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INCREASED SUBSCRIPTION PRICE

The subscription price for the QUARTERLY OF APPLIED MATHEMATICS has remained the same since the foundation of this periodical in 1943. The steady increase in the cost of printing now makes it necessary to raise the subscription price to $8.00 per volume (April-January). For prices of single issues and back volumes see inside front cover.

BOOK REVIEWS

On numerical approximation. Edited by Rudolph E. Langer. The University of Wisconsin Press, Madison, 1959. x + 462 pp. $4.50.

This book presents the 21 papers which were delivered at the Symposium on Numerical Approximation conducted by the Mathematics Research Center, United States Army, at the University of Wisconsin from April 21 to 23, 1958. The objective of this symposium was the presentation and discussion of recent developments in the field of numerical approximation centered around three general themes: linear approximation, extremal approximation, and algorithms.

In "On Trends and Problems in Numerical Approximations" A. M. Ostrowski surveys, entertainingly, developments in interpolation theory, in methods of successive approximation and in the application of statistics to the analysis of computational error. R. Creighton Buck, in "Linear Spaces and Approximation Theory," discusses approximation theory from the point of view of functional analysis. In "Operational Methods in Numerical Analysis Based on Rational Approximation" Z. Kopal approximates non-polynomial difference and differential operators by ratios of polynomial operators and applies this to numerical quadrature and the numerical solution of ordinary differential equations. Philip J. Davis, "On the Numerical Integration of Periodic Analytic Functions," compares the remarkably efficient trapezoidal rule with Gaussian quadrature and shows that widespread analyticity of the integrand favours the former, singularities near the integration interval the latter. Herbert E. Salzes states, in "Some New Divided Difference Algorithms for Two Variables," criteria which such algorithms should fulfill and proposes two examples. Preston C. Hammer's "Numerical Evaluation of Multiple Integrals" is an expository paper on his and his collaborators' recent results, especially on integration over spheres, cubes, cones and simplexes. The general problem studied in "Optimal Approximation and Error Bounds" by M. Golomb and H. F. Weinberger is stated by the authors to be "how to derive from a given amount of information about the unknown function u the total range of possible values of the functional $F(u)$." The authors discuss this general problem of approximation, particularly when and to what extent approximation is possible; they completely solve the case where the information consists of the values of a finite set of linear functionals $F_i(u)$ and a bound for a non-negative quadratic functional in u. These results are practically evaluated and many special cases, applications and generalisations given. "The Rationale of Approximation" by Arthur Sard is a general exposition of the theory of approximation of functionals. The following problem in Tchebycheff approximation is, for instance, considered: how to choose a linear operator T from a given linear manifold of bounded linear operators on a Hilbert space X such that $||Tg - h||$ attains a minimum over T for fixed g and $h \in X$. Joseph L. Walsh writes "On Extremal Approximation" and reviews recent results on the structure of extremal polynomials. In "Numerical Methods of Tchebycheff Approximation" E. Stiefel first outlines the basic theoretical tools required and then discusses old and new algorithms for solving the classical problem of linear Tchebycheff approximation. A survey of recent interpolation methods available to the user of tables and desk machines is contained in L. Fox's "Minimax methods in table construction." T. S. Motzkin discusses very briefly and without proofs the "Existence of Essentially Nonlinear Families Suitable for Oscillatory Approximation," in particular of families which have some of the advantages of polynomials for approximating given functions. "On Variation Diminishing Approximation Methods," by I. J. Schoenberg, recapitulates some of the author's work over past years. Let $g(x) = L(f(x))$ be a method furnishing the approximation $g(x)$ to $f(x)$. If $v(f)$ is, in a certain sense, the number of changes of sign of $f(x)$ in the range [0, 1], then the method is said to be variation diminishing if it has the property $v(g) \leq v(f)$. The author shows, for example, that approximation by Bernstein polynomials has this property and thus avoids unnecessary oscillations of the approximating function such as often appears in least squares and Tchebycheff approximation; he also investigates approximation of a given function by power and by trigonometric polynomials from this point of view when the approximation is generated by a linear functional operator. M. Golomb, in "Approximation by Functions of Fewer Variables," investigates both least squares and Tchebycheff approximation of a function of n independent variables by functions of fewer variables, thus attacking a difficult practical problem and giving hope for the emergence of efficient numerical procedures. Practical applications of polynomial approximation are dealt with by J. C. P. Miller in "Extremal approximation—A Summary."

