

## BOOK REVIEWS

*Randwertprobleme der Mikrowellenphysik.* By Fritz E. Borgnis and Charles H. Papas. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1955. xv + 266 pp. \$11.50.

This book is an excellent short treatise on the modern applications of electromagnetic theory to antennas, wave guides, and scattering. The well-organized collection of problems appears for the first time in book form, although much of it could be found by an exhaustive search through the journal literature.

The connecting link between these various problems seems to be the nature of the mathematical techniques needed in their solutions. There is extensive use of Green's functions and their various representations, integral equations, and stationary principles of the "Levine-Schwinger" type. Computational details are often omitted but numerical results are given in graphical form and there is usually a careful discussion of the physical significance of these results.

The very recent and important results on high frequency scattering (work of Keller, Rubinow and Wu, Kodis) apparently could not be included in time for publication. These papers must have reached the authors in proof since there is a footnote throwing doubt on the formula (8.7 on page 51) for the scattering cross section of a circular cylinder. Indeed this result must be discarded and replaced by the one found by Rubinow and Wu. It would also seem that the integral equation (5.11) might be discussed somewhat more carefully since the kernel, which is a second derivative of the Green's function at the singularity, does not exist. The proper limiting procedure in this case has been explained by Bouwkamp in his review article in the "Proceedings of the Royal Society."

Mention should be made of the authors' lucid style; their German is surprisingly easy to follow. The printing is of the highest quality and altogether this volume represents a successful contribution to the literature in the field.

I. STAKGOLD

*Numerical analysis.* Edited by John H. Curtiss. Proceedings of Symposia in Applied Mathematics, Vol. 6. McGraw-Hill Book Company, Inc., New York, 1956. vi + 303 pp. \$9.75.

The volume contains all but two of the papers presented at the Sixth Symposium in Applied Mathematics of the American Mathematical Society (Santa Monica City College, August 1953). As the space available for this review rules out a discussion of the individual contributions, the diversified contents of the volume is indicated by the following list of papers: Computational Problems in the Theory of Dynamic Programming (R. Bellman), Some Methods for Solutions of Boundary-value Problems of Linear Partial Differential Equations (S. Bergman), Computational Aspects of Certain Combinatorial Problems (R. H. Bruck), Numerical Results on the Shock Configuration in Mach Reflection (D. R. Clutterham and A. H. Taub), Some Applications of Gradient Methods (J. W. Fischbach), Some Qualitative Comments on Stability Considerations in Partial Difference Equations (S. P. Frankel), Approximations in Numerical Analysis—A Report on a Study (C. Hastings, Jr., J. Hayward, and J. P. Wong, Jr.), The Conjugate-gradient Method for Solving Linear Systems (M. R. Hestenes), Number Theory on the SWAC (E. Lehmer), The Assignment Problem (T. S. Motzkin), The Method of Steepest Descent (P. C. Rosenbloom), Function Spaces and Approximation (A. Sard), Some Computational Problems in Algebraic Number Theory (O. Taussky), Machine Attacks on Problems Whose Variables Are Permutations (C. B. Tompkins), Best-approximation Polynomials of Given Degree (J. L. Walsh), Recent Results in Numerical Methods of Conformal Mapping (S. E. Warschawski), On the Asymptotic Transformation of Certain Distributions into the Normal Distribution (W. Wasow), Error Bounds for Eigenvalues of Symmetric Integral Equations (H. Wielandt), On the Solution of Linear Systems by Iteration (D. Young).

W. PRAGER

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*Creep and fracture of metals at high temperature.* Proceedings of a Symposium held at the National Physical Laboratory, Teddington, England. Her Majesty's Stationery Office, London, and British Information Services, New York, 1956. iv + 420 pp. \$5.60.

This book is a summary of the proceedings of an international meeting concerned with the creep and fracture of metals at high temperatures. The meeting was held at the National Physical Laboratory, Teddington, England in May and June 1954. There are twenty-two papers falling into four categories:

Section I, Deformation Processes In Simple Materials; Section II, Creep Resistance of Complex Materials; Section III, Theory of Fracture; Section IV, Tertiary Creep and Fracture.

These proceedings provide an excellent summary of a cross section of the present ideas (on an international scale) about creep and fracture at elevated temperatures. In the two years since this conference there have been some significant advances in the understanding of grain boundary motion.

