# QUARTERLY

OF

# APPLIED MATHEMATICS



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Manuscripts submitted for publication in the Quarterly of Applied Mathematics should be sent to the Managing Editor, Professor W. Prager, Quarterly of Applied Mathematics, Brown University, Providence 12, R. I., either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to engineers. Authors will receive galley proofs only. The authors' institutions will be requested to pay a publication charge of $5.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

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# SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

**Manuscripts:** Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

**Titles:** The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

**Mathematical Work:** As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter $O$, between the numeral one (1), the letter $l$ and the prime ($'$), between alpha and $a$, kappa and $k$, mu amd $u$, nu and $v$, eta and $n$.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t\,(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

**Cuts:** Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

**Bibliography:** References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354-372 (1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

**Footnotes:** As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

**Abbreviations:** Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

## CONTENTS

Hence if

$$\alpha \leq \text{Min } [1/c_9 \, , \, 1/(c_4 + c_5 + c_6 + c_7)], \tag{26}$$

we have uniform convergence of $z_N$ to a function $z$ over the entire $(w_1 \, , \, w_2)$ plane and the infinite $t$-interval, $0 \leq t \leq \infty$. It follows from the uniform convergence that we can pass to the limit as $N \to \infty$ under the integral sign in (3), obtaining (1). Differentiation of (1) yields the original equation.

The uniqueness is now established in the standard fashion.

# BOOK REVIEWS

*Table of the Bessel functions $Y_0(z)$ and $Y_1(z)$ for complex arguments.* Prepared by the Computation Laboratory, National Bureau of Standards. Columbia University Press, New York, 1950. xi + 427 pp. $7.50.

This volume supplements the earlier volume of tables of $J_0(z)$ and $J_1(z)$ for complex arguments [see Q. of Appl. Math. **2**, 276 (1944) and **6**, 95 (1948)]. The main tables give $Y_0(\rho e^{i\varphi})$ and $Y_1(\rho e^{i\varphi})$ to ten decimal places for $\rho = 0(.01)10$ and $\varphi = 0(5°)90°$. Auxiliary tables give $Y_0(\rho e^{i\varphi}) - (2/\pi)J_0(\rho e^{i\varphi}) \log \rho$ and $Y_1(\rho e^{i\varphi}) - (2/\pi)J_1(\rho e^{i\varphi}) \log \rho + (2/\pi\rho)e^{-i\varphi}$, the complex zeros of Bessel functions, and five-point Lagrangian interpolation coefficients.

W. Prager

*The inelastic behavior of engineering materials and structures.* By Alfred M. Freudenthal. John Wiley & Sons, Inc., New York and Chapman & Hall, Limited, London, 1950. xvi +587 pp. $7.50.

The amazing scope of the book and its detailed coverage of so many facets of inelastic action bear eloquent testimony to the author's wide-spread reading and his own research. Quantum statistics, conventional metallurgy, mathematical theories of plasticity, visco-elasticity, stress analysis solutions, and design criteria for metals and concrete are all presented from a unified and extremely interesting point of view. The reader is made to feel equally familiar with electron clouds, simple and complex mechanical models of the behavior of real materials, Brownian motion, limit design, and testing machines.

The only objection to be noted is that little indication is given at the highly controversial nature of the field. Opinions are often stated as facts. For example, this reviewer believes that much of the material on thermodynamics and the mechanical equation of state is based on demonstrably over-simple and probably incorrect assumptions about the dissipated work. However, read with an open and skeptical mind the book is invaluable.

D. C. Drucker

*Electromagnetic fields.* Theory and application. Volume I: Mapping of Fields. By Ernst Weber. John Wiley & Sons, Inc., New York and Chapman & Hall Limited, London, 1950. xiv + 590 pp. $10.00.

The author has divided electromagnetic theory into static electric and magnetic fields on one hand

and dynamic electromagnetic fields on the other hand. This book (Volume I) deals with static electric and magnetic fields or in other words with the methods of potential theory. The list of contents: (1) The Electrostatic Field, (2) The Magnetostatic Field, (3) General Field Analogies, (4) Fields of Simple Geometries, (5) Experimental Mapping Methods, (6) Field Plotting Methods, (7) Two-Dimensional Analytic Solutions, (8) Three-Dimensional Analytic Solutions and Appendices.

The book is very carefully written throughout and the reviewer feels that particular attention should be called to Chapter 7 which contains an excellent discussion of conformal transformations and mapping. Chapter 6 on field plotting methods contains a rather thorough discussion of the method of images both electric and magnetic. Under numerical methods of field plotting the author includes a discussion of relaxation methods.

Altogether the reviewer feels that this book should be of real interest to a rather large group of physicists, engineers, and applied mathematicians.

ROHN TRUELL

*The evolution of scientific thought. From Newton to Einstein.* By A. d'Abro. Second edition, revised and enlarged. Dover Publications, Inc., 1950. xx + 481 pp. $3.95.

This "semi-popular book", using "non-technical language" (?) hopes to "serve as a general introduction" to the theory of relativity, "to whet the appetite for further knowledge" (the quotes are from Author's Preface). Its four parts deal with pre-relativity physics, the special theory of relativity, the general theory of relativity, and the methodology of science. We already have half-a-dozen short and masterly treatments of the same field by Einstein, Weyl and Eddington, and this one suffers badly in comparison. This very long book has seemed consistently boring to this reader. As an introduction to serious work it is far from helpful, as it contains no index, no bibliography, and quotations are made throughout without page reference. There are fifteen portraits of mathematicians (without indication of origin) but the pleasure the reader should have in looking at them is completely spoiled by fourteen of them being crowded together at the center of the book. It is to be hoped that this publisher's experiment, which no doubt made life easier for the make-up man, will not be repeated.

P. LE CORBEILLER