{12}\left[(q^2+1)(q^2+4) + p(5p^3-12p^2-15p+30) + pq^2(12-10p)\right],$$

$$\mathfrak{R}eA_1(P) = \frac{1}{12}\left[p(p+1)(p^2-4)(p-3) + q^2(-10p^3+12p^2+21p-8) + q^4(5p-2)\right],$$

$$\mathfrak{I}mA_1(P) = \frac{q}{12}\left[(q^2+4)(q^2+3) + p(5p^3-8p^2-21p+16) + pq^2(8-10p)\right],$$

$$\mathfrak{R}eA_2(P) = \frac{1}{24}\left[-p(p^2-1)(p-3)(p+2) - q^2(-10p^3+6p^2+21p-1) + q^4(1-5p)\right],$$

$$\mathfrak{I}mA_2(P) = -\frac{q}{24}\left[5p^4-4p^3-21p^2+2p+6+q^2(q^2+7)+2pq^2(2-5p)\right],$$

$$\mathfrak{R}eA_3(P) = \frac{p}{120}\left[(p^2-4)(p^2-1) + 5q^2(q^2-2p^2+3)\right],$$

$$\mathfrak{I}mA_3(P) = \frac{q}{120}\left[(q^2+4)(q^2+1) + 5p^2(p^2-2q^2-3)\right].$$