ROHN TRUPELL

*Introduction to plasticity.* By Aris Phillips. The Ronald Press Company, New York, 1956. ix + 230 pp. \$7.00.

The first six chapters, which form one-half of the book, deal with the one-dimensional problem of the plastic design of beams and frames, the plastic bending of beams with and without axial forces, the calculation of deflections in beams and frames from the moment-curvature relation, and an elementary theory of curved bars. In the preface the author states that the book is written from the point of view of the stress analyst. Accordingly, approximately twenty pages of Chapters 2-6 are devoted to the listing of results for various cases which may arise in particular problems.

The Prandtl-Reuss stress-strain relations are stated in Chapter 7 and are applied in the remaining two chapters to problems involving combined stresses. Except for a brief discussion of plane strain, the problems treated have the simplifying feature of axial or radial symmetry. The treatment throughout is at a low mathematical level and is at times somewhat inelegant.

R. T. SHIELD

*Mathematische Methoden* (Volumes I and II of Handbuch der Physik). Edited by S. Flügge. Springer-Verlag, Berlin-Göttingen-Heidelberg. Vol. I: 1956, vii + 364 pp. \$17.25; Vol. II: 1955, vii + 520 pp. \$20.95.

The volumes contain the following articles. Volume I: Fundamental Concepts of Classical Analysis, Ordinary Differential Equations, Theory of Functions (J. Lense, Technische Hochschule München, 89 pp.)—Partial Differential Equations (J. Lense, 30 pp.)—Elliptic Functions and Integrals (J. Lense, 27 pp.)—Special Functions of Mathematical Physics (J. Meixner, Technische Hochschule Aachen, 71 pp.)—Boundary Value Problems (F. Schlögl, Universität Köln, 135 pp.); Volume II: Algebra (G. Falk, Technische Hochschule Aachen, 116 pp.)—Geometry (H. Tietz, Technische Hochschule Braunschweig, 81 pp.)—Functional Analysis (I. N. Sneddon, University College of North Staffordshire, 151 pp.)—Numerical and Graphical Methods (L. Collatz, Universität Hamburg, 122 pp.)—Modern Computers (H. Bückner, General Electric Company, Schenectady, N.Y., 28 pp.). A critical discussion of these articles, or even a mere listing of section headings, would require more space than is available for this review. Since, however, Sneddon's article, which is in English, may interest a greater number of readers of this Quarterly than the other articles, which are in German, its scope is indicated by the following headings and subheadings: Integration and Abstract Spaces (Introduction—Banach Space)—Integral Transforms (The Laplace Transforms—The Fourier Transforms—The Mellin Transform—The Hankel Transform—Finite Transforms—Approximate Methods of Evaluating Transforms)—Hilbert Space—Schwartz's Theory of Distributions—Variational Methods in Functional Analysis.

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## BOOK REVIEWS

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*The theory of functions of real variables.* By Lawrence M. Graves. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1956. xii + 375 pp. \$7.50.

Published ten years ago, the first edition of this book is now widely known and has been reviewed in detail: see, e.g., *Math. Rev.*, 8 (1947), 319-20, or *Bull. Amer. Math. Soc.*, 54 (1948), 487-9. Except for a few rewritten passages, the second edition differs from the first only in that two new chapters have been appended, and we shall confine comment to these.

Chapter XIII, on sets and transfinite numbers, begins with an amplified account of sets and functions (= relations) from the non-formal "common-sense" point of view adopted by the author in Chapter I. In the same spirit there follows a discussion of order leading up to definitions of ordinal and cardinal number which are essentially von Neumann's: an ordinal is a set which (i) is well-ordered by the class-membership relation and (ii) has each of its members as a subset; and a cardinal is an initial ordinal in a cardinal-similarity class of ordinals. Next, the maximality principle (Zorn's Lemma) is stated and studied in a variety of equivalent forms. The chapter closes with an introduction to ordinal and cardinal algebra. Chapter XIV discusses, for metric spaces, basic facts concerning convergence, completeness, connectedness, separability, compactness, and category. Here the references for further study might well have included such recent items as J. L. Kelley's *General Topology*, 1955, as well as the older works listed. Compared with the rest of the book both of the added chapters are notable for their profusion of well selected exercises.

TRUMAN BOTTS

*Automata studies.* Edited by C. E. Shannon and J. McCarthy. Princeton University Press, New Jersey, 1956. viii + 285 pp. \$4.00.

