PLASTIC FLOW IN A DEEPLY NOTCHED BAR WITH SEMI-CIRCULAR ROOT*

BY

ALEXANDER J. WANG

Brown University

Summary. The unsteady motion problem of a circular-notched bar pulled in tension in plane strain is considered. The theory of perfectly plastic solids is used. Large strains are analyzed so that the material can also be considered as plastic-rigid. The basic equations governing stress and velocity are integrated independently in the characteristic plane. The results are used to construct the boundary change in a step-by-step manner. The problem is greatly simplified because at each step the new free boundary of the plastic region can be approximated by a circle. The final shape of the boundary of an initially semi-circular notch is presented when plastic flow has reduced the initial connection at the root to a line contact between the shanks.

1. Introduction. We consider the plastic flow in a deeply notched bar under tension with semi-circular root. For a deep notch, the plastic flow is localized in the vicinity of the root of the notch and the parts remaining elastic prevent appreciable lateral contraction, thus allowing us to consider the plastic flow to be in plane strain. The present paper follows a series of other papers on the deformation of notched bars under tension with V-shaped roots [1]** and rectangular roots [2]. The former is a quasi-steady case in which the configuration maintains geometrical similarity. The latter is an unsteady case in which the analysis can be readily made by building up some known solutions of slip line fields. The present treatment is another attempt at an unsteady case. Furthermore, since actual specimens have circular fillets at the corners, the result can also be extended to interpret such tension experiments. Thus this solution may help to examine more completely the singularities created by sharp corners in the previous cases.

The present paper limits itself to the case when the plastic region extends only to the circular part of the notch, Fig. 1 where $C_0'BC_0$ is the circular part and C_0D, $C_0'D'$ are the linear parts of the free boundary. The case when the plastic region extends to

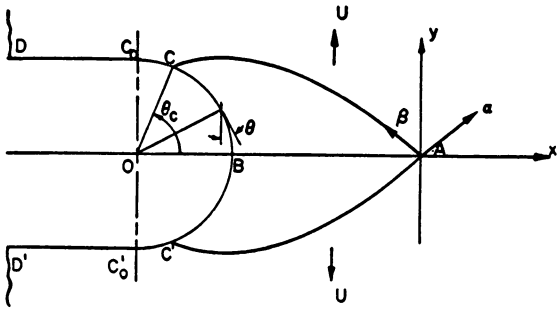

FIG. 1.

*Received February 11, 1953. This work was sponsored by Watertown Arsenal Laboratory under Contract DA-19-020 ORD-1117.

**Numbers in square brackets refer to the references at the end of the paper.

the linear part of the notch and the case of a circular fillet will be considered in later work.

We shall use plastic rigid theory. It is valid here as well as in most metal forming problems, since plastic flow needs to be analyzed in the regions of large flow only. We shall also consider the material to be perfectly plastic, i.e., plastic flow occurs at a constant stress limit. It is not equivalent to neglecting work-hardening, but rather to averaging its effect over the field of flow. The basic theory of such analysis has been fully discussed in the recent literature [3], [4], [5]. Accordingly, only a brief résumé of the final equations is given below.

2. Equations governing plastic flow in plane strain. In the plastic region the stress has to satisfy the yield condition and the equilibrium equations. In plane strain these three equations lead to a pair of first order hyperbolic equations having two orthogonal sets of real characteristics commonly called slip lines. The equations expressed in terms of these lines as coordinates (called the canonical form in the theory of partial differential

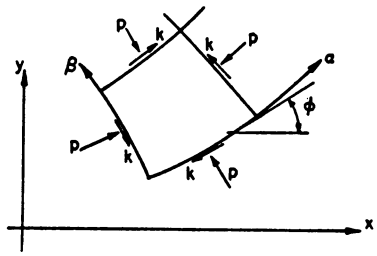

FIG. 2.