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R. Creighton Buck presents a valuable "Survey of Recent Russian Literature on Approximation," with a bibliography of 135 items, most of them with their Mathematical Reviews reference. "The Quotient-Difference and Epsilon Algorithms" are compared by F. L. Bauer and formulas given connecting the Stieljes continued fraction with its power series expansion. J. Barkley Rosser, in "Some Sufficient Conditions for the Existence of an Asymptotic Formula or an Asymptotic Expansion," derives the asymptotic behaviour, for large z, of $\int_A^B f(x) \exp(zg(x))\, dx$, giving numerical illustrations of his results. John W. Tukey, in "The Estimation of (Power) Spectra and Related Quantities" presents various topics in the spectral analysis of time-series. Aliasing, resolvability and variability are discussed and the generalised Fourier analysis approach is contrasted with the ensemble approach. The salutary effect of prewhitening is described, as are various other computational techniques useful in the empirical determination of the spectrum. "Approximation in Partial Differential Equations," by L. Collatz, deals with the following problem. Let E be a linear or quasi-linear second order differential operator of elliptic, parabolic or mixed (e.g. Tricomi) type, and D an n-dimensional region with boundary C. The problem is to solve $E[u] = 0$ in D with $F[u] = 0$ on C. It is attempted, in particular, to find a v satisfying $E[v] = 0$ only, such that $\max_D |u - v|$ is small, and examples of the technique are given. Finally, John Todd outlines a graduate course on the role of "Special Polynomials in Numerical Analysis," embracing Weierstrass' theorems, Tchebycheff polynomials, the theorems of the Markoffs and related topics.

This volume will be greatly appreciated both by persons interested in the pure mathematical aspects of approximation theory and by those interested in applying the theory to particular situations.

WALTER FREIBERGER

Théorie des Circuits Non-linéaires en Régime Alternatif. By V. Belevitch. Librairie Universitaire, Louvain, Belgium, and Gauthier-Villars, Paris, 1959. 293 pp. $8.45.

This is a sequel to an earlier text by the author, "Theory of Telecommunication Circuits," which dealt primarily with linear time-invariant circuits of the type used in communication systems. The present text is concerned for the most part with the analysis of rectifiers, modulators and oscillators. The material is oriented toward the needs of the circuit designer rather than the mathematically inclined engineer. There are hardly any differential equations in the text, the author's main tools of analysis centering on the use of Fourier series. Although these are employed skillfully and effectively, the range of applicability of the author's methods is quite limited.

An interesting part of the book is a nine-page section describing in brief recent contributions to the subjects discussed in the text. One wishes that some of these contributions were discussed more fully in the main body of the work.

All in all, this text is an impressive testimony to the author's high competence. Its narrow scope, however, robs the book of much of its value to those who have a broad interest in the theory of nonlinear systems.

L. A. ZADEH

Methods based on the Wiener-Hopf technique. By B. Noble. Pergamon Press, New York, London, Paris, Los Angeles, 1958. x + 246 pp. $10.00.

The Wiener-Hopf method is a fascinating and useful technique whose consistent exploitation in the treatment of boundary value problems and integral equations has evolved during the last fifteen years, despite its invention in 1931. The method depends only on classical theorems in the theory of functions of a complex variable and the author reviews all such necessary background material in Chapter I of this book. In Chapter II he utilizes three different formalisms, each depending on the

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Wiener-Hopf idea, to treat the Summerfeld half-plane diffraction problem. The reviewer strongly supports the author's view that here, and in other problems whose basic formulation includes a partial differential equation, the direct attack on the differential equation is far less laborious than the alternative procedures which require the choosing of a Green's function and the formulation of an integral equation as intermediate steps. This is especially true when the differential equation is not one of those few for which the appropriate Green's function is known *a priori*. Also included in Chapter II is an outline of the treatment of several problems whose geometry and analysis closely parallel those of the diffraction problem.

Chapter III is concerned with the details of the application of this technique to other problems in which the geometry is somewhat more complicated. Typical of these are the diffraction of waves by a pair of semi-infinite planes and the transmission of sound from a semi-infinite pipe. It is to be noted that the method becomes no more difficult in principle for such problems but the algebraic labor increases enormously.