The recent development of high-speed digital computers and of electronic control equipment has revived interest in questions on the ultimate limits of a "mechanical brain" and on how much of animal or human mental activity can be described in physical terms. The present collection of studies represents some of the contributions of mathematics to these perennially interesting fields of speculation.

The articles on the properties and limitations of computers are based on the work of Turing, in which he showed that the action of most digital computers is equivalent to an appropriate sequence of actions of a machine with simply describable properties, the limitations of which could be studied by known mathematical methods. The articles in this collection presuppose a knowledge of Turing machines and carry further the analysis of their behavior.

The papers more specifically related to the analysis of mental action are concerned with the properties of "automata," machines built up of "neurons" having a simple, all-or-none behavior analogous to living nerve cells. The first paper of the collection, by S. C. Keene, gives a readable introduction to the subject. Some of the later papers discuss limitations and extensions of the basic ideas.

Of course an automaton, built up of infallible "neurons" could perhaps duplicate some of the human brain's abilities in the development of formal logic. To duplicate actual learning and memory we have to introduce the possibility of failure of an element from time to time. The study of automata with unreliable elements is of considerable interest, for it may point the way to methods of improving the reliability of the machine as a whole, whether it be one of our all-too-fallible present day computers or our own brains. The basic article on this subject, in the collection, is by von Neumann. One can hope that others will explore further the ideas he outlines here.

The articles are organized into three sections: the first on Finite Automata, the next on Turing Machines and the last on Synthesis of Automata. The papers are recommended to anyone interested in this general field, though some of them make pretty hard reading. If the interesting Introduction, by John McCarthy and Claude Shannon, had been expanded by a dozen pages or so to provide a review of of previous work and a bibliography of important papers, this volume would have constituted a much-needed review of the field. As it is, the collection is useful and thought-provoking.

PHILIP M. MORSE

*Elementary differential equations.* By William Ted Martin and Eric Reissner. Addison-Wesley Publishing Co., Inc., Cambridge, Mass., 1956. xi + 260 pp. \$5.50.

This book is intended primarily as a text book for advanced undergraduate science and engineering students. The book contains six chapters relating to ordinary differential equations, one chapter on finite difference equations and one chapter on partial differential equations. Emphasis is on methods of solution for which the authors attempt to give some motivation although this is clearly rather difficult at times. There are numerous examples and problems including several examples to illustrate that the method being discussed does not always succeed. These latter examples show to the student better than words could describe that the subject goes much deeper.

GORDON F. NEWELL

*Advances in mechanics: Volume IV.* Editors: H. L. Dryden, Th. von Kármán. Managing Editor: G. Kuerti. Associated Editors: F. H. Van den Dungen, L. Howarth, J. Pérès. Academic Press, Inc., New York, 1956. x + 413 pp. \$10.00.

The present volume is the first to appear since the death of Richard von Mises, the initiator of this series; it contains the following articles: The Turbulent Boundary Layer, by Francis H. Clauser (51 pp.)—Nonlinear Elasticity, by T. C. Doyle and J. L. Ericksen (63 pp.)—Physical and Statistical Aspects of Fatigue, by A. M. Freudenthal and E. J. Gumbel (42 pp.)—Three-Dimensional Boundary Layer Theory, by Franklin K. Moore (70 pp.)—Dislocation Theory of Plasticity of Metals, by G. Schoeck (51 pp.)—The Poincaré-Lighthill-Kuo Method, by H. S. Tsien (69 pp.)—On the Concept of Elastic Stability, by Hans Ziegler (53 pp.).

W. PRAGER

*Les Épreuves sur Échantillon.* By Maurice Dumas. Centre d'Études Mathématiques en vue des applications, of the Centre National de la Recherche Scientifique, Paris, 1955. 170 pp. \$2.90.

This is a short monograph on sampling inspection intended for scientists and engineers who are supposed to have the mathematical background, but not the available time, to read up on the relevant theory. The presentation is thought-provoking, and perhaps a little complicated. Certainly it is far more sophisticated than that of the numerous American books and manuals on practical statistical quality control which have been appearing in profusion lately. Incidentally, the preface is by no less a celebrity than Professor Maurice Fréchet.