equations) take on a particularly simple form. Referring to the notation of Fig. 2, the equilibrium equations become

$$\begin{aligned} p + 2k\varphi &= \text{constant along an } \alpha\text{-line}, \\ p - 2k\varphi &= \text{constant along a } \beta\text{-line}, \end{aligned} \tag{1}$$

and so

$$\frac{\partial^2 \varphi}{\partial \alpha \, \partial \beta} = 0. \tag{2}$$

Equation (2) governing the slip line field can also be expressed in terms of the radii of curvature R, S of the α and β lines respectively, giving the alternative relations

$$\begin{aligned} dS + R\, d\varphi &= 0 \text{ along an } \alpha\text{-line}, \\ dR - S\, d\varphi &= 0 \text{ along a } \beta\text{-line}. \end{aligned} \tag{3}$$

The condition of incompressibility and the relation between stress and strain-rate lead to a similar set of equations for velocity components. They have the same characteristics. If u and v denote the velocity components along the α and β-lines, respectively, we have

$$\begin{aligned} du - v\, d\varphi &= 0 \text{ along an } \alpha\text{-line} \\ dv + u\, d\varphi &= 0 \text{ along a } \beta\text{-line}. \end{aligned} \tag{4}$$

We shall use the convention that R, S are positive in the directions of increasing β and α, respectively. If the curvatures of the α, β net are as shown in Fig. 2, R will be positive and S negative, then

$$\varphi = \alpha + \beta + \text{constant}, \tag{5}$$

$$\frac{p}{k} = -\alpha + \beta + \text{constant}, \tag{6}$$

$$\frac{\partial S}{\partial \alpha} = -R, \qquad \frac{\partial R}{\partial \beta} = S. \tag{7}$$

We then have for (3) and (4)

$$\text{(a)} \quad \frac{\partial^2 R}{\partial \alpha\, \partial \beta} = -R, \qquad \text{(b)} \quad \frac{\partial^2 S}{\partial \alpha\, \partial \beta} = -S, \tag{8}$$

$$\frac{\partial^2 u}{\partial \alpha\, \partial \beta} = -u, \qquad \frac{\partial^2 v}{\partial \alpha\, \partial \beta} = -v, \tag{9}$$

In the following, (8) will be referred to as the stress equations and (9) as the velocity equations. They are all of the form of the telegraph equation [6].

The transformation from the characteristic plane to the physical plane is achieved by integrating the following relations:

$$\begin{aligned} \frac{\partial x}{\partial \alpha} &= R \cos \varphi, \qquad \frac{\partial x}{\partial \beta} = S \sin \varphi, \\ \frac{\partial y}{\partial \alpha} &= R \sin \varphi, \qquad \frac{\partial y}{\partial \beta} = -S \cos \varphi. \end{aligned} \tag{10}$$

3. Analytical solutions. Finding the plastic flow inside the plastic region requires the solutions of equations (8) and (9) with their respective boundary conditions. The solutions of (8) will give the shape of the slip lines at any instant. Since the slip lines are the trajectories directed along the plane of maximum shear stress, we can find the stress distribution throughout the plastic region. The solutions of (9) will in turn give the strain-rate distribution at any instant. The two sets of equations can be solved separately in the characteristic plane; thereafter they are transformed back to the physical plane.

When the unsteady motion of the initially semi-circular notched root is investigated, it is found that the free-surface of the plastic region maintains very closely the form of a circular arc. To carry out the complete analysis at all stages in the deformation, it is therefore only necessary to evaluate the stress and velocity fields for a free boundary having circular form. This basic problem is considered in detail below.

First of all, we shall determine the constants in equations (5) and (6). By symmetry we may consider one half of the bar. Let the free surface of the plastic region subtend an angle $2\theta_c$ of a circular arc, and take the origin at the center of the bar, Fig. 1. Equation (5) becomes

$$\varphi = \alpha + \beta + \frac{\pi}{4}. \tag{11}$$

On the free boundary $p = -k$. This determines the constant in (6), giving

$$\frac{p}{2k} = -\alpha + \beta - \theta_c - \frac{1}{2}. \tag{12}$$

The free boundary BC therefore has the equation

$$-\alpha + \beta = \theta_c\,. \tag{13}$$

On the boundary we also have the relation,

$$\alpha + \beta = \theta. \tag{14}$$

Next, we look for the solution to the stress Eqs. (8). Because of symmetry it is necessary to solve for only one of the terms R or S. Hence, for S

