

## BOOK REVIEWS

*Ten lectures on theoretical rheology.* By Markus Reiner. Nordemann Publishing Co., Inc. New York, 1943. iv+164 pp. \$4.50.

It is customary to define Rheology as the science of flow and deformation of matter. If this definition is taken literally, Rheology becomes practically identical with Mechanics of Continua, and it becomes hard to understand why a new term has been coined. It seems that a more adequate definition would state Rheology to be a Mechanics of Continua in which the ideal elastic body and the perfect fluid are almost as systematically disregarded, as they are over-emphasized in classical Mechanics of Continua. This undue prominence which the classical scheme gives to the ideal elastic body and the perfect fluid tends to be reflected in textbooks of Mechanics as well as in engineering curricula. The necessity of developing and studying the mechanics of other solids and fluids must therefore be stressed, even to the extent of creating a new term for this part of Mechanics of Continua which, up to a fairly recent past, has been so badly neglected.

There is a definite need for a treatise covering the entire field of Rheology rather than parts, such as plasticity or dynamics of (Newtonian) viscous fluids. The present book is in the nature of an introduction to such a treatise. Four chapters (1-3, 9) deal with the analysis of stress and strain and the important decomposition of the tensors of stress and strain into isotropic and deviatoric parts. The author bows to convention in defining the strains as  $e_{xx} = \partial u / \partial x, \dots, e_{xy} = \partial v / \partial x + \partial u / \partial y, \dots$ . This procedure, a remnant from the time when the tensor character of strain and its geometrical implications was not yet fully realized, makes it necessary to denote the components of the strain tensor by  $e_{xx}, \dots, \frac{1}{2} e_{xy}, \dots$ , and deprives many relations of their natural symmetry. Four further chapters (4, 6, 8, 10) are devoted to the discussion of certain rheological idealizations, viz. the ideal elastic solid, the (Newtonian) viscous fluid, and the materials customarily named after Maxwell (viscous fluid with relaxation of stresses), Voigt (visco-elastic solid), Saint-Venant (perfectly plastic solid), and Bingham (viscous fluid with yield limit). The remaining two chapters (5, 7) are concerned with the solution of special problems (tension and simple flexure of a prismatical bar), Einstein's law of the viscosity of sols, and rheological models.

Theoretical Rheology is a subject which cannot be treated satisfactorily without using the tool of tensor analysis. The author did obviously not want to suppose the reader to be familiar with this tool. On the other hand, limitations originally imposed on the time available for his lectures seem to have prevented him from presenting even the basic conception of tensors in a precise form. One wonders whether, under these circumstances, it would not have been preferable to restrict the discussion to the mechanical behavior of solids and liquids in pure shear. As it is, the uninitiated reader cannot fail to get the impression that any nine quantities neatly arranged between double vertical bars constitute a tensor. The laws of transformation are touched upon in chapter 9 only, and are nowhere stated to form an integral part of the definition of tensor. On the other hand, the reader familiar with tensor analysis might wish to have a tensorial expression of the yield condition of plastic materials which is more adequate than the cryptic relation  $p_0 = \vartheta$  (p. 111, Eq. (4)), where  $p_0$  denotes the stress deviator and  $\vartheta$  is defined as "the yield stress." Many other instances could be cited where the clarity of exposition has obviously suffered from the tendency to cram too much material into a text which, according to the preface, is intended as a brief introduction. In spite of such occasional shortcomings the book, which fills a patent need, will prove very useful.

W. PRAGER

*Table of reciprocals of the integers from 100,000 through 200,000.* Prepared by the Mathematical Tables Project, Work Projects Administration of the Federal Works Agency; conducted under the sponsorship of the National Bureau of Standards. Official Sponsor: Lyman J. Briggs; Technical Director: Arnold N. Lowan. Columbia University Press. New York, 1943, viii+201 pp. \$4.00.