The author arranges his book in this way: First there is a summary in a telegraphic style of the main formulas of the book and procedures for applying them. Then there is a careful chapter on the terminology, techniques and rationale of sampling inspection. This chapter includes a brief but clear non-mathematical discussion of the relations between statistics, practical probabilities, and mathematical theory of probability. After this comes a general description, with numerical examples, of the four categories of problems which the author intends to discuss: Estimation, acceptance inspection, comparison of lots with each other and with fixed standards, and statistical quality control in the Shewhart sense.

In the next chapter (we have now arrived at Chapter III), detailed procedures for attacking these four types of problems are given for the case of sampling by attributes. In the last chapter, Chapter IV, methods for the case of sampling by variables are given. The idea of the arrangement is that a reader who wants only to get a general idea of what sampling inspection is all about, can do it by reading only the first part of the book, omitting the last two chapters.

Mathematical notations and formulas are used liberally, but there are no proofs. For these, the reader is referred chiefly to two large works which stand behind this one: a six-hundred and thirty page work on industrial statistical methods written by the author in collaboration with P. Maheu, and a work called *Technique modernes de contrôle des fabrications*, by J. Mothes. There is a paucity of acknowledgments to those whom the author calls "the Anglo-Saxons", although much of the terminology is drawn directly from Fisher, Dodge and Romig, Neyman and Pearson, and Shewhart. (By the way,

Dr. Shewhart's name is apparently never mentioned.) This is certainly excusable in view of the nature of the book, but Dumas indicates in his brief bibliography that he has another reason for not sensing a debt to the Anglo-Saxons too keenly. He says that anyhow, in a 1925 *Mémorial de l'Artillerie Française* entitled *Sur une interprétation des conditions de recette*, he anticipated the relevant portions of the Dodge-Romig and Neyman-Pearson theories.

The author generally limits himself to strictly classical procedures and formulas. There are short sections on sequential analysis, and some of the tests of Barnard are given, but otherwise this book could have been written at least twenty-five years ago. There is, for example, nothing about non-parametric analysis and nothing about sampling by variables when lot quality is measured by a percentage of non-conforming items. Perhaps the author should read more Anglo-Saxon articles on mathematical statistics.

However there is an interesting emphasis on a posteriori probability, which this reviewer considers to be of importance in sampling inspection from a pragmatic rather than from a philosophical point of view. Almost all sampling inspection is performed on lots which themselves can be considered as probability samples from a larger population, and one way to take this into account is to use Bayes' Theorem. The essentially stochastic nature of inspection lots does not seem to be sufficiently emphasized in the current Anglo-Saxon literature of the subject.

In the case of sampling by variables, the author deals at some length with the technique of plotting observed frequency distributions on normal probability paper. He calls such a plot a "graphique de Henry". In this connection, readers of the monograph under review might do well to look at an article entitled "Use of Normal Probability Paper", by Chernoff and Lieberman, in the *Journal of the American Statistical Association*, vol. 49, 1954, pp. 778-785, which discusses the question of optimal choices of the ordinates for various problems.

All in all, this book should be quite successful in meeting its aims, especially as regards French readers. In fact, some Anglo-Saxons who meet the specifications of the intended audience, and who can read the rather turgid prose which seems to be in favor nowadays in French scientific circles (many semi-colons and colons) might find the book useful.

J. H. CURTISS

*Zur Ermittlung von Unterschallströmungen mit der Transformationsmethode bei quadratischer Approximation der Adiabate.* By Hans Schubert and Erich Schincke. Akademie-Verlag, Berlin, 1955. 32 pp. \$.55.

This monograph contains a treatment of the problem of finding the compressible inviscid flow past a symmetrical two-dimensional object when the pressure-density relation is approximated by such a function that the equation for the dimensionless stream function  $\psi$  becomes  $\Delta\psi + k^2\psi = 0$ . Here  $k$  is a constant and the independent variables are distorted hodograph variables. It is shown that this  $p$ - $v$  law agrees with the usual isentropic relation to quadratic terms in their series in powers of the velocity. Solutions are then expected as the sum of a singular term closely related to the singular part of the hodograph solution of the corresponding incompressible problem and a series of non-singular terms whose coefficients are determined by the boundary conditions in the physical plane and which are therefore to be extracted as the solution of a non-linear algebraic problem. The authors note that the last step is very laborious.

G. F. CARRIER

*Asymptotic expansions.* By A. Erdélyi. Dover Publications, New York, 1956. vi + 108 pp. \$1.35.

