

where  $A$  is the shaded area shown in Fig. 1, i.e.

$$j_+(0) = \frac{1}{\pi T_{mj}} \tan^{-1} \left\{ \left( \frac{1-\gamma}{1+\gamma} \right)^{1/2} \right\}. \quad (25)$$

For the process  $A_\gamma$ ,

$$j_+(0) = \frac{1}{\pi \tau_0(1-\gamma)} \tan^{-1} \left\{ \left( \frac{1-\gamma}{1+\gamma} \right)^{1/2} \right\}. \quad (26)$$

As  $\gamma$  approaches one,  $j_+(0)$  becomes infinite. This will be true not only for zero but for all  $x$ . The implication is plain. The Fokker-Planck process is a degenerate process in which the one sided current density of the system is infinite. A Fokker-Planck model for the velocity motion of a colloid particle would describe an infinite number of changes of direction of the particle per unit time. Such a model used to describe voltage fluctuations would imply an infinite number of polarity reversals per second. Since a process  $A_\gamma$  will afford the same correlation function and equilibrium distribution, and finite polarity reversal frequency, it is suggested that such a model may better describe noise, and that the number of zero crossings be regarded as an independent macroscopic physical quantity on an equal footing with  $\tau_0$ ,  $E_0$ .

The author thanks Dr. Franz Stumpers of Phillips Eindhoven and Dr. Edwin Akutowicz for their interest and encouragement.

### EVALUATION OF CONSTANTS IN CONFORMAL REPRESENTATION\*

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In using the Schwarz-Christoffel transformation [1],

$$dz = K \prod_{i=1}^n (\zeta - \zeta_i)^{(\alpha_i/\pi)-1} d\zeta = Kf(\zeta) d\zeta$$

whereby the upper half  $\zeta$ -plane is mapped into a simple connected polygon, the evaluation of the unknown constant  $K$  (if complex  $K = ce^{i\lambda}$ ,  $c, \lambda$  real), is oftentimes tedious. We shall show a simple method of evaluating the unknown constant  $K$  by examples, proving first a

**THEOREM:** *By the Schwarz-Christoffel transformation if  $\zeta_i$  in the  $\zeta$ -plane corresponds to two points  $P_i, Q_i$  in the  $z$ -plane and  $\zeta = \zeta_i$  is a simple pole of  $f(\zeta)$ , then*

$$K = \frac{\text{dist}(P_i, Q_i)}{\pi i R(\zeta = \zeta_i)}$$

$R$ , denoting residue and  $\text{dist}(P_i, Q_i)$ , denoting the distance between the two points  $P_i$  and  $Q_i$ .

\*Received May 8, 1953.

Proof: Since

$$\begin{aligned} dz &= Kf(\zeta) d\zeta \\ \int_{P_i}^{Q_i} dz &= \text{dist}(P_i, Q_i) \\ &= \lim_{\delta \rightarrow 0} \int_0^\pi Kf(\zeta_i + \delta e^{i\theta}) i \delta e^{i\theta} d\theta \\ &= K \lim_{\delta \rightarrow 0} \int_0^\pi f(\zeta_i + \delta e^{i\theta}) i \delta e^{i\theta} d\theta, \end{aligned}$$

where  $\zeta - \zeta_i = \delta e^{i\theta}$ .

Since  $f$  has a simple pole at  $\zeta$ , the Laurent expansion [2], is

$$f(\zeta) = \frac{R}{\zeta - \zeta_i} + g(\zeta)$$

with  $g(\zeta)$  analytic near  $\zeta = \zeta_i$ . Now,

$$f(\zeta_i + \delta e^{i\theta}) = \frac{R}{\delta e^{i\theta}} + g(\zeta_i + \delta e^{i\theta}).$$

Therefore,

$$\begin{aligned} \lim_{\delta \rightarrow 0} \int_0^\pi f(\zeta_i + \delta e^{i\theta}) i \delta e^{i\theta} d\theta &= \lim_{\delta \rightarrow 0} \left[ Ri \int_0^\pi d\theta + i\delta \int_0^\pi \sum_{k=0}^{\infty} c_k \delta^k e^{(k+1)i\theta} d\theta \right] \\ &= i\pi R \end{aligned}$$

whence

$$K = \frac{\text{dist}(P_i, Q_i)}{\pi i R(\zeta = \zeta_i)}.$$

Consider the transformation described by Milne-Thomson [3], whereby we map the infinite strip on the upper half  $\zeta$ -plane. The Schwarz-Christoffel transformation gives

$$\frac{dz}{d\zeta} = K\zeta^{-1} \quad \text{or} \quad z = K \int \frac{d\zeta}{\zeta} + L.$$

$L = 0$  for  $z = 0$  corresponds to  $\zeta = 1$ .

By the theorem,

$$K = \frac{\text{dist}(P_i, Q_i)}{\pi i R(\zeta = 1)} = \frac{ai}{\pi i(1)} = \frac{a}{\pi}$$

In this case comparison of real and imaginary parts with infinities is avoided.

#### REFERENCES

1. Zeev-Nehari, *Conformal mapping*, McGraw Hill, 1952, pp. 189 ff.
2. E. C. Titchmarsh, *The theory of functions*, 2nd Ed., Oxford University Press, 1950, p. 91.
3. L. M. Milne-Thomson, *Theoretical hydrodynamics*, 2nd. Ed., The Macmillan Company, London, 1950, p. 275.