BOOK REVIEWS

Theoretical elasticity. By A. E. Green and W. Zerna. Oxford University Press, New York, 1954. xiii + 442 pp. $8.00.

All serious students of the mathematical theory of elasticity will welcome this book. It does not make any claim to being a treatise on the subject. Rather, it represents both in choice of subject and presentation the personal interests and point-of-view of the authors. Indeed many of the topics discussed are ones which are to a great extent the results of Professor Green's own researches. Much of the material has not been previously presented in a unified notation or in book form.

The subjects covered are drawn both from the finite and infinitesimal elasticity theories. In presenting the basic concepts of these theories, finite elasticity theory is developed first, and the fundamental formulation of the infinitesimal theory is derived as a first approximation to the more general theory. Although this no doubt adds to the formal complexity of the presentation, it has the undoubted advantages of rigor and conceptual clarity.

The finite elasticity theory is used to solve a number of simple problems which admit of exact general solution without further approximation or assumption—except that of incompressibility of the material. The theory of the superposition of infinitesimal deformations on finite deformations is then developed.

The infinitesimal theory is developed for the cases of plane strain and generalised plane stress and is applied to the discussion of a wide range of problems for both isotropic and anisotropic materials. Complex variable methods are used consistently in a manner somewhat similar to that adopted by Muskhelishvili. In the final four chapters of the book the general bending theory of shells and membrane theory are briefly discussed, together with their particularisation to cylindrical shells and shells of revolution.

Throughout the book tensor notation with convected coordinates is used. Although this makes for great elegance of presentation, it will no doubt limit to a large degree the extent to which the book will be used.

R. S. RIVLIN

Stochastic models for learning. By Robert R. Bush and Frederick Mosteller. John Wiley and Sons, Inc., New York, and Chapman & Hall, Ltd., London, 1955. xvi + 365 pp. $9.00.

This book is primarily concerned with models for analysing learning experiments in which the learner is exposed repeatedly to the same stimulus. The essential ingredients of the model are relations between stimuli, responses and events. It is postulated that there is a set of probabilities associated with the various responses to the given stimulus. Following each response a certain event takes place such as for example a rewarding or an absence of a reward for the subject and this changes the response probabilities for future stimuli. The event will generally depend in some preassigned way upon the response, for example there may be a one to one correspondence between events and the responses. The events are represented by linear transformations to be applied to the response probabilities.

Some very interesting mathematical problems arise in connection with the determination of the asymptotic behavior of the response probabilities after a large number of repetitions of the stimulus. Although the events themselves are represented by linear transformations on the probabilities, which event takes place depends upon the response to the previous stimulus; this in turn is a function of the response probabilities. The probabilities after any event are generally quadratic functions of those after the previous event.

Although the book contains many details that will be useful only to a psychologist who wants perhaps actually to apply the model to some data, it also contains numerous unsolved problems that make it very interesting reading for the non-specialist and it should greatly stimulate future study by both mathematicians and psychologists.

G. F. NEWELL

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BOOK REVIEWS

(*Continued from p. 170*)

Tables of integral transforms. Based, in part, on notes left by Harry Bateman. Compiled by Staff of the Bateman Manuscript Project. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1954. Volume I, xx + 391 pp., $7.50. Volume II, xvi + 451 pp., $8.00.

The two volumes in question arise from the Bateman Manuscript Project at the California Institute of Technology under the support of the Office of Naval Research. Prepared in part from notes left by the late Harry Bateman by A. Erdelyi, W. Magnus, F. Oberbettinger, and F. G. Tricomi, these two volumes contain transform tables arranged as follows: *Volume I*. Fourier Sine, Cosine and Exponential Transforms 118 pp., Laplace Transforms 173 pp., Mellin Transforms 59 pp. *Volume II*. Bessel Transforms. Hankel Transforms of the form $\int_0^\infty f(x)J_\nu(xy)(xy)^{\frac{1}{2}}\,dx$, 83 pp., and the same for $1/\nu(xy)$, $K_\nu(x, y)$ and $H_\nu(x, y)$, 75 pp., Kontrovich-Lebedev Transforms, 3 pp. Miscellaneous Transforms. Fractional Integral, Stieltjes and Hilbert Transforms, 78 pp. Integrals of Higher Transcendental Functions. Orthogonal Polynomials, Gamma functions and related functions, Legendre functions, Bessel functions and Hypergeometric functions, 152 pp.