This is an unaltered republication of Technical Report 3, prepared under Contract Nonr-220(11) for the Office of Naval Research.

*An Essay on the foundations of geometry.* B. A. W. Russell. Dover Publications, New York, 1956. xviii + 201 pp. \$3.25.

Except for an added foreword by M. Kline (7 pp.), this is an unaltered edition of the 1897 edition.

*Mechanik der Kontinua.* By G. Hamel. Edited by I. Szabó. B. G. Teubner, Stuttgart, 1956. 210 pp. \$7.10.

In spite of its sweeping title, the book is primarily concerned with mechanics of fluids. The first part (114 pp.) is devoted to perfect fluids and contains chapters on foundations, one-dimensional motions of air (sound, detonation), potential flows without free surfaces, vortex motion, steady two-dimensional flows of compressible fluids, and potential flows of incompressible fluids. The second part (66 pp.) deals with viscous fluids and has chapters on the Navier-Stokes equations, slow flow, exact solutions, boundary layer theory and turbulence. The third part (19 pp.) has the heading "On more general deformable systems"; its only chapter has the title "Elastic vibrations" but is mostly devoted to the plane flow of a perfectly plastic solid. Though statements in the editor's preface would seem to contradict this impression, the last part strongly suggests that the author's death interrupted work on a more ambitious treatise, leaving us with two completed parts and miscellaneous notes.

W. PRAGER

*Rayleigh's principle and its applications to engineering.* By G. Temple and W. G. Bickley. Dover Publications, Inc., New York, 1956. 152 pp. \$1.50 (paper bound).

This is an unabridged and unaltered republication of the book originally published by the Oxford University Press in 1913.

*Proceedings of the conference on high-speed aeronautics.* Edited by Antonio Ferri, Nicholas J. Hoff, and Paul A. Libby. Polytechnic Institute of Brooklyn, Brooklyn, New York, 1955. vi + 392 pp. \$5.25.

This volume is a collection of papers by distinguished aeronautical scientists given as part of the centennial celebrations of the Polytechnic Institute of Brooklyn. Most contributions are on aerodynamic topics but important structural fields are also covered. About one half of the papers are in the nature of reviews while the remainder describe new developments.

On the aerodynamical side von Kármán gives a very broad but connected criticism of High Speed Flow Theory in its latest stage of development. Of particular interest are his presentation of linearized theory in a logical order and his lucid explanation of transonic and hypersonic flow phenomena. A second review paper by Dryden covers the complete development of the theory of transition; this emphasizes the role of spots of turbulence in causing the breakdown of laminar flow. In the group of research papers Crocco gives a theory of shock boundary layer interaction applicable to the base pressure problem, Oswatitsch and Keune describe a new method of treating transonic flow past bodies and Ferri discusses conical fields with unusual mixed flow patterns in transverse planes. There are three papers on Hypersonic Flow; two, by Libby and Bloom and by Smelt respectively, describing experimental facilities and a third, by Kantrowitz, giving an all too brief survey of physical phenomena. The remaining papers on fluid mechanics are by Busemann on the relation between noise and wave drag, by Jones on the efficient operation of high speed aircraft, by Schlichting on Cascade Theory, and by Michel and Sirieix on Transonic Lift.

On the structural side Broglio gives a complete account of the balance method in thermal stress analysis while Hoff develops a long and clear discussion of creep phenomena. Two further papers, by Duberg and Horton respectively, deal with heating effects.

M. HOLT

*Numerical integration of differential equations.* By Albert A. Bennett, William E. Milne, and Harry Bateman. Dover Publications, Inc., New York, 1956. 108 pp. \$1.35 (paper bound).

This is an unabridged and unaltered republication of a report of the Division of Physical Sciences of the National Research Council originally published in 1931.

*Integral transforms in mathematical physics.* By C. J. Tranter. John Wiley & Sons, Inc., New York, and Methuen & Co., Ltd., London, 1956. ix + 133 pp. \$2.00.

This is a new edition of a member of the Methuen Monograph Series, first published in 1951 and reviewed in Vol. X, No. 2 of this Quarterly.

The present edition differs from the first in the addition of a chapter on dual integral equations, the correction of misprints and an enlargement of the bibliography.

The work given in the new chapter is largely of recent origin and contains many of the author's own contributions. The equations discussed arise in the solution of boundary-value problems in which the condition on one boundary is a "mixed" one. It is welcome that an account of transform methods for such problems is thus made more widely available.