The remainder of the book is concerned primarily with variations on, limitations of, and approximations within, this technique. The reader will find that he needs only Chapters I and II to understand both the basis of the method and the manner in which it can be used. The remaining material, including the rather complete bibliography, gives an excellent view of its exploitation and range of applicability. The book should be useful to those who need an introduction to the subject and to those who want a compendium of its accomplishments and limitations.

GEORGE F. CARRIER

Nichtlineare Mechanik. By Hans Kauderer. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1958. xi + 684 pp. $15. 51.

In the book under review, the author has produced the type of volume which would be described in the English literature as a treatise on the vibrations of nonlinear systems. However, in addition, one aspect of nonlinear elasticity is treated as well.

The first part (amounting to less than one quarter of the total volume), deals with elastostatics under nonlinear stress-strain relations but infinitesimal strains. A section dealing with theoretical aspects of elasticity theory under nonlinear (single-valued) stress-strain relations is followed by a discussion of problems of plane strain, torsion, and bending of thin plates. Nonlinearities arising from sources other than the stress-strain relation are not considered.

Except for a short final section, the remainder of the book is devoted entirely to the vibrations of systems which are governed by ordinary nonlinear differential equations. More than two-thirds of this portion deals with systems having a single degree of freedom; the remainder contains a discussion of multi-degree-of-freedom systems.

A feature of the book which strikes the reviewer as particularly pleasing is the logical arrangement of the subject matter.

Vibrations in a single degree of freedom are subdivided into autonomous and non-autonomous systems. In the section on autonomous systems one finds, in that order, conservative, damped, and self-excited systems. The section on non-autonomous systems deals with the forced harmonic and subharmonic vibrations in the Duffing problem, with forced vibrations of systems capable of self-excitation, and with auto-parametrically excited systems. The stability theory of autonomous systems is an integral part of the section treating the response of these systems, while the stability of non-autonomous systems is treated in a special section.

The last ninety pages are devoted to the vibrations of multi-degree-of-freedom systems. This portion is divided into systems having two, and having an infinity of degrees of freedom; each is again subdivided into autonomous and non-autonomous systems. The treatment of dual-mode non-autonomous systems contains a discussion of forced vibrations of (otherwise) conservative, and of damped systems under simple harmonic excitation. The section of infinitely many degrees of freedom consists

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of some classical problems when the stress-strain relations are slightly nonlinear, and nonlinearities arising from boundary conditions.

The subject matter is interspersed with interesting and novel examples, and all "analytical," numerical and graphical methods which are described are illustrated by examples most of which are original.

A book review of conventional length can not do justice to so comprehensive a book as that under review. Therefore, some general impressions will have to suffice.

The level of the book is such that a reader familiar with elementary calculus and with ordinary differential equations is prepared to read it. In fact, the author has purposely addressed himself to the practitioner. Advanced mathematical techniques were avoided by ignoring questions of existence of the solutions which are desired, and by assuming that the methods used in their construction will converge. On the one hand, this approach ignores many results now available; on the other, it is quite realistic because the present state of theoretical knowledge of nonlinear differential equations is totally inadequate to establish rigorously the validity of so-called analytical solutions that have served so well to predict experimentally observed phenomena.

Many sections of this book contain original work by the author, most of it not published heretofore. Moreover, the author has shunned no efforts in presenting a great many very detailed solutions to specific problems. While these may make for tedious reading for those more interested in theoretical work, they will be welcome additions for the practitioner.

The author's presentation follows the German tradition and is not easy to read even for the native German; it may be unduly difficult for the American reader. The typography and the illustrations are exceptionally appealing and the text seems virtually free from typographical errors which so frequently mar first printings.

The reviewer's preference would have been to delete entirely the section on nonlinear elasticity. For one thing, this section does not deserve the promising title which it bears since it deals only with nonlinear stress-strain relations, and for another, the connection between this part and the remainder of the book is tenuous. The space gained in this manner could then have been used to present additional material now omitted.