Tables are very well arranged. The notation is explained in appendix to each volume. The printing job is really excellent.

ROHN TRUELL

Proceedings of the International Conference of Theoretical Physics. Kyoto and Tokyo, September, 1953. The Organizing Committee, International Conference of Theoretical Physics, Science Council of Japan, Ueno Park, Tokyo, 1954. xxviii + 942 pp. $10.00. (postage $1.00 extra).

This book of more than nine hundred pages is a detailed account of a conference held in Japan in September 1952 which covered almost all of the currently interesting topics of theoretical physics. No attempt will be made to mention all of the authors who contributed to the conference. In fact it is unfortunately impossible even to list the titles of all of the papers. There were about 125 papers presented. Because all of the topics covered were represented by the recognized leaders in these fields the relative emphasis in various fields is indicated below by the number of papers presented and the number of pages taken up in the proceedings. The main topics were: I. *Field Theory and Elementary Particles*. Papers and discussion concerned with Field Theory, Cosmic Rays and V-particles, Pions, Coupling Theory, and Nuclear Forces occupy approximately one third of this book. Forty five papers were presented in this area of physics with the result that these proceedings contain probably the best summary available of recent work. II. *Nuclear Physics*. Nuclear Reactions. Shell Structure and Beta Decay were the subject matter of a dozen papers and eighty pages of these proceedings. III. *Statistical Mechanics*. The Theory of Polymers, Liquids, Transport Phenomena, Irreversible Processes and General Methods of Statistical Mechanics were topics covered in twenty one papers and 180 pages of the proceedings.

IV. *Molecules and Solids*. Dislocations, Molecular Theory, Metals, Electron Theory of Intrinsic Magnetization, Antiferro- and Ferrimagnetism, Magnetic Resonance, Dielectrics, and color centers were the subjects in that section of the conference dealing with solid state theory. Thirty seven papers cover 311 pages of the proceedings.

V. *Liquid Helium and Superconductivity*. Theories of Liquid Helium and Superconductivity are discussed in ten papers covering about sixty pages of the proceedings.—This conference and the proceedings appear to have made a large contribution by putting into print an outline of the present state of theoretical physics as it is seen by a large number of physicists from all countries.

ROHN TRUELL

Introduction to theoretical mechanics. By Robert A. Becker. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1954. xiii + 420 pp. $8.00.

This is a text book on a level appropriate for junior or senior year courses. It is based on a course for students in engineering physics, and the choice of topics, examples, and problems has a flavor of physics rather than of engineering. Some 80 worked and 400 unworked problems are given. Vectors are used throughout, but tensor or matrix concepts are not mentioned. The chapters on plane and general motion of a rigid body seem especially well written and complete, considering the elementary level of mathematics used. The chapter on vibrating systems, however, seems inadequate since it is confined mainly to properties of normal coordinates, and the standard techniques of combining normal modes are not introduced. The principle of virtual displacements appears not to be given the importance it deserves as an analytical tool in both statics and dynamics (in deriving Lagrange's equations, for example). Apart from these minor criticisms the reviewer was well impressed by this as a carefully written and reliable basic text.

P. S. Symonds

Geometrical mechanics and de Broglie waves. By J. L. Synge. Cambridge University Press, 1954. vi + 167 pp. $4.75.

This monograph presents a rather unique amalgamation of several disciplines from the author's unusual collection of interests. The result of this synthesis is an approach to relativistic particle dynamics based upon Hamilton's methods of geometrical optics, the Minkowski geometry of special relativity and the particle-wave concept of de Broglie.

