QUARTERLY

OF

APPLIED MATHEMATICS

QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Contents

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}]\ \text{is preferable to}\ e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)}\ \text{is preferable to}\ \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$
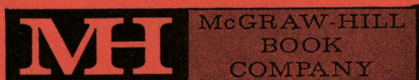
STATISTICAL DECISION THEORY

By LIONEL WEISS, Cornell University. *McGraw-Hill Series in Probability and Statistics.* 195 pages, $7.50.

A text describing and developing modern statistical decision theory at an intermediate mathematical level. The first four chapters develop the necessary probability theory. The next four chapters cover stastical decision theory, including linear programming as a computational tool and problems involving making a sequence of decisions over time. The final chapter develops the standard techniques of conventional statistical theory as special cases of statistical decision theory. Requires elementary calculus.

ELEMENTS OF QUEUING THEORY WITH APPLICATIONS

By THOMAS L. SAATY, Office of Naval Research. Available in November, 1961.

This book presents a variety of queuing ramifications, methods of treatment, and in general provides a broad account of the rapid development in this challenging field. Most of the fundamental ideas of queues are discussed and developed. Many applications are described and discussed, in addition to a discussion of both Poisson and non-Poisson queues with different queuing disciplines. Bibliography of queues included.

INTRODUCTION TO MATRICES AND VECTORS

By JACOB T. SCHWARTZ, New York University. 163 pages, $5.50.

An elementary, practical introduction to matrix algebra designed for the senior high school or early college student and intended to bring the relatively inexperienced student to a point where he can appreciate some sophisticated approaches to mathematics. Covers algebra of matrices; the minimal equation and its use in inverting matrices; systems of linear equations; geometry of vectors in 2, 3, and n-dimensions; and some special additional topics in the algebra and analysis of matrices.

Academician V. I. Smirnov's LINEAR ALGEBRA AND GROUP THEORY

By V. I. SMIRNOV, U.S.S.R.; revised, adapted, and edited by RICHARD A. SILVERMAN, Formerly of MIT and New York University. 480 pages, $12.50.

In this unique work, an internationally known authority offers several features of special interest to English-language readers. In addition to a detailed treatment of linear algebra, it also gives an excellent introduction to group theory and an extensive discussion of group representations. Also rarely encountered in first year courses in higher algebra is the material included on infinite-dimensional spaces and continuous groups.

SURVEY OF NUMERICAL ANALYSIS

Edited by JOHN TODD, California Institute of Technology. Available in January, 1962.

The work of 14 nationally known authors, this book covers numerical analysis, both classical and modern, together with accounts of certain areas of mathematics and statistics which support it yet are not adequately covered in current literature. The first third of the book provides a basic training in numerical analysis and the remainder of the text is devoted to accounts of current practice in solving, by high speed equipment, special types of problems in the physical sciences, engineering and economics.

NONLINEAR DIFFERENTIAL EQUATIONS

By RAIMOND A. STRUBLE, North Carolina State College. *International Series in Pure and Applied Mathematics.* Available in January, 1961.

Prepared for a one-semester advanced undergraduate or beginning graduate course in nonlinear differential equations. With the needs of the applied mathematician, engineer, and physicist in mind, the book provides for a rapid contact with the majority of the mathematically significant concepts of nonlinear differential equations while imposing but modest demands on the reader for previous mathematical experience.

McGraw-Hill Book Company, Inc.
330 West 42nd St., N. Y. 36, N. Y.

BOOK REVIEWS

Introduction to linear programming. By Walter W. Garvin. McGraw-Hill Book Co., Inc., New York, Toronto, London, 1960. xiv + 281 pp. $8.75.

The book grew out of a course for selected personnel of the Standard Oil Company of California. It does not make great demands on the mathematical background of the reader and explains linear programming in elementary algebraic terms.

Part I (82 pp.) deals with the general linear programming problem. The simplex method is developed and sensitivity analysis is discussed. An illustrative gasoline-blending problem is treated in considerable detail. Part II (65 pp.) is devoted to problems with the structure of the transportation problem, in which the triangularity of the basis leads to simplifications in the computational procedure. A generalized transportation problem with semi-triangular basis is formulated and a method for solving it is presented. Part III (118 pp.) treats a variety of special questions such as the presence of upper bounds, statistical and parametric linear programming, the revised simplex method, the resolution of degeneracy, and duality.

W. Prager

An introduction to the calculus of finite differences and difference equations. By Kenneth S. Miller. Henry Holt & Co., New York, 1960. viii + 167 pp. $4.50.

This book is not for one whose only interest is a working knowledge of finite differences for application to numerical computation. Rather, it is an elegant and rigorous presentation of the theory, developed with emphasis on the beautiful analogy with differential and integral calculus and differential equations.