Most notable among these omissions is the use of phase-plane methods for the discussion of the transient solutions of non-autonomous systems of one degree of freedom. In addition, the method of Poincaré for the discussion of solutions as functions of a parameter might well have been included. In fact, some of the examples could have been treated more easily and elegantly by this method than was done in the text.

These remarks are not meant as criticisms. In a lesser book they would have been out of place. However, in this book the coverage is so complete, the treatment is so thorough, and considerations of space seem to have been so liberal, that the reviewer wishes that he could find all of the important material under a single cover. In any case, he can recommend this book very highly as an excellent reference work and a welcome addition to the growing literature in the field of nonlinear vibrations.

R. M. Rosenberg

Refined iterative methods for computation of the solution and the eigenvalues of self-adjoint boundary value problems. By M. Engeli, Th. Ginsburg, H. Rutishauser, E. Stiefel. Birkhäuser Verlag, Basel, Stuttgart, 1959. 107 pp. $3.95.

Numerical analysis resembles a physical science in that its theories lose or gain luster in confronting observations. But in both cases the observational data, which for numerical analysis is concerned with accuracy, computational speed, generality of application and, frequently, ease of coding, must be skillfully marshaled to be convincing. The first three of these criteria have been applied to a variety of computational methods, using a fourth-order plate problem as the principal test, and the authors have provided an extremely useful evaluation of the methods examined.

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Professor Stiefel's introduction illustrates that the self-adjointness of a continuous problem is most naturally preserved in its discrete analogue, not by approximating a partial differential equation and its boundary conditions, but by approximating and minimizing the underlying energy integral. Rutishauser's essay summarizes the theory of the gradient methods to be tested, and Ginsburg reports on the results of his experiments. Engeli deals compactly with both the theory and the numerical consequences of overrelaxation.

The verdict, set down in the last chapter, is generally speaking for overrelaxation on Dirichlet-type problems and elimination otherwise; for eigenvalue problems, a champion among the gradient methods is designated, but the authors make it plain that their investigation did not include some other competitive methods.

The clarity is unimpaired by minor errors in spelling and usage, but a misplaced formula on pp. 52–53 is more serious, and the notations cosh and Ch both appear.

This example of a valuable service which can be performed "collectively" by a computation laboratory unquestionably deserves to be followed.

W. Gilbert Strang

Probability and statistics. Edited by Ulf Grenander. The Harald Cramér Volume. Almqvist & Wiksell, Stockholm, and John Wiley & Sons, New York, 1959. 434 pp. $12.50.

This book contains nineteen papers dedicated to Harald Cramér in honor of his 65th birthday. Most of the contributions are survey papers of recent developments and collectively they cover a wide range of selected topics in mathematical statistics and probability. The contributors are T. W. Anderson, M. S. Bartlett, J. L. Doob, G. Elfving, W. Feller, E. Fix, J. L. Hodges, E. L. Lehmann, U. Grenander, M. Kac, D. G. Kendall, P. Lévy, P. Masani, N. Wiener, J. Neyman, H. Robbins, M. Rosenblatt, C. O. Segerdahl, T. W. Tukey, S. S. Wilks, and H. Wold. The quality of the papers is as one would expect from such a distinguished group of authors and is a fitting tribute to Cramér.

G. F. Newell

Matrices. By William Vann Parker and James Clifton Eaves. The Ronald Press Co., New York, 1960. viii + 195 pp. $7.50.

This is a textbook on elementary matrix algebra for introductory college courses, probably most suitable for students majoring in applications of mathematics. It is clearly written and produced, and includes many worked and unworked examples.

W. Freiberger

Introduction to mathematical statistics. By Robert V. Hogg and Allen T. Craig. The Macmillan Co., New York, 1959. ix + 245 pp. $6.75.

This is one of several new text books on mathematical statistics to appear this year apparently as a consequence of the trend toward inclusion of this subject in the mathematics curriculum of most American universities at the advanced undergraduate level. The book covers an exceptionally wide range of topics including such things as sufficient statistics, most powerful tests and other topics not always included in an elementary book. To accomplish this, however, the author must sacrifice mathematical elegance. Some theorems from other branches of mathematics are quoted without proof, and the proofs of some key theorems in probability theory are incomplete. Some people will also object to the use of a probability density function for discrete and continuous random variables instead of the

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distribution function. If one takes the attitude that one should try to cover as many topics as possible and not avoid certain subjects just because a good mathematical treatment of them is either too difficult or too long, then this would certainly serve as a satisfactory introduction.

