$$\int_{x_0}^{x} f(\alpha) I_0(2[\lambda(y - y_0)(x - \alpha)]^{1/2})\, d\alpha, \qquad \int_{y_0}^{y} g(\beta) I_0(2[\lambda(x - x_0)(y - \beta)]^{1/2})\, d\beta \tag{4}$$

will also be solutions of (2) for arbitrary functions $f(\alpha)$ and $g(\beta)$. This enables us to write down directly a solution of (2) for prescribed values of $v(x, y)$ along the characteristic lines $x = x_0$ and $y = y_0$. For we have from (4) by superposition

$$v(x, y) = \int_{x_0}^{x} f(\alpha) I_0(2[\lambda(y - y_0)(x - \alpha)]^{1/2})\, d\alpha + \int_{y_0}^{y} g(\beta) I_0(2[\lambda(x - x_0)(y - \beta)]^{1/2})\, d\beta + v(x_0, y_0) I_0(2[\lambda(x - x_0)(y - y_0)]^{1/2}), \tag{5}$$

where

$$f(x) = \frac{\partial v(x, y_0)}{\partial x}, \qquad g(y) = \frac{\partial v(x_0, y)}{\partial y}, \tag{6}$$

the prescribed functions $v(x, y_0)$, $v(x_0, y)$ being assumed suitably well-behaved to carry out the operations indicated.

As an instance of (5) we remark that it furnishes directly a closed form solution of the boundary value problem treated by Mason [3] in connection with heat transfer in cross-flow and explains the mysterious appearance of $I_0(2[abxy]^{\frac{1}{2}})$ in his solution, arrived at by means of Laplace transform methods.

References

1. E. T. Copson, *Some applications of Riesz's method*, Proceedings of the Conference on Differential Equations, University of Maryland Press, 1956, p. 113
2. J. B. Diaz and G. S. S. Ludford, *On the Euler-Poisson-Darboux equation*, Ibid., p. 82
3. J. L. Mason, *Heat transfer in cross-flow*, Proceedings of the Second U. S. National Congress of Applied Mechanics, 1954, p. 801

Corrections to the paper

THERMAL INSTABILITY OF VISCOUS FLUIDS

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BY CHIA-SHUN YIH (*University of Michigan*)

Equation (27) should read

$$[\sigma - \Pr (D^2 - b^2 - a^2)]\, (D^2 - a^2) f = -R \Pr D\theta$$

The second sentence following Eq. (27) should then be changed to: "Since Eqs. (26) and (27) would be identical to Eqs. (20) and (21) if $\sigma + b^2$ is replaced by σ, any variation of the flow with y will invariably make the flow more stable, if u_2 remains strictly zero, and the boundary conditions on f itself are not relaxed."

With the periodic variation in the y-direction provided, the boundary conditions on f itself can be relaxed, and the fluid may be more unstable. This instability in fact corresponds to the root zero of the characteristic equation

$$\tan R^{1/4} = \tanh R^{1/4}$$

given in the paper. The author is indebted to Mr. R. A. Wooding of Cavendish Laboratory of Cambridge for pointing out this most unstable mode.

ON THE SOMMERFELD HALF-PLANE PROBLEM*

By BERNARD A. LIPPMANN (*Lawrence Radiation Laboratory, University of California, Livermore, California*)

Abstract. A simple derivation of the Sommerfeld solution to the problem of the diffraction of a plane, scalar wave by a half-plane is given. The discusson is of interest mainly because of the simplicity of the argument; however, it is felt that the ansatz that forms the core of the argument is probably more generally applicable.

I. The method of deriving the Sommerfeld solution to the problem of the diffraction of a plane, scalar wave by a half-plane, presented here, became apparent during a study of the utility of conformal mapping in diffraction problems. The discussion is presented mainly because of the simplicity of the argument employed to deduce Sommerfeld's well-known result, but, beyond this, there are two other features of interest: first, one feels that the ansatz that constitutes the core of the argument is probably applicable more generally; and second, although the explicit conformal mapping used is trivial, the manner in which the mapping enters into the formulation of the boundary conditions in the ansatz may be suggestive in clarifying the relationship between conformal mapping and diffraction theory.

II. We seek a function, u, that is a solution of the wave equation[1]

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2\right)u = 0, \tag{1}$$

corresponding to the incident wave[2]

$$\exp\left[-ikr\cos(\theta - \theta')\right], \tag{2}$$

and satisfying certain other conditions to be stated presently. These conditions, the wave equation, and the form of the incident wave, show that u is a function of $\theta - \theta'$ only; we may therefore allow θ' to approach zero and use

$$u_0 = \exp(-ikr\cos\theta) \tag{3}$$

as the incident wave, providing that, in the final result for u, we replace θ by $\theta - \theta'$ everywhere.

It is instructive to consider the problem simultaneously as it appears in Fig. 1, which shows the actual half-plane, coincident with the positive x-axis, and Fig. 2, which

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[1]We shall use the coordinates x, y; r, θ; and $z = y + iy$, $z^* = x - iy$, as convenience dictates.

[2]In contrast to the usual convention, note that this incident wave is traveling towards the x-axis, from above, at an angle θ'.