The calculus of finite differences is introduced in the first chapter, the theory of infinite products and its application to that calculus are demonstrated in the second, and Bernoulli polynomials and the Euler-Maclaurin formula treated in the third. The last chapter is devoted to linear difference equations in the real domain.

Each chapter concludes with a large number of problems. Although written with all the pains a pure mathematician might demand, the book will appeal to applied mathematicians and numerical analysts who wish to gain a deeper knowledge of finite differences.

Walter Freiberger

Bessel functions—zeros and associated values. Part III. Royal Society Mathematical Tables, Volume 7. Edited by F. W. J. Oliver. Cambridge University Press, New York, 1960. lx + 77 pp. $9.50.

It is well-known that the numerical determination of the zeros of a rational polynomial presents difficulties of much higher order than the evaluation of the function at a set of values of the argument. This is even more true of transcendental functions like Bessel functions, and this volume therefore differs from its predecessors in a significant way. The 60-page introduction describes in fascinating detail how the difficulties inherent in this problem have been brilliantly overcome, and is well worth reading for its own sake as a stimulating essay in a baffling field of numerical analysis.

For the practitioner, the tables will be invaluable. The main tables give the zeros of J_n and of Y_n with corresponding values of J_n' and Y_n'; the zeros of J_n' and Y_n' with corresponding values of J_n and Y_n; and the zeros of the spherical Bessel functions $\sqrt{\{\pi/2x\}}\, J_{n+1/2}$ and $\sqrt{\{\pi/2x\}}\, Y_{n+1/2}$ with the corresponding values of their derivatives.

The lay-out and typography are as exemplary as one has come to expect from the Cambridge University Press.

Walter Freiberger

BOOK REVIEWS
(Continued from p. 194)

The control of multivariable systems. By Mihajlo D. Mesarovic. Technology Press of
   M. I. T., Cambridge, Mass., and John Wiley & Sons, Inc., New York and London,
   1960. xi + 112 pp. $3.50.

The monograph is divided into two parts. The first part contains some general philosophical discussion culminating in the observation that control of a complex system can be materially improved if more is known of the internal structure. To illustrate this point, the author considers several different types of structures together with their transfer functions and control policies.

The second part is devoted to the synthesis problem. This is first formulated as the problem of minimizing a functional of a collection of state variables, and then is simplified to that of minimizing a quadratic functional. A number of specific calculations are then carried out.

There is no doubt that the author has some interesting ideas in this modern fascinating field of control theory. However, the book is so loosely organized and the results so imprecisely stated that it is exceedingly difficult to discern what they are. This is a book which required a strong editorial hand, and did not receive it.

RICHARD BELLMAN

Wave propagation and group velocity. By Leon Brillouin. Academic Press, New York
   and London, 1960. xi + 154 pp. $6.00.

The purpose of this book is to bring to the fore some of the fundamental old papers on the subject of wave propagation. The title is perhaps misleadingly short, since the book deals with the distinction and relations between phase velocity, group velocity and signal velocity of a propagating wave. The author presents the subject against the background of its historical developments, as he is indeed well qualified to do.

The structure of the book is somewhat unusual. The first chapter gives several basic definitions concerning wave propagation, followed by a summary of the theoretical situation around 1910. Particular emphasis is placed on the difficulties that were encountered, in those days, in making a clear distinction between group velocity and signal velocity in dispersive media. The following four chapters cover the fundamental work in this area by Sommerfeld and by Brillouin, which culminated in a clarification of the above concepts. This part is presented in the form of four papers, originally published between 1914 and 1932; these papers are reprinted with very little editorial comment. Finally, the last chapter discusses briefly the application of the ideas developed in these papers to guided waves (both acoustic and electromagnetic).

The work on this subject by Summerfeld and by Brillouin is, of course, a classic in the field. Presenting it in this form, under one cover, will undoubtedly facilitate the task of those who are interested in the complete, original treatment.

C. ELBAUM

Weight-strength analysis of aircraft structures. By F. R. Shanley. Dover Publications,
   Inc., New York, 1960, xiii + 404 pp. $2.45.

This is an unabridged reprint of the first edition (McGraw-Hill, 1952), to which a new preface and bibliographies on optimum design of structures and creep buckling have been added.

BOOK REVIEWS

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Stochastic processes: problems and solutions. By Lajos Takacs. Methuen & Co. Ltd., London, and John Wiley & Sons, Inc., New York, 1960. xi + 137 pp. $2.75.

A true description of this book is given in the Introduction as follows: "This book $\cdots$ is primarily a collection of problems and their solutions, and is intended for readers who are already familiar with probability theory. Its aim is to summarise the fundamental notions and theorems of stochastic processes. The proofs of the theorems are generally omitted or only a brief outline is given. $\cdots$ The scope of this book extends over the theory of Markov chains, Markov processes, stationary stochastic processes, recurrent processes and secondary stochastic processes. The problems are taken from the fields of natural sciences, engineering and industry."

