value of this relation is that, together with a steady input calculation, it permits an estimate to be made of the base temperature level at the surface, upon which the effect of the $(m+1)$st pulse is superimposed. The character of the $(m+1)$st temperature surge can thus be calculated without the necessity of summing the separate effects of the m preceding pulses.

NOTE ON RAYLEIGH'S METHOD AND THE NON-UNIFORM STRUT*

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For a strut of the non-uniform flexural rigidity $B=EI(x)$ under end thrust,[1] Rayleigh's method consists of estimating the lowest critical load P_1 by using an assumed deflection y and computing the expression

$$P = \frac{\int_0^L B(y'')^2 dx}{\int_0^L (y')^2 dx}. \tag{1}$$

If P_1'' denotes the lowest value of P obtained for an assumed deflection y which satisfies the end conditions, it can be proved that $P_1' > P_1$, i.e., Rayleigh's method always leads to an overestimate.

An equivalent procedure may be based on the relation

$$P = \frac{\int_0^L (y')^2 dx}{\int_0^L \left(\frac{y^2}{B}\right) dx}. \tag{2}$$

Equation (2) is obtained by eliminating y'' from (1) through use of the governing differential equation $By''+Py=0$. The lowest critical load obtained from (2) for an assumed deflection y is denoted by P_1'.

It is commonly taken for granted that (2) yields (a) an overestimate, and (b) a lower value than (1). These statements, however, are not obvious consequences of the fact that (1) gives an overestimate and no proof of them seems to have been given. A simple proof is presented here, i.e., it is shown that $P_1''>P_1'>P_1$. The deflection y is expanded in the form $y=\sum_1^\infty A_N Y_N(x)$ where $Y_N(x)$ is the deflection function corresponding to the nth mode and so satisfies $BY_N''+P_N Y_N=0$. Here P_n is the nth critical load. The functions Y_n are supposed ordered so that Y_1 and P_1 correspond to the first mode.

To simplify the discussion let

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[1] For a detailed discussion see R. V. Southwell, *Theory of elasticity*, Oxford, 1946, pp. 446–453.

$$I_N = A_N^2 \int_0^L [Y_N']^2 dx.$$

The required integrals are then:

$$\int_0^L (y')^2 dx = \sum_1^\infty I_N, \qquad \int_0^L \left(\frac{y^2}{B}\right) dx = \sum_1^\infty \frac{I_N}{P_N}, \qquad \int_0^L B(y'')^2 dx = \sum_1^\infty P_N I_N.$$

The integrals depend on the relations

$$\int_0^L (Y_M')(Y_N') dx = \int_0^L \frac{(Y_M)(Y_N)}{B} dx = \int_0^L B(Y_M'')(Y_N'') dx = 0, \quad [m \neq n]$$

$$\int_0^L B[Y_N'']^2 dx = P_N \int_0^L [Y_N']^2 dx, \qquad \int_0^L \frac{[Y_N]^2}{B} dx = \frac{1}{P_N} \int_0^L [Y_N']^2 dx,$$

the details of which are presented by R. V. Southwell[1] when the strut is pinned. Other common end conditions are accounted for by imagining the strut as extended so that the pinned ends correspond to two adjacent intersections of the centerline of the strut with the line of thrust.

To establish the inequalities, we first write (2) in the form

$$P_1' = \frac{I_1 + I_2 + \cdots}{(I_1/P_1) + (I_2/P_2) + \cdots} = P_1 \frac{I_1 + I_2 + \cdots}{I_1 + I_2(P_1/P_2) + \cdots}.$$

Then since $P_1/P_N < 1$ for each integer $N \geqq 2$ the term in brackets exceeds unity and so $P_1' > P_1$.

To establish the inequality $P_1'' > P_1'$ it is sufficient to show that

$$\frac{P_1 I_1 + P_2 I_2 + \cdots}{I_1 + I_2 + \cdots} > \frac{I_1 + I_2 + \cdots}{(I_1/P_1) + (I_2/P_2) + \cdots}.$$

But this is equivalent to

$$[P_1 I_1 + P_2 I_2 + \cdots] \left[\frac{I_1}{P_1} + \frac{I_2}{P_2} + \cdots\right] > [I_1 + I_2 + \cdots]^2.$$

For each m and n corresponding cross terms are:

$$I_M I_N \left[\frac{P_N}{P_M} + \frac{P_M}{P_N}\right] \quad \text{and} \quad 2 I_M I_N, \qquad (m \neq n).$$

When $m = n$ the terms of form I_N^2 which appear on each side of the inequality are equal.

The inequality is established if for each m and n

$$\left[\frac{P_N}{P_M} + \frac{P_M}{P_N}\right] > 2 \qquad (m \neq n),$$

but multiplying by $P_n P_m$ this reduces to $(P_N - P_M)^2 > 0$ and the result follows.[2]

[2] Professor G. F. Carrier called the author's attention to the fact that this technique was applied to vibration problems in the paper *Ueber das Gegenstueck zum Rayleighschen Verfahren der Schwingungslehre* by K. Hohenemser and W. Prager, Ing.-Arch., **3**, 306–310 (1932).