A NOTE ON THE PAPER OF MILLER, BERNSTEIN AND BLUMENSON*

By F. A. J. FORD (*Admiralty Research Laboratory, Teddington, England*)

In connection with the above authors' recent paper [1] on generalized Rayleigh distributions, it may be of interest to show how the cumulative distribution function of the first order distribution of such a process can be expressed in terms of tabulated functions. The cumulative distribution function in question is

$$\int_0^R \frac{A}{\psi_0}\left(\frac{R}{A}\right)^{N/2} \exp\left[-(R^2 + A^2)/2\psi_0\right] I_{(N-2)/2}\left(\frac{RA}{\psi_0}\right) dR,$$

which may be written as

$$\frac{(2\psi_0)^{n/2}}{A^n}\, p_n\left\{\frac{R}{(2\psi_0)^{1/2}},\ \frac{A}{(2\psi_0)^{1/2}}\right\},$$

where

$$p_n(x, y) = \int_0^x 2u^{n+1} \exp\left[-(u^2 + y^2)\right] I_n(2uy)\, du \qquad n = (N-2)/2.$$

When N is odd the modified Bessel function I_n can, as is pointed out in [1], be expressed in terms of Hyperbolic functions, so that $p_n(x, y)$ can be expressed in terms of Error functions. On the other hand, if N is even (so that n is integral), $p_n(x, y)$ can still be expressed in terms of tabulated functions.
For, on integration by parts, it follows that

$$p_n(x, y) = y p_{n-1}(x, y) - x^n \exp\left[-(x^2 + y^2)\right] I_n(2xy) \qquad (n \geq 1).$$

Thus

$$\sum_{r=0}^{n-1} \left[y^r p_{n-r}(x, y) - y^{r+1} p_{n-1-r}(x, y)\right] = p_n(x, y) - y^n p_0(x, y)$$
$$= -\exp\left[-(x^2 + y^2)\right] \sum_{r=0}^{n-1} x^{n-r} y^r I_{n-r}(2xy).$$

This result may be written

$$p_n(x, y) = y^n p_0(x, y) - \exp\left[-(x^2 + y^2)\right] \sum_{s=1}^{n} x^s y^{n-s} I_s(2xy). \tag{1}$$

Now $p_0(x, y)$ has already been tabulated (a list of tables is given in [2], so that by using Eq. (1) numerical values of $p_n(x, y)$ can be conveniently obtained, at least for small values of n, from the tables of $p_0(x, y)$ and of the modified Bessel functions.

The author is indebted to the reviewer for suggesting a simpler derivation of Eq. (1) than that originally presented.

References

1. K. S. Miller, R. I. Bernstein and L. E. Blumenson, *Generalized Rayleigh processes*, Quart. Appl. Math. **16**, 137–145 (1958)
2. F. A. J. Ford, *On certain indefinite integrals involving Bessel functions*, J. Maths. & Phys. **37**, 157 (1958)

*Received November 20, 1958.