$$\frac{\partial^2 S}{\partial\alpha\,\partial\beta} + S = 0. \tag{15}$$

The boundary conditions on BC are:

$$R = -S = +2^{1/2}a.$$

Using (7), we have

$$\frac{\partial S}{\partial\alpha} = -2^{1/2}a.$$

Making use of the fact that BC is a circular arc, we find

$$\frac{\partial S}{\partial\beta} = 2^{1/2}a.$$

The boundary conditions are therefore,

$$S = -2^{1/2}a, \qquad \frac{\partial S}{\partial\alpha} = -2^{1/2}a, \qquad \frac{\partial S}{\partial\beta} = 2^{1/2}a. \tag{16}$$

This is a Cauchy problem since two boundary conditions are given on a curve which is nowhere tangent to a characteristic. The solution is obtained by Riemann's method [6]. The Riemann's function in this case is $J_0[2(\xi - \alpha)^{1/2}(\eta - \beta)^{1/2}]$.

The solution is obtained by considering the general formulation of Green's theorems which in this case reduces to the following

$$\oint \left\{ J_0 \frac{\partial S}{\partial\beta}\, d\beta + S \frac{\partial J_0}{\partial\alpha}\, d\alpha \right\} = 0.$$

With the path PTQ as indicated in Fig. 3, we have,

$$\begin{aligned} S_p(\xi, \eta) &= S_Q - \int_{TQ} \left\{ J_0 \frac{\partial S}{\partial\beta}\, d\beta + S \frac{\partial J_0}{\partial\alpha}\, d\alpha \right\} \\ &= -2^{1/2}a - \int_{\eta}^{\xi+\theta_c} J_0[2i(\xi - \beta + \theta_c)^{1/2}(\beta - \eta)^{1/2}]2^{1/2}a\, d\beta \\ &\qquad - \int_{\eta-\theta_c}^{\xi} J_0'\left[i\left(\frac{2(\alpha + \theta_c - \eta)}{\xi - \eta}\right)^{1/2}\right] a\, d\alpha \\ &= -2^{1/2}a\{1 + P_I + P_{II}\} \\ &= -2^{1/2}a\left\{1 + 2\sum_{m=0}^{\infty} (-)^m \frac{B^{m+1}}{m!(2m+1)} [I_m(2B) + I_{m+1}(2B)]\right\}, \end{aligned} \tag{17}$$

where $2B = \theta_c - \eta + \xi$. The last step is obtained through the help of Sonine's integral representation of Bessel's function [7].

We know, however, from consideration of symmetry that the slip lines form logarithmic spirals with radii of curvature

$$S = -2^{1/2}r = -2^{1/2}ae^{\theta_c-\theta_p} = -2^{1/2}ae^{2B}. \tag{18}$$

The numerical identification of (17) and (18) is readily established, but the reduction of (17) to (18) through Bessel function recurrence relations appears to be quite complicated.

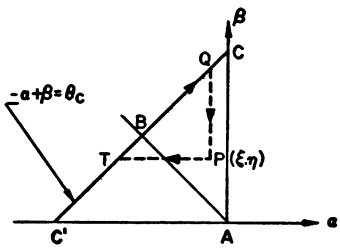

FIG. 3.

By symmetry,

$$R(\xi, \eta) = -S(-\eta, -\xi). \tag{18a}$$

The pressure distribution in the plastic region can be found easily, being especially simple across the line of symmetry AB. On AB, $-\alpha = \beta$ so that

$$\frac{p}{2k} = -2\alpha - \theta_c - \frac{1}{2},$$

$$\sigma_y = -p + k \qquad \text{(tension)} \qquad = 2k(1 + \theta_c + 2\alpha).$$

With

$$x = a(e^{\theta_c} - e^{\theta_p}),$$

$$\theta_p = \alpha + \beta = 2\alpha + \theta_c\,,$$

$$\theta_p = \log\left(e^{\theta_c} - \frac{x}{a}\right).$$

We obtain (see Fig. 4)

$$\frac{p}{2k} = -\log\left(e^{\theta_c} - \frac{x}{a}\right) - \frac{1}{2},$$

$$\sigma_y = 2k\left[1 + \log\left(e^{\theta_c} - \frac{x}{a}\right)\right]. \tag{19}$$

These relations are plotted in Fig. 4.