G. F. NEWELL

Modern probability theory and its applications. By Emanuel Parzen. John Wiley & Sons, Inc., New York and London, 1960. xv + 464 pp. $10.75.

This book is intended as a text for an undergraduate course in probability theory. It is novel and may cause controversy but is sufficiently well written that it is not likely to be ignored. It is perhaps the most significant introductory book written on this subject since Feller's famous book in this same Wiley series.

It is claimed that about three quarters of the book can be covered in 39 lectures to undergraduates with one year of calculus. Since the author has undoubtedly done so one must believe that it can be done but certainly the average student at this level must find this pace very strenuous. The book seems more suited for advanced undergraduates or first year graduate students.

The style of writing through about the first half of the book reminds one of Feller's book and reflects the author's comment that "probability theory is a subject with great charm and intrinsic interest of its own." The routine material on combinatorial problems, simple types of distributions, etc. is described as if it were alive and the author makes copious use of examples the answers to which are contrary to intuition. About a third of the way through the book, probabilities on real number sets are introduced and the second half of the book deals mainly with continuous random variables including a discussion of limit theorems for sums of independent random variables.

Since "modern probability theory" is based upon measure theory and involves rather sophisticated theorems, it is not possible to include in an undergraduate text a treatment that is both rigorous and reasonably complete. Here the author uses modern terminology and at least sketches the line of reasoning behind the key theorems of probability theory with frequent use of such phrases as "in more advanced treatments one can prove—." Although the intuitive meaning of such things as Borel sets and random variables are explained, precise definitions are frequently lacking. Thus the book is a compromise between the treatise that gives detailed proofs of key theorems and the more usual elementary text that just avoids them.

The book contains numerous references including the most recent and in most cases the best available books or papers (many of which, however, would be too advanced for an undergraduate to understand), many exercises and problems, and many interesting historical sketches. It establishes a style of undergraduate course that many teachers will wish to follow and perhaps this first edition will be the prelude to still better ones.

GORDON F. NEWELL

Les Transformations de Mellin et de Hankel. By S. Colombo. Centre National de la Recherche Scientifique, Paris, France, 1959. 99 pp. $2.05.

This short book is directed to physicists wishing to learn something of the integral transforms associated with the names of Mellin and Hankel. The impression it gives is that it is designed for French-speaking scientists, since it does not seem to contain anything not already available in books (in English) cited by the author in the bibliography.

The first chapter is an introduction to the study of integral transforms in general, reference being made to the Fourier and Laplace transforms whose principal properties are listed and tables of which are provided. Next, the Mellin transforms are introduced, their main properties discussed and their

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use illustrated in the calculation of definite integrals. The third chapter, entitled "Les Formules Réciproques de Hankel," is misnamed since it contains not, as we should suppose, a derivation of the Hankel inversion theorem, but an account of the formal properties of Hankel transforms (the main theorem being stated without proof). The next chapter is concerned with applications of these transforms to the solution of certain simple boundary value problems—Laplace's equation for a wedge by the Mellin transform, Laplace's equation and the diffusion equation by the Hankel transform. All of these problems have been discussed in earlier books by Titchmarsh, Tranter and the present reviewer. The book ends with a brief discussion of the solution of the classic pair of "Titchmarsh" dual integral equations.

The book is written clearly and provides for the French-speaking world an admirable introduction to these transforms. Anyone whose native language is English would, however, be better advised to read C. J. Tranter's "Integral Transforms in Mathematical Physics" (Wiley, New York, 1951).

I. N. SNEDDON

Incompressible aerodynamics. Editor Bryan Thwaites. Oxford University Press, New York, 1960. xx + 636 pp. $12.00.

The present book is an account of the theory and observation of the steady flow of incompressible fluid past airfoils, wings, and other bodies of aeronautical interest. Written by sixteen collaborators, it is the first of a new series sponsored by the Aeronautical Research Council to succeed the well-known pair of works edited by Sydney Goldstein and by Leslie Howarth. The present volume, together with a second volume on laminar boundary layers and a third on turbulence yet to appear, will recover, in the main, the topics in the Goldstein volumes.