WALTER F. FREIBERGER

*The theory of ordinary differential equations.* By J. C. Burkill. Oliver and Boyd, Ltd., Edinburgh and London, and Interscience Publishers, Inc., New York, 1956. ix + 101 pp. \$1.55.

This little text-book is a useful addition to the University Mathematical Texts series. The book outlines the theory of ordinary differential equations in a concise but lucid manner, with the intention of providing the basic results in the most useful rather than the most general form.

The basic existence theory for first order equations is proved in Chapter I, and Chapter II treats the general linear equation of order  $n$ . Chapter III deals with the oscillation theorems on the zeros of solutions of the homogeneous second order equation and briefly introduces eigenfunctions and expansions. The remainder of the book considers ordinary differential equations, almost exclusively linear and of the second order, in complex variables. Chapters IV, V and VI are concerned with solutions in series, singularities of equations, and solutions expressed in integrals along a contour. The main results and formulae for Legendre and Bessel functions are developed in Chapters VII and VIII. The last chapter gives a brief introduction to asymptotic series and asymptotic solutions of differential equations. A number of illustrative examples and problems with answers are given throughout the text.

Although by no means intended to replace more comprehensive books on the title subject, the text should provide a good basis for a second course on differential equations.

R. T. SHIELD

*Random processes in automatic control.* By J. Halcombe Laning, Jr., and Richard H. Battin. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1956. ix + 434 pp. \$10.00.

That this book is an outgrowth of a course given to aeronautical engineers is rather surprising. The objective is to present a background for study of automatic control of aircraft, of fire control, etc. and many of the examples lean in this direction. The main part of the text, however, describes the subject of random processes and controls in quite general terms. Approximately half of the book is devoted to probability theory and the description of random processes. Although this material is quite concentrated and would require careful reading by someone lacking preliminary training in probability or statistics, the presentation is clear, concise and well organized. It is indeed remarkable how the authors manage to give such an extensive survey of probability theory in such a short space. The mathematical rigor does not approach that of advanced mathematical texts, but the exposition certainly could not be classed as a popularized version.

The last half of the book is more applied. There is a chapter on analysis of effects of time-invariant linear systems on stationary processes (filtering and prediction with noise) and another chapter on optimum smoothing and prediction (Wiener theory) both of which are well developed subjects. Two other chapters deal with generalizations to non-stationary systems and optimum operation with finite data. Much of the material in these latter two chapters is original, but the algebra is rather involved in places. The last half of the book does not, unfortunately, measure up to the standards of the first part either as to subject matter or presentation. As a whole, however, the book is well written and a very worthwhile contribution to the literature.

GORDON F. NEWELL

*Applied analysis.* By Cornelius Lanczos, Prentice Hall, Inc., Englewood Cliffs, N. J., 1956. xx + 539 pp. \$9.00.

Some ten years ago, satisfactory textbooks on numerical analysis in English could have been counted on the fingers of one hand. Since then, the spectacular development of high-speed computing equipment has revived the interest of mathematicians in this field. The reappearance of numerical analysis in the curricula of many colleges and universities was accompanied by the publication of several outstanding texts. These range in treatment from the strictly practical to the highly sophisticated, and in scope from collections of the authors' favorite methods to comparative surveys of methods suitable for various types of computing devices. In these terms of reference, the present book would have to be described as striking an excellent balance between the practical and the sophisticated points of view, but restricting the discussion to a rather small number of methods. This statement, however, would fail to convey an idea of the stimulating character of the book. As is indicated by their hyphenated names, many methods of numerical analysis have a long history of development by small improvements. If such a method were merely described and proved to furnish the desired result, the reader might be left wondering how the complex procedure could ever have been invented. By concentrating on principal points and letting the reader fill in the details, the present author not only gains the space needed for a more adequate coverage of the development of the various methods, but actually makes the reader feel that he has participated in this development.