Next, we look at the solution to the velocity equations (9). By symmetry we may consider u only. Thus, we have

$$\frac{\partial^2 u}{\partial\alpha\,\partial\beta} + u = 0 \tag{20}$$

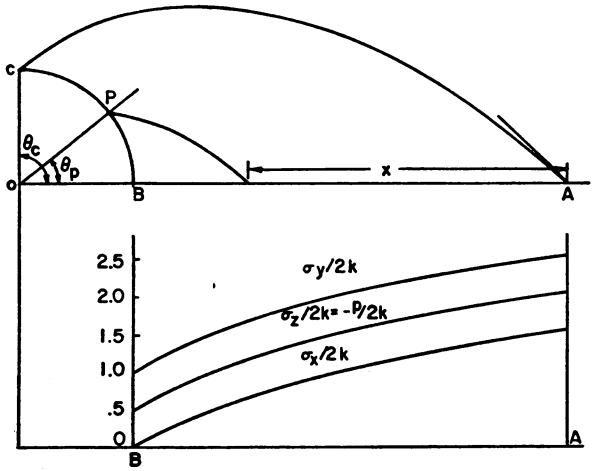

FIG. 4.

With the boundary conditions

$$u = \sin\varphi \qquad \text{along } AC,$$

$$v = -\cos\varphi = \frac{\partial u}{\partial\alpha} \qquad \text{along } AC', \tag{21a}$$

where U has been taken as the unit of velocities.

The last relation is obtained from Eq. (4). At the point A, we see that there is a jump in u across AC' of magnitude $2^{1/2}$. Integrating the last relation we find that, along AC',

$$u = -(\sin\varphi - 2^{1/2}). \tag{21b}$$

Note that the boundary conditions on the plastic rigid boundaries AC and AC' depend only on φ and not on the exact shape of the curve. Therefore, the result can be applied for any shape of plastic rigid boundary, the particular discontinuity of $2^{1/2}$ in u requiring the body to be symmetric at least about one of the x and y axes.

The problem of solving (20) and (21) is a problem of the Riemann type since we are given one condition on each of the two characteristics. The Riemann's function is again $J_0[2(\xi - \alpha)^{1/2}(\eta - \beta)^{1/2}]$. Using

$$\oint \left[J_0 \frac{\partial u}{\partial\beta}\, d\beta + u \frac{\partial J_0}{\partial\alpha}\, d\alpha \right] = 0$$

with the path $PQAT$ as indicated in Fig. 5, we have

$$u_P(\xi, \eta) = u(\xi, 0) - \int_{QA} J_0 \frac{\partial u}{\partial \beta} d\beta - \int_{AT} u \frac{\partial J_0}{\partial \alpha} d\alpha$$

$$= 2^{1/2} - \sin\left(\xi + \frac{\pi}{4}\right) + \int_0^\eta J_0[2\xi^{1/2}(\eta - \beta)^{1/2}] \cos\left(\beta + \frac{\pi}{4}\right) d\beta$$

$$+ \int_\xi^0 \left[2^{1/2} - \sin\left(\alpha + \frac{\pi}{4}\right)\right] \frac{\partial J_0[2\eta^{1/2}(\xi - \alpha)^{1/2}]}{\partial \alpha} d\alpha.$$

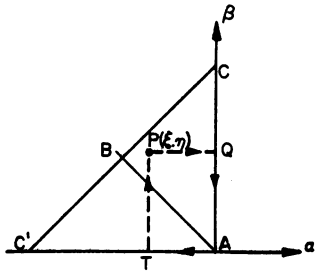

FIG. 5.