This is an ideal book for anyone who is already familiar with elementary probability theory and would like a brief introduction to the above topics and their applications. By avoiding difficult topics and proofs, the author manages to scan an amazing range of subjects for such a small book. The treatment of Markov chains and processes, though highly condensed is very well done but the subject of stationary stochastic processes is perhaps too brief to be very instructive.

G. F. Newell

Tables of $\ln \Gamma(z)$ *for complex argument.* By A. A. Abramov. Translated from the Russian by D. G. Fry. Pergamon Press, New York. 331 pp. $17.50

These are six-figure tables of $\ln \Gamma(x + iy)$ for $x = 1 (.01) 2$, $y = 0 (.01) 4$, that were compiled at the Institute of Precision Mechanics and Computer Technology of the U. S. S. R. Academy of Sciences. Formulas for computing $\ln \Gamma(z)$ outside this rectangle are given. A loose sheet furnished with the book contains a nomogram facilitating interpolation.

Technische Schwingungslehre. By Karl Klotter. Springer-Verlag, Berlin, Göttingen, Heidelberg, 1960. xv + 483 pp. $14.95.

This second volume of a two-volume series on vibrations deals with vibrations of multiple-degree-of-freedom systems. It is intended as a textbook rather than as a handbook, so the arrangement of material is primarily pedagogic; moreover, emphasis is on fundamental ideas rather than applications, and certain more sophisticated topics such as Laplace transforms or non-linear differential equations are deliberately omitted. The topics are: Part I: meaning of degrees of freedom, with many examples; electrical-mechanical analogies; equations of motion, and their matrix form; Lagrange's equations and Hamilton's principle, with numerous examples; normal modes, with graphical methods—first for two degrees of freedom, and subsequently more generally; properties of linear differential equations; Routh stability criterion, and examples; quadratic forms, and small displacements; damping; practical examples. Part II: torsional vibrations and shafts; difference methods; iteration; graphical methods; bounds; matric algebra; more complex problems; literature survey.

From the student's viewpoint this is an exhaustive and exhausting book (almost 500 pages). It contains an interminable number of examples, many of which are useful and instructive (the book is particularly rich in graphical methods—with the surprising omission of phase plane methods, however), but which are excessive in number. For textbook use the reviewer would prefer a concise text on methodology with a reasonable number of selected examples; also, the omission of non-linear systems seems unwise.

C. E. Pearson

BOOK REVIEWS

Lectures on differential and integral equations. By Kosaku Yosida. Interscience Publishers,
   Inc., New York and London, 1960. ix + 220 pp. $7.00

This text is the English edition of a volume published originally in Japanese. It is intended to be a
self-contained exposition of the theory of ordinary differential equations and of integral equations.
The subject of partial differential equations lies outside the scope of the book. Chapters, 1, 2 and 5
deal with ordinary differential equations, while Chapters 3, 4 and 6 are concerned with integral equa-
tions. Chapter 1 begins by treating the initial value problem $dy/dx = f(x, y)$, $y(x_0) = y_0$, for a single
equation and for a finite system, both in the real and complex domain. After dealing with linear differ-
ential equations of the nth order, the chapter ends with a succint treatment of second order differential
equations of the Fuchs type, with the special cases of Gauss, Legendre, and Bessel being analyzed in
detail. Chapter 2 covers the Sturm-Liouville boundary value problems for linear second order differential
equations, Green's functions; generalized Green's functions, the Hilbert-Schmidt theory of integral
equations with a symmetric kernel, and Liouville's method for obtaining asymptotic expressions
for the eigenvalues and the eigenfunctions. Chapter 5 is devoted to an elementary exposition of the
general expansion theorem (Weyl-Stone-Titchmarsh-Kodaira) relative to the differential equation
$y'' + (\lambda - q(x)) y = 0$, $a < x < b$, where $q(x)$ is a real valued continuous function and no assumption
is made about the behavior of $q(x)$ as $x \to a$ or as $x \to b$ (this is the general "singular" case). Let $y_1(x, \lambda)$,
$y_2(x, \lambda)$ be a fundamental system of solutions determined by initial conditions at a number c with
$a < c < b$: $y_1(c, \lambda) = 1$, $y_1'(c, \lambda) = 0$; $y_2(c, \lambda) = 0$, $y_2'(c, \lambda) = 1$. Then, for "appropriate boundary
conditions at $x = a$ and at $x = {}_bb$", every real valued continuous function $f(x)$ in $a < x < b$ with
$\int_a f^2(x)dx < + \infty$ can be "expanded" as follows:

$$f(x) = \int_{-\infty}^{+\infty} d_u \left\{ \sum_{j,k=1}^{2} \int_0^u y_j(x, u) \, dp_{jk}(u) \int_a^b f(s)y_k(s, u) \, ds \right\}.$$