Chapter I deals with algebraic equations, a "movable strip" version of Horner's scheme and Bernoulli's method of moments being at the center of the discussion. Chapter II treats matrices and eigenvalue problems with welcome emphasis on geometrical aspects. Chapter III is devoted to the large-scale linear systems that result from the algebraisation of boundary value problems. In particular, the author's "spectroscopic method" of determining the real eigenvalues of large matrices and the corresponding method of solving large systems of linear equations are developed. Chapter IV is concerned with harmonic analysis, the emphasis being on the interpolation aspects of the Fourier series. Among the special features of this chapter are discussions of the use of " $\sigma$  factors" for smoothing out the Gibbs oscillations, numerical aspects of the Fourier and Laplace transforms with applications to circuit design, and an improved method of searching for hidden periodicities. Under the title "Data Analysis", Chapter V deals with interpolation and smoothing. Chapter VI treats quadrature methods, more than half of the chapter being devoted to Gaussian quadrature and several useful modifications of this. In particular, a quadrature formula is developed that employs only the values of the function and its derivatives up to a certain order at the two endpoints of the interval; the use of this formula in boundary value problems is illustrated. The final chapter "Power Expansions" emphasizes the use of Chebyshev polynomials, discussing the author's " $\tau$ -method" among other applications.

W. PRAGER

*Momentum transfer in fluids.* By William H. Corcoran, J. B. Opfell and B. H. Sage. Academic Press, New York, 1956. xi + 394 pp. \$9.00.

This is a book mainly written for the Chemical Engineer who does not have or need much previous knowledge in the other branches of Fluid Mechanics. It consequently stresses the theoretical treatment of the phenomena encountered in internal shear flows. In Chapter I the concept of momentum transfer is introduced and then applied to a number of cases involving laminar flow. It is concluded with discussion of transition to turbulent flow and friction coefficients. Chapters II to IV deal with the properties, macroscopic characteristics and some analysis of turbulent flows, treating such topics as the similarity hypothesis, the momentum transfer hypothesis, the vorticity transfer hypothesis and their applications to flows in channels. In Chapter V the basic equations of general Fluid Mechanics are developed. Chapter VI defines and discusses such terms as correlations, spectrum, etc. in the treatment of turbulence and the final chapter introduces the reader to the concept of the boundary layer and its various applications. While the main body of the book does not make use of either vector or tensor analysis, an exhaustive and rigorous introduction to tensor analysis and the statistical theory of turbulence is added as an Appendix. It is the reviewers opinion that, while the book seems to be designed mainly for chemical engineers, some of its chapters strike such a happy compromise between mathematical treatment and physical discussion, as to warrant its use by other branches of engineering dealing with fluid mechanics.

P. MAEDER



*Elements of pure and applied mathematics.* By Harry Lass. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1957. xi + 491 pp. \$7.50.

This is another addition to the growing collection of text books that give a very brief introduction to numerous topics. The chapter headings are: 1. linear equations, determinants, and matrices, 2. vector analysis, 3. tensor analysis, 4. complex-variable theory, 5. differential equations, 6. orthogonal polynomials, Fourier series and Fourier integrals, 7. the Stieltjes integral, Laplace transform, and calculus of variations, 8. group theory and algebraic equations, 9. probability theory and statistics, 10. real-variable theory. Each of these is about 40 pages except for the last chapter that is about 100 pages.

As compared with most other books intended as text books for an advanced undergraduate or graduate level, this book covers more subjects in more condensed form and in a more abstract way although not in an extreme way. The book is partly "pure" mostly by virtue of the choice of topics; the presentation is not consistently more rigorous than other books. One annoying feature is the frequent use throughout the first nine chapters of mathematical concepts discussed in the tenth chapter. Aside from this, the book is fairly well organized and well written. It is perhaps just the type of book that some instructors would like to use, this being largely a matter of personal preference.

G. F. NEWELL

*The theory of hydrodynamic stability.* By C. C. Lin. Cambridge University Press, New York, 1955. xi + 155 pp. \$4.25.

This monograph presents a unified survey of the theory of stability with respect to infinitesimal disturbances of laminar flows of a homogenous viscous fluid. This is a subject to which the author has made many fundamental contributions and to which he has been a continual source of stimulation for further work by both his students and colleagues. In addition to the presentation of the general theory and a careful exposition of the mathematical questions raised by this theory there are separate chapters on Couette, Plane Poiseuille, and Boundary Layer flows, and on the use of stability theory to problems in Astro- and Geophysics. The various contributions of the many workers in the field are specifically delineated and the book ends with an exhaustive bibliography, complete through 1955.