Integrating the second integral by parts and changing the variables, we obtain

$$u_P(\xi, \eta) = 2^{-1/2} I_0[2(-\xi\eta)^{1/2}] + \int_0^\eta I_0[2(-\xi z)^{1/2}] \cos\left(\eta + \frac{\pi}{4} - z\right) dz$$

$$+ \int_0^{-\xi} I_0[2(\eta z)^{1/2}] \cos\left(\xi + \frac{\pi}{4} + z\right) dz$$

$$= 2^{-1/2} I_0[2(-\xi\eta)^{1/2}][1 + \eta - \xi] + 2^{-1/2} \sum_{p=1}^{\infty} I_{2p}[2(-\xi\eta)^{1/2}]$$

$$\cdot \left\{\left(\frac{\eta}{\xi}\right)^p \left[\frac{\eta}{2p+1} + 1 - \frac{\xi}{2p-1}\right] - \left(\frac{\xi}{\eta}\right)^p \left[\frac{\xi}{2p+1} + 1 - \frac{\eta}{2p-1}\right]\right\}. \tag{22}$$

The last step is obtained through the help of Sonine's integral representation of Bessel's function [7]. The convergence of the series in the range considered is very good. By symmetry we have

$$v(\xi, \eta) = -u(-\eta, -\xi). \tag{22a}$$

In the step-by-step construction of the boundary, the following expressions are useful. On the boundary

$$\xi = \tfrac{1}{2}(\theta - \theta_c), \qquad \eta = \tfrac{1}{2}(\theta + \theta_c)$$

Hence, the radial velocity

$$U_r = 2^{-1/2}(u - v) = (1 + \theta_c) I_0[(\theta_c^2 - \theta^2)^{1/2}]$$

$$- \sum_{p=1}^{\infty} I_{2p}[(\theta_c^2 - \theta^2)^{1/2}] \left[\left(\frac{\theta + \theta_c}{\theta - \theta_c}\right)^p \left(\frac{\theta - 2p\theta_c}{4p^2 - 1}\right) - \left(\frac{\theta - \theta_c}{\theta + \theta_c}\right)^p \left(\frac{\theta - 2p\theta_c}{4p^2 - 1}\right)\right] \tag{23}$$

and the tangential velocity

$$U_t = 2^{-1/2}(u + v) = \sum_{p=1}^{\infty} I_{2p}[(\theta_c^2 - \theta^2)^{1/2}]\left[\left(\frac{\theta + \theta_c}{\theta - \theta_c}\right)^p - \left(\frac{\theta - \theta_c}{\theta + \theta_c}\right)^p\right] \tag{24}$$

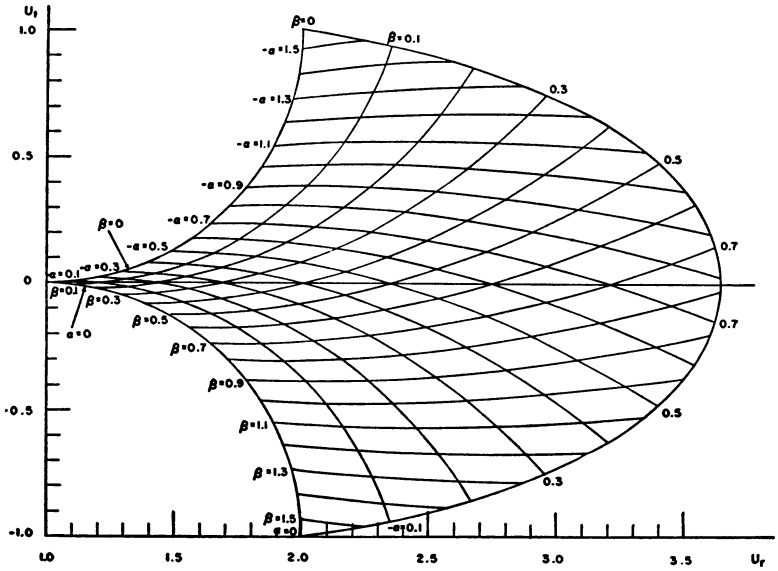

FIG. 6.