Following an inauspicious introductory chapter, the second chapter presents a brief modern discussion, from a utilitarian point of view, of the computation of chiefly turbulent boundary-layer characteristics, including the results of D. Coles and F. Clauser. The next chapter compares the results given by theoretical models of real flows with those observed. Delightful photographs here illustrate the need for fast exposures to depict what actually happens, for example, in the wake of a disk, the more familiar pictures being revealed as smears of the phenomena. The usual two-dimensional airfoil theories are briefly but nicely set out in Chapter IV. Lighthill's theory for rendering perturbation results uniformly valid at stagnation points is included, as is necessary in a modern treatment, and the chapter goes on to treat other recent results. The pristine theories of separated flow remind us (and perhaps we need it) that the two-dimensional, incompressible flow past airfoils is still a living, vital subject, providing, in fact, a framework suitable for the study of separated flow. The next chapter on the uniform viscous flow past airfoils includes a critical generalized view of the Kutta condition; it deals with its subject from the point of view (maintained throughout the volume) of successive approximation to the actual viscous flow via the first inviscid approximation, the first viscous approximation (the boundary layer), the second inviscid approximation (displacement effects), the second viscous approximation, etc.

Following two interesting chapters on boundary-layer control and the thickness effects of three-dimensional wings, the subject of three-dimensional lifting wings, probably the central chapter in such a book, is treated. This is introduced with some remarkable idealized sketches of the separated flow pattern and associated trailing vortex system produced by a flat, circular disk at varying incidences. These lead to the Lanchester-Prandtl vortex systems and to the recent Roy model for the separated delta wing. The classical linearized lifting-surface theory and slender-body theory are then presented, emphasizing more recent developments; this material overlaps somewhat with Volume VII of the Princeton Series. Chapter IX on bodies of revolution follows the same pattern. Wing-wing and wing-body interference is treated in the next Chapter. The discussion of the importance of nonlinear effects (in the boundary conditions) near the junction of two thin rectangular wings in cruciform arrangement is very interesting. Incompressible flow in axial-flow compressors, a pleasant surprise in a book of this type, is next considered, most of the chapter being devoted to actuator-disk theory. Concluding Chapter XII

is concerned with miscellaneous fringe topics, notably recent and important jet flap research and shear flows past simple bodies.

This thoroughly modern treatment is distinguished for its attention to observation, its revealing three-dimensional sketches and photographs, its carefully reasoned formulation of idealized models, and its ever present experimental verification. If, indeed, the book has a categoric shortcoming, it is in an incomplete interpretation of the results of the analysis itself in respect to the feedback on the *a priori* physical reasoning and to the deeper connections between related analyses. As an example, consider the leading-edge singularities given by linearized airfoil theory. In accord with the point of view of the book, the formal manner in which the Lighthill theory for uniform validity manages to tuck the leading-edge singularity just the proper distance inside the wing by expanding the independent variables deserves interpretative discussion. For various cases, Goldstein and Van Dyke (dubbed Dyke in the book) have proposed multiplying the linearized solution by a factor which is the ratio of the deduced correct behavior at the leading edge to the linear behavior. This does not change the behavior of the solution elsewhere, within the approximation, and leads, as is mentioned in the book, to the same correct result as the Lighthill theory at the singularity. One wishes to see established the connection between the two diverse approaches, especially considering the success Lighthill has had in providing similar physical interpretations of his theory for cases of supersonic wave propagation (volume VI of the Princeton series).

Didactically viewed, it isn't likely that the book will be used as a text, but it will serve well to flesh out with supplementary reading briefer treatments of the subject. The work can be read by a first-year graduate student without difficulty or need of classroom support. At the other extreme from the Princeton Series, the chapters are so thoroughly integrated and otherwise worked over by the editor that, like the Goldstein volumes, the names of the contributors are not associated with their original contributions. Attributed articles carry the concomitant features of individual recognition and responsibility, whereas the need to coordinate the efforts of collaborators is also obvious. One wonders whether the integrated but signed chapters of the Howarth volumes do not represent the best compromise solution to the problem of producing compendia.

JOSEPH H. CLARKE

Statistics. By Earl K. Bowen. Richard D. Irwin, Inc., Homewood, Illinois, 1960. xi + 415 pp. $6.50.

