From the above results it may be safely concluded that the first eigenvalue of Eq. (1) is $\lambda_1 = 723.907$, correct to at least six figures.

It is theoretically possible to obtain not only the first, but also the higher, eigenvalues by this method. However, meaningful results can only be obtained for the higher ones if a very high degree of accuracy in the coefficients exists. With the present figures, the second highest roots of Eqs. (9) and (10) are respectively, .276 and .313. The latter value is probably correct to at least two places; the corresponding value of α_2^2 is 32.0.

It may be added that this same equation has been treated by Purday [3], also using a series in the independent variable. The results obtained there are

$$\alpha_1^2 = 2.83, \qquad \alpha_2^2 = 32$$

which agree with the values of the present analysis.

References

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(3) H. F. P. Purday, *An introduction to the mechanics of viscous flow*, Dover, p. 149, 1949

A USEFUL INTEGRAL FORMULA FOR THE INITIAL REDUCTION OF THE TRANSPORT EQUATION*

By RICHARD L. LIBOFF (*Army Chemical Center*)

A theorem pertaining to spherical harmonics** states that if an angle β is determined by two directions, whose coordinates on the surface of a unit sphere are (θ, α) and (θ', α'), and $F(\beta)$ is a function expandable in a series of spherical harmonics in argument $(\cos \beta)$,

$$F(\beta) = \sum_l \frac{2l+1}{4\pi} k_l P_l(\cos \beta), \tag{1}$$

then

$$\int_{4\pi} F(\beta) Y_l(\theta, \alpha) \, d\omega = k_l Y_l(\theta', \alpha'). \tag{2}$$

$Y(\theta, \alpha)$ may either be spherical surface harmonic, a tesseral harmonic, or a spherical harmonic.

The theorem has been noted because of its applicability in diffusion theory, for problems which militate use of the transport equation.

The transport equation for various types of scattering (e.g., neutron, gamma, light) may in general, be written,

$$\frac{1}{\mu} \mathbf{s} \cdot \nabla N(\mathbf{r}, \mathbf{s}) = -N + \frac{1}{4\pi} \int_{4\pi} p(\beta) N(\mathbf{r}, \mathbf{s}') \, d\omega' + S. \tag{3}$$

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**Hobson, *Spherical and Ellipsoidal Harmonics*, Cambridge, 1931, p. 146.

Equation (3) states that the spatial rate of change of the distribution function $N(\mathbf{r}, \mathbf{s})$ at the position $\mathbf{r}$, in the direction of the unit vector $\mathbf{s}$, is due to three effects which appear on the right side of the equation. They are, in order of their appearance, first, radiation lost through absorption, and scattering out of the direction $\mathbf{s}$; secondly radiation which is scattered from all 4π solid angle into the direction $\mathbf{s}$. $p(\beta)$ is the angular differential cross section for the specific process involved. Thirdly there is the source contribution S. It is assumed in the above equation that a scatter is not accompanied by a change in wave length. μ is the total narrow beam absorption coefficient.

When the solution of (3) is through expansion processes, Eq. (2) becomes a useful integral formula for reducing the integral of (3) to its equivalent summation.

For instance, consider the problem which permits the unit vector $\mathbf{s}$ to be replaced by the ordinate angle θ (e.g., spherical symmetry and infinite plane source). In this case the distribution function may be expanded as

$$N(\mathbf{r}, \mathbf{s}') = \sum_l \frac{2l+1}{4\pi} a_l(\mathbf{r}) P_l(\cos \theta'). \tag{4}$$

Substitution of this expansion into the kernel of (3) will transform the original integral into a series of integrals, each term of which is similar to expression (2). From this we may write for the integral of (3),

$$\sum_l \frac{2l+1}{4\pi} a_l(\mathbf{r}) k_l P_l(\cos \theta), \tag{5}$$

k_l is the Legendre coefficient of the expansion of $p(\beta)$.

Following this with the replacement of $N(\mathbf{r}, \mathbf{s})$ by its equivalent expansion (4) into (3), will reduce the original integro-differential transport equation to a system of differential equations involving the sequence $\{a_l(\mathbf{r})\}$, knowledge of which completely determines $N(\mathbf{r}, \mathbf{s})$.

For more complicated geometries, where expansions in tesseral, or spherical surface harmonics are called for, the formula may be used in like manner, reducing the integral term of the transport equation to its corresponding summation, whence usually, a reduced system of equations is easily derivable.

TWO REMARKS ON HEISENBERG'S THEORY OF ISOTROPIC TURBULENCE*

By WILLIAM H. REID[1] (*Trinity College, Cambridge, England*)

1. Introduction. The exact dynamical equation for the rate of change with time of the energy spectrum function $E(k)$ in isotropic turbulence may be written in the form

$$\partial E(k)/\partial t = T(k) - 2\nu k^2 E(k), \tag{1}$$

where $T(k)$ is the transfer function usually denoted by this symbol. The incompleteness of Eq. (1) is well known and, in the past, several so-called "physical transfer theories" have been proposed in which a further relationship between $T(k)$ and $E(k)$ is postulated;

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