These quantities are plotted in Fig. 6 for θ_c ranging from 0 to $\pi/2$. At the point B,

$$(U_t)_B = 0,$$

$$(U_r)_B = 2^{1/2}u_B = I_0(\theta_c) + 2\sum_{p=0}^{\infty}(-)^p I_{2p+1}(\theta_c).$$

Since $\sinh z = 2\sum_{p=0}^{\infty} I_{2p+1}(z)$ (being transformed from the expression for $\sin z$ [7]), we have

$$2^{1/2}u_B = I_0(\theta_c) + \sinh\theta_c - 4\sum_{p=1}^{\infty} I_{4p-1}(\theta_c).$$

The convergence of the last expression is particularly rapid; for 4 significant figures one term in the summation is enough, i.e.,

$$2^{1/2}u_B = I_0(\theta_c) + \sinh\theta_c - 4I_3(\theta_c) \tag{25}$$

The transformation functions are obtained in the following manner. With

$$S = -2^{1/2}ae^{\theta_c-\beta+\alpha}, \qquad R = 2^{1/2}ae^{\theta_c-\beta+\alpha}$$

we can integrate Eq. (10) obtaining

$$x = 2^{1/2}a\int e^{\theta_c-\beta+\alpha}\cos\left(\alpha + \beta + \frac{\pi}{4}\right)d\alpha + f(\beta)$$

or

$$= -2^{1/2}a \int e^{\theta_c-\beta+\alpha} \sin\left(\alpha + \beta + \frac{\pi}{4}\right) d\beta + g(\alpha)$$

or

$$x = ae^{\theta_c-\beta+\alpha} \cos(\alpha + \beta) + C_1 .$$

At $\alpha = \beta = 0$, $x = 0$ so $C_1 = -ae^{\theta_c}$. Therefore,

$$x = ae^{\theta_c}[e^{\alpha-\beta} \cos(\alpha + \beta) - 1].$$

Similarly,

$$y = a[e^{\theta_c-\beta+\alpha} \sin(\alpha + \beta)]. \tag{26}$$

They appear to be quite simple, however, with α, β and θ_c all varying with time, the transformation functions for a generic t are very complicated.

4. Approximate solutions. Since the analytical solutions above are not in closed form, the dependence of θ_c, α, β on t becomes very complicated and it is much simpler to analyze the unsteady motion by approximate means. If we represent a boundary curve in intrinsic coordinates R and S and if we know the normal and tangential velocity components along this curve, then the subsequent shape of the boundary will depend only on

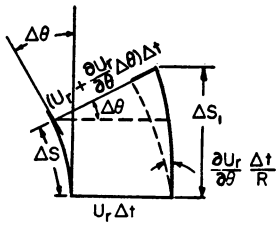

FIG. 7.

the normal velocity component. Referring to Fig. 7, we have, after the time Δt,

$$R_1 = R + \Delta t\left[U_r - \frac{1}{R}\frac{\partial}{\partial\theta}(R)U_r' + U_r''\right] + O(\Delta t^2), \tag{27}$$

where we denote differentiation with respect to θ by a prime.

For an originally circular boundary, $\partial R/\partial\theta = 0$; $U_r + U_r''$ was numerically computed for $\theta_c = \pi/4$. The result shows that if Δt is taken as 1, the deviation of the new boundary from a circle with the same original center is within $\pm 2.0\%$ and if Δt is taken as 0.1, the deviation is within $\pm 0.4\%$. Furthermore, we can change the position of the center of the circle with the resulting error of 0.33% for $\Delta t = 1$ and 0.14% for $\Delta t = 0.1$. In the following step-by-step construction, a circle is drawn through the new positions of B and C with the center remaining on the x-axis. The validity of approximating the new boundary by such a circular arc is checked all along the step-by-step process for θ_c ranging from $\pi/2$ and zero. The maximum error is about 0.2%. It is considered to lie within our error of graphical construction.