After a short introductory chapter, the author devotes about half of the book to a generalization of the Hamilton methods to the four-dimensional Minkowski space-time. Although the methods are described as Hamilton's methods of optics, it is pointed out that these differ only formally from Hamilton's analysis of classical dynamics. The former, however, is in a form more easily adapted to the generalization in question. In this generalization, the rays of optics (trajectories of dynamics) become time-like world lines of particles. The waves of optics (2-dimensional surfaces) become 3-waves in the Minkowski space describing the history of a 2-wave.

One of the very interesting results of this procedure is that two velocities, a ray velocity and a wave velocity emerge from the theory in a natural way. These are interpreted to correspond respectively to the particle and wave velocities of de Broglie waves. These two velocities are obtained, however, solely from geometrical considerations without any discussion of wave interference or other concepts customarily involved in the association of particle velocity with a group velocity.

Not until chapter IV does the author attempt to assign a phase to these 3-waves. He does so then only for the purpose of incorporating into the theory a "primitive quantization" in much the way that one obtains physical optics from geometrical optics by the assignment of a phase to the wave front. Several special applications of this relativistic quantum mechanics are considered, including the hydrogen atom quantization, the Zeeman effect and interference of a particle from two holes.

The final chapter is concerned with the further generalization of the methods to the two body problem in an 8-dimensional product of two Minkowski spaces.

The author points out that his primary concern is to develop a "coherent mathematical theory"; that physical interpretation is considered of secondary importance. Indeed the theory is not all inclusive from a physicist's point of view, for there is no obvious way of incorporating into the theory the concepts of spin, or "second quantization" and even the "first quantization" according to this scheme has been tested on only a few problems. Where the theory is known to give correct or good approximate results, however, this approach to the physical problem furnishes a very elegant geometrical interpretation.

The purpose of the theory is well stated, the postulates are well defined and the development proceeds in a systematic fashion. Although this reviewer thinks that the book gives an excellent presentation of the ideas that it attempts to convey, he fears that it will not receive the circulation it deserves because it does not seem to belong specifically to any of the many narrow channels of current scientific interest. The physical principles underlying the theory have been known since 1928. If this book could have been published then, it would undoubtedly have been an immediate sensation.

Gordon F. Newell

Modern developments in fluid dynamics. Two Volumes. Edited by L. Howarth. Oxford University Press, London, 1953. xvi + 875 pp. $17.00 per set.

These two volumes were developed as companion volumes to "*Modern developments in fluid dynamics*" edited by S. Goldstein, which appeared in 1938. These latter books have become indispensable to all those working in fluid dynamics, since they bring together in a concise and accurate form the accumulated understanding of fluid mechanics phenomena as of the time of their publication. The present companion volumes on high speed flow are also sponsored by the fluid motion panel and attempt to similarly summarize developments in this latter field.

The publication of two new volumes instead of a revision of the older volumes is possible because much of the new work in fluid mechanics which grew out of wartime problems has been in the field of high velocities. However, any publication in this latter field must be carried out under the disadvantage of attempting to describe the important phenomena without violation of security restrictions which arose during the war and which have only been partially relaxed. As in the previous volumes, a group of distinguished authors have contributed sections to the new work.

Roughly, the first volume is devoted to discussions of theory and the second volume to questions of experimental results. The first two chapters are devoted to a general introduction to the various high speed phenomena and to the equations in both vector and tensor notation required for their description. In these chapters, as throughout the book, the editor has attempted to make clear the distinction between flows in which the entropy is constant for each particle, so that its change of state is isentropic, and flows in which the entropy is constant everywhere. These latter are described as homentropic. While this distinction is clearly a significant one, the reviewer was sometimes confused by the words used and was left with the feeling that the change was somewhat of a fetish.

Chapter 3 on the characteristics method is followed by Chapter 4 in which shock waves and their formation and various interactions are described. After thus treating fundamentals in some detail, Chapter 5 presents the few available exact solutions, vortex flow, radial flow, spiral flow and Prandtl-Meyer flow including their various applications.