An important service has been performed by the careful indication throughout the book of what parts of the theory can be considered as known and what parts are controversial. This is particularly useful in view of the many physical and delicate mathematical questions that have been raised over the various simplifying assumptions and mathematical procedures that have been used in the development of the subject. The monograph should serve as an excellent source book and stimulus to anyone wishing to enter an active and interesting field. There are many sections which end with a remark such as "adequate methods.....are still lacking" or "a complete theory.....is as yet far from being completely developed."

PETER CHIARULLI

*General relativity and cosmology.* By G. C. McVittie. John Wiley & Sons, Inc., New York, 1956. x + 198 pp. \$9.00.

This is the fourth volume of the International Astrophysics Series, which aims to provide authoritative volumes, suitable for both specialists and students, dealing with the main branches of astrophysics and radio astronomy. On leafing through the book, two things catch the eye. First, a welcome rarity in technical literature, the author explains at some length his philosophical views about science. Secondly, he presents details which one is glad to have readily available. Thus, we find Dingle's formulae for the Einstein tensor calculated for any orthogonal line element, and tables of observed results for the bending of a light ray grazing the sun and for red-shifts, solar and cosmological. The introductory paragraphs to the various chapters are excellent, and it was only when I started to study the mathematics of the book that I found myself out of tune with the author.

Perhaps this is due to two related views I hold about the general theory of relativity: (1) it is a

thing of beauty, the essential simplicity of which must at all costs be made to shine through those formal manipulations which cannot be avoided in applying it, and (2) we must develop and use to the full our geometrical intuitions about space-time in the same way as mathematical physicists, through the ages, developed and used their kinematical intuitions about space and time in the Newtonian scheme. I can best describe the book in a negative sense by saying that those views are not the author's. He has no use for geometrical intuition in space-time, but, in supporting his own scepticism, he should not mislead the reader by telling him (p. 31) that in a Riemannian 2-space it is impossible to write down the curvature tensor, the Ricci tensor and the curvature invariant in terms of the Gaussian curvature; the very simple formulae which do this will be found on pp. 89, 96 of Synge and Schild, *Tensor Calculus*.

The following list of chapter headings will show the scope of the book: I. Introduction. II. The tensor calculus and Riemannian geometry. III. Newtonian mechanics and special relativity. IV. The principles of general relativity. V. The Schwarzschild space-time. VI. Approximations to Einstein's equation and Newtonian gas-dynamics. VII. Special cases in Newtonian gas-dynamics. VIII. Theory of uniform model universes. IX. Model universes and the system of galaxies. There are references and notes at the end, and a good index.

This is not, in my opinion, a book for a beginner in relativity. But for one who understands the theory already in its general outlines, and seeks additional special information, particularly in relation to cosmological problems, the last two chapters will provide much food for thought.

Here are some minor criticisms. The derivation of the equations of a null geodesic (p. 22) seems unsound, because it supposes falsely that all curves adjacent to a null geodesic are null to the first order. The view (p. 35) that the cosmological constant is essentially a constant of integration stems from the author's curious way of looking at the matter. In eq. (3.204) read  $mp$  for  $p$ , where  $m$  is molecular weight. It is a pity (p. 51) to cry down Minkowski's great achievement by referring to him as "one of the mathematicians who first employed" the idea of flat space-time; or is there another claimant to priority?

J. L. SYNGE

*Introduction to statistical analysis.* By Wilfrid J. Dixon and Frank J. Massey, Jr. McGraw-Hill Book Co., Inc., New York, London, Toronto, 1957. xiii + 488 pp. \$6.00.

As the title might suggest, this is an undergraduate text for the non-specialist. The authors (who work in biostatistics) have been careful to select varied material from industry, educational measurements, and commercial fields, as well as from their specialty; and every problem and exercise is in language comprehensible to the average undergraduate. So many topics are touched upon, and often dismissed with so few sentences, that the "introduction" is sometimes little more than a distant glimpse. There are few demands upon mathematical proficiency. In other respects the section headings pretty well cover the range of current statistical interest, save for the notable omission of the specialized topics relating to business forecasting and index numbers. Almost a quarter of the book consists of well-chosen tables. This second edition includes a brief terminal chapter on a posteriori probability. A useful feature of the form of presentation is the separate listing of "Discussion Questions," "Class Exercises," and "Problems." A chapter on "Microstatics" deals with the theory of small samples. Many exercises carry on in succession with data found by the individual student in sampling either by use of random-number tables or from populations of tags, disks, or beads. No new principles are developed in this text.

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