5. Step-by-step process. After a small time increment Δt, an originally circular

arc boundary BC will take the form B_2C_2, Fig. 8. We shall pass a circle through B_2 and C_2 with center O_2 on the x-axis. Hence,

$$(r + \Delta r)^2 = \left[a \cos \theta_c + \left(\frac{dx}{dt}\right)_c \Delta t - \Delta x_0\right]^2 + \left[a \sin \theta_c + \left(\frac{dy}{dt}\right)_c\right]^2, \tag{28}$$

$$r + \Delta r + \Delta x_0 = r + 2^{1/2} u_B \, \Delta t. \tag{29}$$

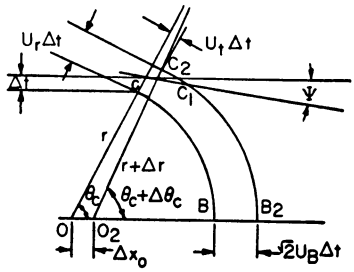

FIG. 8.

For the values at C, we can make use of the boundary conditions. Then,

$$\left(\frac{dx}{dt}\right)_c = u_c \sin\left(\frac{\pi}{4} - \theta_c\right) - v_c \cos\left(\frac{\pi}{4} - \theta_c\right)$$

$$= \cos \theta_c + \sin \theta_c ,$$

$$\left(\frac{dy}{dt}\right)_c = 1 + \sin \theta_c - \cos \theta_c .$$

Solving (28) and (29) for Δx_0 and Δr, we obtain

$$\Delta x_0 = \frac{2^{1/2} u_B - 1 - \sin \theta_c}{1 - \cos \theta_c} \Delta t + O(\Delta t^2), \tag{30}$$

$$\Delta r = \frac{-2^{1/2} u_B \cos \theta_c + 1 + \sin \theta_c}{1 - \cos \theta_c} \Delta t + O(\Delta t^2), \tag{31}$$

$$\theta_c + \Delta\theta_c = \log \frac{r e^{\theta_c} - \Delta x_0}{r + \Delta r}$$

$$= \theta_c + \frac{\Delta x_0}{r}(1 - e^{-\theta_c}) - \frac{2^{1/2} u_B}{r} \Delta t + O(\Delta t^2). \tag{32}$$

Since these algebraic equations are expressed in terms of θ_c and $2^{1/2}u_B$ only, where $2^{1/2}u_B$ is represented by Eq. (25), it is easier to work with these equations than to interpolate the results from Fig. 6.

An example for an initial $\theta_c = \pi/2$ is carried out. The free boundary is found after a small time increment Δt by using eqns. (30), (31) and (32). The first step is to replace θ_c by $\pi/2$, put $2^{1/2}u_B(\pi/2) = 3.644$ in these eqns. and find the position of the new center and the values of the new radius and of θ_{c1}. For this new value of θ_{c1} we can calculate the corresponding $2^{1/2}u_B$. The new values of θ_c and $2^{1/2}u_B$ are then substituted in the three equations to find out the next θ_c and so forth. The magnitude of the time incre-

ment is chosen to provide as rapid a procedure as possible in conjunction with satisfactory accuracy. In the present example $\Delta t = 0.050$ with $r = 1$ at that instant. The accuracy of such a step is checked by taking $\Delta t = 0.025$ for two steps and comparing the result with that of one step of $\Delta t = 0.050$. The resulting points are indistinguishable. However, similar comparison between two steps of $\Delta t = 0.05$ and one step of $\Delta t = 0.10$ shows that $\Delta t = 0.10$ yields considerable error. Therefore, the step of $\Delta t = 0.05$ is taken and the resulting error is no greater than the unavoidable graphical errors. The