This text represents an introductory course for students of business administration, industrial management, and economics. Realistic problems are integrated with a systematic presentation of principles and methods and no formal training in higher mathematics is needed for understanding it. There are many questions and problems illustrating the material of each chapter and an extensive bibliography. The book should make interesting and useful reading for anyone wishing to acquaint himself with the multitudinous uses to which statistics can be put in the context of economic problems.

WALTER F. FREIBERGER

Mathematical methods of operations research. By Thomas L. Saaty. McGraw-Hill Book Co., New York, Toronto, London, 1959. xi + 421 pp. $10.00.

After a preliminary chapter entitled "History and Concepts," the book is divided into four parts. The first of these contains a chapter discussing scientific method in operations research, a chapter on mathematical existence proofs and symbolic logic, and a chapter summarizing, with some operations research examples, the use in problem formulation and solution of such mathematical techniques as differential equations (Lanchester's application to combat), generating functions (probability), integral equations (renewal problems), and others.

Part Two, entitled "Optimization, Programming and Game Theory," contains a chapter each on these subjects. The first of these is a collection (65 pages) of mathematical methods which the operations researcher should welcome, including the solution of inequalities, the use of Langrange multipliers, and the calculus of variations with inequality constraints. A chapter on linear and quadratic programming

is followed by a short chapter on the theory of games. A special section is appended to Part Two listing some mathematical exercises.

Part Three contains a chapter on probability and another treating some applications in inventory processes and system reliability. There follows a review chapter on the elements of classical statistics, and finally the resume of queueing theory which the author published in *Operations Research* (vol. 5, no. 2, April 1957, pp. 161–200). A set of exercises is appended.

In Part Four, an essay chapter entitled "Some Thoughts on Creativity" is followed by a collection of 95 problems. These problems include mathematical and logical teasers of the sort popular in scientific journals in recent years, as well as some suggested projects in operations research.

The chapter-by-chapter bibliographies are well done. The interest of the books is, as the title implies, in mathematical methods, and it should be found useful by the applied mathematician seeking a collection of this size of topics relevant to operations research.

HERBERT P. GALLIHER

Turbulent flows and heat transfer. Editor: C. C. Lin. Princeton University Press, New Jersey, 1959. xv + 549 pp. $15.00.

This book is intended to deal with the problems of turbulent flow and conductive, convective, and radiative heat transfer. Certainly many of the articles appearing in the volume attest to the high standard of excellence which may be expected from some of the outstanding contributors to it. On this basis alone it is to be recommended to those people who wish to obtain an appreciation of the subjects treated, particularly of the complex area of turbulence and transition to turbulence.

As a connected work the volume falls far short of what might have been expected. This is particularly true of those sections concerned with heat transfer. The separate sections in the book span a range which varies from a handbook engineering style to highly sophisticated theoretical discussions. The caliber of the articles has an equally wide spread going from poor to excellent. Furthermore, as a result of the disconnected presentation there results considerable duplication from different points of view and as a consequence the reader is oftimes left with a confused and, in some cases, erroneous impression of the field.

The nine articles in the book in their order of presentation are:

A. Transition from laminar to turbulent flow, Hugh L. Dryden; B. Turbulent flow, Galen B. Schubauer and C. M. Tchen; C. Statistical theories of turbulence, C. C. Lin; D. Conduction of heat, M. Yachter and E. Mayer; E. Convective heat transfer and friction in flow of liquids, R. G. Deissler and R. H. Sabersky; F. Convective heat transfer in gases, E. R. van Driest; G. Cooling by protective fluid films, S. W. Yuan; H. Physical basis of thermal radiation, S. S. Penner; I. Engineering calculations of radiant heat exchange, Hoyt C. Hottel.

It is in Sections A to C that this reviewer found a sensible and orderly presentation of the material. Section A opens the volume with a discussion of transition from laminar to turbulent flows essentially from an experimental point of view, and is followed in B with a discussion of turbulent shear flow from a semi-empirical and experimental point of view, while in C turbulence is discussed from the purely deductive statistical point of view.