Chapter 6 and 7 treat the one-dimensional approximation with applications to nozzles and jets, and the various techniques available by use of the hodograph plane. Then comes several chapters dealing with approximate methods. The various linearization procedures and numerous of their airfoil, planform, tube, and jet applications are followed by available solutions to unsteady problems, primarily unsteady airfoil characteristics.

Chapter 10, the last one in the first volume, treats the general theory of boundary layers including skin friction, aerodynamic heating, heat transfer, stability, and boundary layer-shock wave interaction. The first volume contains 475 pages.

The second volume begins with Chapter 11 devoted to experimental methods which is broken down into four sections. The first, wind tunnels and moving bodies, discusses general tunnel characteristics and supersonic nozzle design together with very brief statements about intermittent tunnels, shock tubes, and moving body tests. It appears to the reviewer that the latter two methods have been relatively neglected since they receive less than 2 pages of attention.

The second section devoted to uncertainties and corrections arising in tunnel tests is followed by a section on measurements and their reliability, and finally, a section on visualization in which each of the common optical systems is treated in some detail. In Chapter 12, the experimental results available on flow past airfoils and cylinders are reported together with some comparisons with theoretical predictions. In this chapter, the lift, drag moment and their various derivatives are discussed with respect to variation with Mach number, Reynolds number, angle of attack, angle of sweep, section parameters and controls.

Chapter 13, devoted to flow past bodies of revolution is devoted primarily to experimental and theoretical results on projectiles. The base pressure receives very considerable consideration. The range testing technique and the theory of the spinning shell are discussed. Chapters 11, 12, 13, which constitute three-quarters of the second volume are the chapters wherein restrictions on publication make a complete discussion of the problems difficult. The authors have attempted an important piece of writing using generally available information to bring out as clearly as possible the essential features of experimental results in the high-speed aerodynamic field.

The final Chapter 14 on heat transfer would better be considered a revision of what appears as the last chapter with the same title in the earlier volumes. Many sub-sections are identical to the previous volume, although many others have been rather completely rewritten, so as to bring them up to date.

In this heat transfer chapter, there is no attempt to limit the discussions to "high speed flow" although this phase of the subject has similarly been revised from the previous editions.

These two volumes are, the reviewer believes, destined to take their position beside the earlier two volumes as the major publications in the field of fluid mechanics during the middle of our century.

HOWARD W. EMMONS

Vector and tensor analysis. By G. E. Hay. Dover Publications, Inc., New York, 1953. viii + 193 pp. $2.75.

Many excellent books are available on both the vector and tensor calculus and their applications. The present book contains no preface, and accordingly its purpose must be inferred from its contents. The book seeks to develop the essential parts of the vector and tensor calculus together with some discussion of geometrical and physical applications. This, a formidable task to perform within a monograph, is made more difficult by reducing to a minimum the supposed background knowledge of the reader. The greater part (156 pages) of the book is devoted to the vector calculus, covering the usual material up to Green's and Stokes' theorems, and including applications in geometry and rigid body dynamics. This part is well written, and a clear account is given. The remainder (37 pages) of the book is devoted to the tensor calculus with very brief mention of its applications. Here the coverage (this includes weighted tensors and covariant differentiation) is achieved only through extreme condensation of analysis, and the average reader would find it difficult to work through the problems.

The book contains a few incorrect statements and misprints. A partial bibliography is given but it is surprising to find no reference to several important standard texts.

H. G. HOPKINS

Proceedings of the Eastern Joint Computer Conference, held by the Joint Computer Committee of the Association for Computing Machinery, American Institute of Electrical Engineers, and the Institute of Radio Engineers, Inc., in Philadelphia, December 8–10, 1954. American Institute of Electrical Engineers, New York, 1955. 92 pp. $3.00.

The theme of this conference was the design and application of small digital computers. The Proceedings Volume commences with an introductory talk by C. W. Adams, who defines small digital computers by means of examples. He identifies two classes: (1) Those built up from punch card equipment, which he calls "big-little computers", and (2) Those built up around a magnetic drum, which he calls "little-big computers". (The significance of these names for the two classes would perhaps be clearer if the hyphen in each case were moved forward by a word.) Computers of the second class typically have an internal storage capacity of between 10,000 and 40,000 decimal digits.