These tables are a useful supplement to the existing tables of reciprocals. The tabular interval is small enough to permit linear interpolation throughout; the differences decrease slowly from 100 to 25 units of the last place. The arrangement of the tables is very practical. Seven significant figures are given (if a number has  $k$  figures before the decimal point, its reciprocal has  $k-1$  zeros after the decimal

point, before the first significant figure). Moreover, the tables indicate the direction in which the last digit is rounded; this practical device reduces the relative tabular error to  $2.5 \times 10^{-8}$ .

W. FELLER

*Table of the Bessel functions  $J_0(z)$  and  $J_1(z)$  for complex arguments.* Prepared by the Mathematical Tables Project, Work Projects Administration of the Federal Works Agency; conducted under the sponsorship of the National Bureau of Standards. Official Sponsor: Lyman J. Briggs; Technical Director: Arnold N. Lowan. Columbia University Press. New York. 1943, xiv+403 pp. \$5.00.

Many problems of mathematical physics and mechanics lead to Bessel functions of various types. The most important of these problems are sketched in the foreword to the present tables, written by Professor H. Bateman of California Institute of Technology. The number of existing tables of Bessel functions is also legion. The present tables contain a valuable bibliography listing some 65 tables of Bessel functions of orders zero and one. However, the new tables are unique both in range and extent.

The Bessel functions  $J_\nu(z)$  are defined by

$$J_\nu(z) = \sum_{k=0}^{\infty} \frac{(-1)^k z^{\nu+2k}}{k! \Gamma(\nu+k+1) 2^{\nu+2k}}$$

and satisfy the differential equation

$$z^2 u''(z) + z u'(z) + (z^2 - \nu^2) u(z) = 0.$$

One has the recurrence formula

$$J_{\nu+1}(z) = -J_{\nu-1}(z) + 2\nu J_\nu(z)/z$$

which enables one to compute  $J_\nu(z)$  for all integers  $\nu$  given the values of  $J_0(z)$  and  $J_1(z)$ . These are now tabulated to ten decimal places. The entries are written in the polar form  $z = \rho(\cos \phi + i \sin \phi)$ . The functions are tabulated along the rays  $\phi = 0^\circ, 5^\circ, 10^\circ, \dots, 90^\circ$  for values of  $\rho$  from 0 to 10 in the steps of .01. The values of the functions in all other quadrants follow easily by means of simple symmetry relations.

To facilitate interpolation, the book contains also tables of the coefficients in Lagrange's interpolation formula which uses five equally spaced points. The coefficients are tabulated to 10 decimal places, the argument varying in steps of .001. These tables will, of course, be useful for many computations and are quite independent of the main tables.

W. FELLER

*The methodology of Pierre Duhem.* Armand Lowinger. Columbia Univ. Press, 184 pp., 1941. \$2.25.

The French theoretical physicist Duhem (1861–1916) devoted his life-work to thermodynamics, although he is mostly remembered today for his studies in medieval science. He moreover published his views on the methods, aims and significance of physics in a few scattered papers and in one connected account, a book entitled *La Théorie physique, son objet et sa structure* (1906). Mr. Lowinger has given an excellent presentation of Duhem's ideas. These are challenging, for on the one hand Duhem was an eminently "classical" physicist, strongly opposed to atomism, opposed also to Maxwell's electromagnetism, so that from our present point of vantage we can prove him wrong on both these counts; but on the other hand, he believed with Kirchhoff that physical theory is a description, not an explanation; with Mach, that its purpose is intellectual economy, and so was on the way, with these and other correlated ideas, towards present-day scientific pragmatism. Duhem's opinions on physical method were rooted in his metaphysical beliefs, a fact obvious to his readers and critics, but which he was very anxious to deny. Mr. Lowinger's presentation of Duhem is faithful and unbiased; he gives us his own views in a last stimulating chapter. There is a good bibliography of Duhem and his critics, to which should be added the rather important books by A. Rey and P. Humbert mentioned on pp. 15 and 8 respectively.

P. LE CORBEILLER