the Lindstedt procedure furnishes the following values

$$c_2 = 1, \qquad c_3 = 15/16, \qquad c_4 = 13/16.$$

From the following table, reproduced from Shohat's paper cited above, it is tempting to conjecture that the Shohat series converges for all values of λ which are positive. If so, the series should be more widely known.

λ	v computed using (5.2)	v (Van der Pol)
.33	.98	.99
1.0	.93	.90
2.0	.77	.78
8.0	.35	.39
10.0	.30	.31

The Van der Pol values were obtained using graphical techniques.

References

1. R. Bellman, *On the summability of solutions of linear integral equations*, Duke Math. J. **17**, 53-5 (1950)
2. S. Lefschetz, *Lectures on differential equations*, Princeton University Press, 1946
3. J. Lighthill, *A technique for rendering approximate solutions to physical problems uniformly valid*, Phil. Mag. **40**, 1179-1201 (1949)
4. J. Shohat, *On Van der Pol's and related nonlinear differential equations*, J. Appl. Physics **15**, 568-574 (1944)
5. J. J. Stoker, *Nonlinear vibrations*, Interscience, New York, 1950

ON MIDDLETON'S PAPER "SOME GENERAL RESULTS IN THE THEORY OF NOISE THROUGH NON-LINEAR DEVICES"*

By J. S. SHIPMAN (*Laboratory For Electronics, Inc., Boston, Mass.*)

As one of the central results of the title paper [1], Middleton obtained $R_l(t)$, the correlation function for the lth zone, as a function of the input correlation function r_0 in the case of the νth law half-wave rectification of narrow-band normal noise (see, e.g., his equations (7.14) and (7.15)). Unless one resorts to series evaluations, his formulas are not particularly suited for numerical computation as they stand, involving as they do hypergeometric functions which are not well tabulated. For purposes of calculation, then, a reduction of the hypergeometric functions occurring in the formulas to tabulated functions must ordinarily be effected, usually by applying the recursion relations among contiguous hypergeometric functions due to Gauss.

When this reduction is accomplished, the hypergeometric functions in Middleton's formulas are seen to be either polynomials in r_0^2 or combinations of complete elliptic integrals of the first and second kind, provided ν is an integer (see, e.g., Middleton's equations (7.16) and (7.17)). These polynomials and combinations of elliptic integrals turn out to be, in every case so far examined, special cases of "Bennett functions" recently tabulated by the author and his colleagues [2, 3]. In the present note expressions

*Received Sept. 24, 1954. Revised manuscript received Nov. 3, 1954.

for $R_l(t)$ in terms of Bennett functions for the important practical cases $\nu = 1, 2$ for several values of l and for $l = 0$ and $\nu = 3, 4$ are collected. The notation used is that of Middleton except for the symbol $A_{mn}^{(\nu)}$ which is the Bennett function of the νth kind [3]. Thus

$$R_0(t)_{\nu=1} = (\beta^2\psi/2)[\tfrac{1}{2}A_{00}^{(1)}(r_0)], \tag{1}$$

$$R_1(t)_{\nu=1} = (\beta^2\psi/2)(\cos \omega_c t)A_{01}^{(1)}(r_0), \tag{2}$$

$$R_2(t)_{\nu=1} = (\beta^2\psi/2)(\cos 2\omega_c t)A_{02}^{(1)}(r_0), \tag{3}$$

$$R_l(t)_{\nu=1} = 0 \qquad (l = 3, 5, 7 \cdots), \tag{4}$$

$$R_4(t)_{\nu=1} = (\beta^2\psi/2)(\cos 4\omega_c t)A_{04}^{(1)}(r_0); \tag{5}$$

further

$$R_0(t)_{\nu=2} = \beta^2\psi^2[\tfrac{1}{2}A_{00}^{(2)}(r_0)], \tag{6}$$

$$R_1(t)_{\nu=2} = \beta^2\psi^2(\cos \omega_c t)A_{01}^{(2)}(r_0), \tag{7}$$

$$R_2(t)_{\nu=2} = \beta^2\psi^2(\cos 2\omega_c t)A_{02}^{(2)}(r_0), \tag{8}$$

$$R_3(t)_{\nu=2} = \beta^2\psi^2(\cos 3\omega_c t)A_{03}^{(2)}(r_0), \tag{9}$$

$$R_l(t)_{\nu=2} = 0 \qquad (l = 4, 6, 8, \cdots), \tag{10}$$

$$R_5(t)_{\nu=2} = \beta^2\psi^2(\cos 5\omega_c t)A_{05}^{(2)}(r_0); \tag{11}$$

and finally

$$R_0(t)_{\nu=3} = 3\beta^2\psi^3[\tfrac{1}{2}A_{00}^{(3)}(r_0)], \tag{12}$$

$$R_0(t)_{\nu=4} = 12\beta^2\psi^4[\tfrac{1}{2}A_{00}^{(4)}(r_0)]. \tag{13}$$

The functions $A_{mn}^{(\nu)}(r_0)$ are tabulated directly for $\nu = 1, 2$ [2, 3]; for $\nu \geq 3$ recursion formulas are given which enable Bennett functions of higher kind to be obtained from those of lower kind.

The author is indebted to E. Feuerstein and Dr. R. L. Sternberg for suggestions leading to the present note, and to Miss Jean Conway for assistance in the preparation of the manuscript.

References

1. D. Middleton, *Some general results in the theory of noise through non-linear devices*, Quart. Appl. Math. 5, 369 (1948)
2. R. L. Sternberg, J. S. Shipman, and W. B. Thurston, *Tables of Bennett functions for the two frequency modulation product problem for the half-wave linear rectifier*, Quart. J. Mech. and Appl. Math., **7**, 505 (1954).
3. R. L. Sternberg, J. S. Shipman, and H. Kaufman, *Tables of Bennett functions for the two frequency modulation product problem for the half-wave square-law rectifier*, to appear in Quart. J. Mech. and Appl. Math.