The papers presented at such a conference fall into several recognizable categories. First, there is the category of survey papers, which discuss available equipment or current trends in development. In the present case, this category is represented by a good survey of currently available small digital computers presented by A. J. Perlis. Second, there are the papers devoted to engineering design problems. For various reasons, some of which are quite obvious, these have a tendency to be somewhat vague when presented by engineers working for private industry. The volume under review contains three or four papers of this type covering the following subjects: pulse packing density, magnetic core circuits, and automation of information retrieval.

A third recognizable type of paper is the company report which describes a specific proprietary piece of equipment. As is usual at computer conferences held by the engineering societies, this category is well represented in the current volume.

Finally, there are the contributions of those interested in the applications. In the present conference the emphasis here was mainly on business problems. However, there are two papers in the Proceedings Volume with some mathematical content. The first is a paper by H. M. Gurk and Morris Rubinoff on the numerical solution of differential equations. This paper contains some interesting stability charts for the finite difference analogue of the equation $\dot{x} = \lambda\dot{x}$ where λ is complex. The second mathematical paper is a simplified discussion of the use of digital computers in optical ray tracing, by N. A. Finkelstein.

The reviewer found this volume to be a relatively interesting one of its kind and the editorial work was excellent. But this fact did not deflect the reviewer from the pursuit of a project which he has had in mind for some time. The project is to found a Society for the Prevention of the Publication of Proceedings of Symposia. The holding of symposia is a fine thing indeed, but the morning after (which sometimes takes place two years later) is a different matter altogether. The papers with substantial content presented at a symposium should be, and almost always are, published elsewhere than in the Proceedings, with fuller detail. The papers which do not have substantial content should not be published at all.

Membership in the new Society is free. Any member who can prove that he has been instrumental in the suppression of a symposium proceedings volume will be made a Fellow.

J. H. Curtiss

Table of Everett's interpolation coefficients. By E. W. Dijkstra and A. Van Wijngaarden. Excelsior's Photo-Offset, The Hague, 1955. $1.58.

This table was constructed on the electronic computer ARRA of the Mathematisch Centrum in Amsterdam on request of Empresa Nacional Bazan de Construcciones Navales Militares in Madrid. It gives to seven decimal places the values of the coefficients of the second, fourth, and sixth central differences in Everett's interpolation formula.

This table is especially convenient for three reasons; a) Entries for the argument p are given at intervals of 0.0001 so that interpolation is rarely needed in the table itself. b) All the coefficients needed for any given interpolation are printed in the single line of the table, i.e., the coefficients for p and for $1 - p$ are both given on the same line. c) The range extends from $p = 0$ to $p = 1$, so that it is not necessary to change the procedure for $p > 1/2$. As a result of b) and c) all entries appear twice in the table, once at p and once at $1 - p$.

The use of sixth order differences in Everett's formula is equivalent to using eight equally spaced values of the function for the interpolation, or in other words, using a polynomial approximation of seventh degree.

W. E. Milne

Tables of the cumulative binomial probability distribution. By The Staff of the Computation Laboratory. Harvard University Press, Cambridge, 1955. lxi + 503 pp. $8.00.

This, the first set of tables to be computed on the new Harvard Mark IV Calculator, gives values of the cumulative binomial probability distribution

$$E(n, r, p) = \sum_{x=r}^{n} p^x(1 - p)^{n-x}\, n!/x!(n - x)!$$

to five decimal places for integer values of r and n. The unique feature of the tables is that n ranges in varying steps from 1 to 1000 as compared with a previous high of 150. Since only integer r and n are included, the tables are not quite as complete for small n as the old tables by Pearson for the incomplete beta function which included half integer values as well. The incomplete beta function differs from $E(n, r, p)$ only in a trivial way.

The introduction includes seventeen illustrative applications in addition to a discussion of the analytic properties of the function.

G. F. Newell