Take the Fourier sine transform of Eq. (12) and then eliminate the derivatives of u by integrating by parts. This gives

$$w(\xi, t) = \int_0^\infty u(x, t)[a(b - h\kappa\xi^2) \sin \xi x + \xi(a + b - h\kappa\xi^2) \cos \xi x]\, dx + ah\kappa\xi u(0, t) - h\kappa\xi \frac{\partial u(0, t)}{\partial x}. \tag{18}$$

Let

$$\phi(\xi) = \int_0^\infty f(x)[a(b - h\kappa\xi^2) \sin \xi x + \xi(a + b - h\kappa\xi^2) \cos \xi x]\, dx. \tag{19}$$

Then

$$w(\xi, 0) = \phi(\xi) + ah\kappa\xi f(0) - h\kappa\xi f'(0). \tag{20}$$

So

$$w(\xi, s) = \frac{\phi(\xi) + \kappa\xi abg(s) + \kappa\xi ahV}{s + \kappa\xi^2}. \tag{21}$$

Hence

$$w(\xi, t) = [\phi(\xi) + \kappa\xi ahV] \exp(-\kappa\xi^2 t) + \kappa\xi ab \int_0^t g(\tau) \exp[-\kappa\xi^2(t - \tau)]\, d\tau. \tag{22}$$

But

$$w(x, t) = \frac{2}{\pi} \int_0^\infty w(\xi, t) \sin \xi x\, d\xi. \tag{23}$$

Therefore, from Eqs. (12) and (23),

$$u(x, t) = \frac{2}{\pi} \int_0^\infty w(\xi, t) \frac{a(b - h\kappa\xi^2) \sin \xi x + \xi(a + b - h\kappa\xi^2) \cos \xi x}{a^2(b - h\kappa\xi^2)^2 + \xi^2(a + b - h\kappa\xi^2)^2}\, d\xi, \tag{24}$$

where $w(\xi, t)$ is given by Eq. (22), and $\phi(\xi)$ by Eq. (19). From Eq. (3) and (24) one may obtain $v(t)$.

NOTE ON THE SYMMETRICAL PROPERTY OF THE THERMAL CONDUCTIVITY TENSOR*

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According to the Onsager reciprocal hypothesis, the thermal conductivity tensor among other similar properties of matter is symmetrical. In the following note it is demonstrated that for some cases of interest, the property of symmetry according to the Onsager hypothesis is not a necessary piece of information.

Assuming a space filled with a continuous medium at temperature T, having a

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thermal conductivity of K_{ij} such that the conducted flow of heat through an elemental area ds having associated direction cosines l_i is

$$-(K_{ij}T_{,j})l_i\, ds.$$

The net rate of heat flow into a volume bounded by a closed surface "s" then is

$$\int_s (K_{ij}T_{,j})l_i\, ds.$$

The surface integral may be transformed to a volume integral,

$$\int_s (K_{ij}T_{,j})l_i\, ds = \int_v (K_{ij}T_{,j})_{,i}\, dv.$$

It is therefore necessary to investigate the properties of the form

$$(K_{ij}T_{,j})_{,i}\ .$$

Considering the case where

$$K_{ij} = K_{ij}(T),$$

$$(K_{ij}T_{,j})_{,i} = K_{ij}T_{,ji} + \frac{dK_{ij}}{dT}T_{,i}T_{,j}$$

$T_{,ji}$ and $T_{,i}T_{,j}$ are both symmetrical in i and j, therefore only the symmetrical portion of K_{ij} will matter in $(K_{ij}T_{,j})_{,i}$.

Since the case of $K_{ij} = K_{ij}(T)$ applies to a large class of practical applications, it is important to note, that for this case the anti-symmetrical portion of K_{ij} if it existed at all would not contribute to a first law of thermodynamic energy accounting.

A CONVERSE TO THE VIRTUAL WORK THEOREM FOR DEFORMABLE SOLIDS*

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1. Introduction. Consider a continuous body occupying a volume V and bounded by a closed surface S.[1] Any system of stresses σ_{ij} ,[2] satisfying the equilibrium conditions for zero body forces

$$\sigma_{ij,j} = 0, \tag{1}$$

$$\sigma_{ij} = \sigma_{ji}\ , \tag{2}$$

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[1]It is assumed that the body is simply connected and that the surface S is composed of a finite number of pieces of each possessing a continuously turning tangent plane. All of the functions will be assumed to possess as many continuous derivatives in V and on S as are necessary for the theorems which will be used later.

[2]The subscripts range over the values 1, 2, 3 and repeated subscripts will be summed over the entire range. Subscripts following a comma denote partial differentiation with respect to Cartesian coordinates x_i , e.g., $\sigma_{ij,j} = \partial\sigma_{ij}/\partial x_j$.