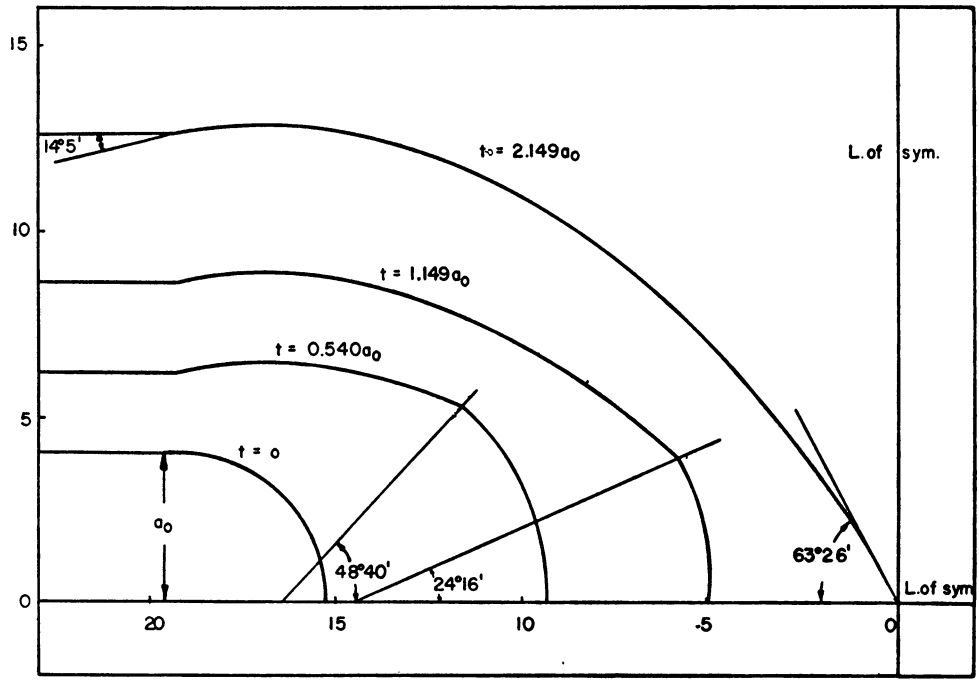

FIG. 9.

final shape along with two intermediate steps are shown in Fig. 9. From Fig. 8 the initial discontinuity in slope is calculated as follows

$$\tan \Psi = \frac{(r + \Delta r) \sin (\theta_c + \Delta\theta_c) - r \sin \theta_c - \Delta t}{\Delta x_0 + (r + \Delta r) \cos (\theta_c + \Delta\theta_c) - r \cos \theta_c}. \tag{33}$$

For $\theta_c = \pi/2$ and $\theta_c + \Delta\theta_c \doteq \pi/2$, $\sin (\theta_c + \Delta\theta_c) \doteq 1$ and $\cos (\theta_c + \Delta\theta_c) \doteq -\Delta\theta_c$

$$\tan \Psi = \frac{\Delta r - \Delta t}{\Delta x_0 - r\Delta\theta_c}$$

$$= \frac{2.644\Delta t - 1.644\Delta t}{(1.644e^{-\pi/2} + 3.644)\Delta t} = \frac{1}{3.986} = .2509,$$

$$\Psi = 14^\circ\ 5'.$$

Note that the boundary forms a recess at the point C which is also the case for a V-notched bar. The final angle of 63° 26′ can be found in a similar manner.

6. Remarks. We note that the problem is conveniently separated into two mutually independent ones, viz., stress and velocity solutions. These are then combined to give the solution to the over-all problem. However, the combination is complicated by the fact that both α and β are functions of time. Complicated integral expressions arise and so step-by-step method is adopted. Fortunately, the approximation of the new boundary by a circular arc is good, making the computation reasonably simple.

References

1. E. H. Lee, *Plastic flow in a V-notched bar pulled in tension*, J. Appl. Mech. 19, 331-336 (1952).
2. E. H. Lee, *The deformation of a bar with rectangular notch*, Report written for Watertown Arsenal under P.O. No. ORDEB 52-988, Brown University, June 1952.
3. R. Hill, *The mathematical theory of plasticity*, Oxford, 1950.
4. W. Prager and P. G. Hodge, Jr., *Theory of perfectly plastic solids*, John Wiley & Sons, 1951.
5. E. H. Lee, *The theoretical analysis of metal forming problems in plane strain*, J. Appl. Mech. 19, 97-103, (1952).
6. Courant and Hilbert, *Methoden der mathematische Physik*, vol. 2, p. 316, Interscience Publishers, 1937.
7. Whitaker and Watson, *Modern analysis*, 4th ed. Cambridge, p. 377.