Dryden's contribution in A is an excellent compilation of all of the presently known experimental facts on the subject of transition to turbulence, and is a fine presentation of how various flow factors influence transition in both incompressible and compressible flows. This reviewer would have liked to have seen more detailed considerations on the three-dimensional nature of the transition process and perhaps brief outlines of some of the major existing theories on stability and transition to turbulence. However, this in no way detracts from the value of this section.

The article by Schubauer and Tchen in B emphasizes the mean properties of turbulent shear flows without discussing the structure of turbulence. Here is to be found an excellent description of the various phenomenological theories of turbulent flow and turbulent boundary layers. A good discussion is given of the wall law and velocity defect law, Clauser's equilibrium boundary layer concept, and Coles' law of the wake. This reviewer was pleased to see the authors' statement in connection with so-called turbulent boundary layer skin friction "theories" that "some empirical form of skin friction coefficient may often be more useful in practice."

To the reader unacquainted with statistical theories of turbulence, C. C. Lin's article in C will indeed

be a welcome addition. The section restricts itself primarily to the case of homogeneous turbulence and the subject is presented from the author's own unique point of view. A clear discussion may be found here of velocity correlations and the spectral theories of turbulence. These methods are then applied to the study of the nature of turbulent motions and similarity considerations are presented. Turbulent motion in a compressible fluid is briefly discussed.

The remaining sections of the book deal with conductive, convective, and radiative heat transfer with special emphasis on high speed flows. The longest section of this part which is intended to analyze convective heat transfer is Section F by van Driest. Unfortunately many of the topics contained therein are unrelated to the topic at hand, so that some of what was presented in A and B on transition and turbulent flows is repeated here in a much less satisfactory manner. The remainder of the section appears to deal almost exclusively with particular problems the author himself has been connected with. After reading this section this reviewer was not left with any clear picture or understanding of the general area of convective heat transfer in high speed flows. The shortest section in the volume is H by S. S. Penner which trys to present in 12 pages the basic ideas of thermal radiation. In this task the author is not successful and the uninitiated reader will find it difficult to actually get a clear picture of the problems involved, particularly in relation to the calculation of gas emissivities.

Section D by Yachter and Mayer gives a brief discussion of conduction in solids covering a variety of topics from a formal mathematical discussion of linear heat conduction problems to practical considerations in minimum weight design. Isolated problem solutions are given and brief remarks of interest to those familiar with the problems involved are given. Deissler in E discusses turbulent heat transfer in liquids from a semi-empirical and experimental point of view with no reference to the discussion of B. In this same section Sabersky gives a brief survey of some problems in boiling heat transfer. Yuan in G discusses film cooling problems from what this author considers to be a rather limited viewpoint. In Section I Hottel presents radiation calculations from a practical engineering approach. This section will force those who are desirous of any understanding of the actual problems involved to seek further information elsewhere.

Although the volume leaves much to be desired it does nevertheless present a great deal of information which the avid reader can certainly seek out.

RONALD F. PROBSTEIN

An introduction to mathematical statistics. By H. D. Brunk. Ginn and Company, Boston, New York, 1960. xi + 403 pp. $7.00.

This text is written for students who have studied calculus but have no formal background in probability theory or mathematical statistics. The first part of the book is devoted to elements of probability theory. A discussion of discrete and continuous random variables follows. The second part on statistics is more ambitious. A listing of some of the chapters in this part follows and indicates the scope of the discussion: random sampling, the law of large numbers, estimation of parameters, central limit theorem, confidence intervals and tests of hypotheses, statistical decision theory, regression, experimental design and analysis of variance, distribution free methods. Of course, some of these topics are at times treated in an abbreviated manner and without proofs. Nonetheless, the discussion is clear and the breadth in coverage startlingly good. Many of the current ideas in statistical practice and theory are touched on. There is a very good set of statistical tables at the end of the text.

M. ROSENBLATT

Exponentially distributed random numbers. By Charles E. Clark and Betty Weber Holz. The Johns Hopkins University Press, Baltimore, Maryland, 1960. vii + 249 pp. $6.50.

The book presents a table of 100,000 random numbers from the distribution with probability density e^{-x}, $0 \leq x < \infty$. The generation of these random numbers was based on uniformly distributed pseudo-random numbers and the validity of the generating process was tested empirically. The tests of randomness and their results, which were satisfactory, are presented in the introduction.

WALTER F. FREIBERGER