This is the Weyl-Stone expansion theorem, which has been completed by Titchmarsh and Kodaira
by giving an explicit formula for the "density function" $p_{jk}(u)$. This general expansion theorem enables
the author to give a unified treatment of classical expansions in terms of special functions, such as the
Fourier series expansion, the Fourier integral, the Hermite polynomials expansion, the Laguerre poly-
nomials expansion, and the Bessel functions expansion, which are worked out in detail as special cases
of the general theorem. For simplicity, the case $a = - \infty$, $b = + \infty$ is considered in the proof of the
general expansion theorem, which is obtained as a limiting case of the previously obtained Hilbert-
Schmidt expansion theorem for the "regular" case of a finite interval. Chapter 3 is dedicated to the
theory of Fredholm integral equations of the second kind and Chapter 4 to the theory of Volterra
integral equations of both the first and the second kind. Chapter 6 deals briefly with non-linear Fredholm
and Volterra equations of the second kind. The author has succeeded in achieving an amazingly com-
pact presentation of the material. This book is remarkable both for its conciseness and for its readability.

                                                                                             J. B. Diaz

BOOK REVIEWS

Mathematical methods for digital computers. Edited by Anthony Ralston and Herbert S. Wilf. John Wiley & Sons, Inc., New York, London, 1960. xi + 293 pp. $9.00.

This collection of articles constitutes a handbook for digital computer programmers, giving algorithms for the solution of standard problems which a programmer should be able to translate into any desired code. Each article adheres to a consistent format, giving the purpose of the program, the mathematical background, the calculation procedure, the flow-chart and its description, specification of required subroutines, a sample problem, and estimates of memory and running time requirements. There is, naturally, much variation in the treatment of each of the above topics by the contributors, some giving detailed, some only large scale flow-charts, some more and some less mathematical background. One might wish that the editors had requested from each contributor a FORTRAN or ALGOL program to accompany the flow charts and thus imposed a uniform standard in this respect. But even without this, the book will be greatly welcomed as a pioneering example of a type of work in great demand amongst applied mathematicians with computer orientation.

There are 26 articles, divided into 6 groups. The first article, alone in its group, deals with the generation of elementary functions. In it, E. G. Kogbetliantz summarises his contributions previously published in the IBM Journal of Research and Development; he discusses power series and rational approximation in a somewhat concise and difficult to read paper which, however, merits the attention it demands.

The second group, on matrices and linear equations, consists of six papers. Direct methods in matrix inversion and related topics are discussed by Alex Orden. He gives an omnibus program which (1) finds the inverse if it exists, (2) solves systems of linear equations, (3) finds the value of a determinant, (4) determines rank and (5) locates a nonsingular minor. The matrix has, apparently, to be small enough for each of its elements to be stored in memory, and dense enough for this to be reasonable—there is no discussion of tape-techniques, a criticism applicable to most of the matrix papers in the volume. In the second paper, R. Van Norton describes a Gauss-Seidel program without special acceleration devices; background theory and convergence criteria are given. Next, F. S. Beckman presents the conjugate gradient method; this iterative process which, unlike Gauss-Seidel, gives the solution after a finite number of steps, was invented by Hestenes and Stiefel in 1953 and in his careful exposition of the algorithm the author shows its accuracy and speed to be high when applied to sparse matrices, but not otherwise. Herbert S. Wilf first published his method of rank annihilation for matrix inversion in the SIAM Journal in 1959; the next paper, by him, is devoted to this method which depends on a formula giving the change of inverse in terms of the change in value of one element. The von Neumann and Ulam Monte Carlo method for matrix inversion is expounded by Florence Jeanne Oswald, with a sample problem. The last paper in this group is by John Greenstadt, on the Jacobi method for the determination of the eigenvalues of a matrix. The recommended version is the "threshold" method of Pope and Tompkins (J. Assoc. Comp. Mach., 1957). The presentation is confined to symmetric matrices, which is a pity since the author and others have recently made progress in the difficult general case; perhaps it is not yet possible to present a general computer algorithm for general non-symmetric matrices— it should be possible, however, if they have a complete set of eigenvectors and can thus be diagonalized.

The next group of four papers is on the numerical solution of ordinary differential equations. First of all, Anthony Ralston gives an informal discussion of multistep methods for initial value problems of first order, including truncation error analysis, variable step-length procedures and stability considerations. Next, Michael J. Romanelli describes Runge-Kutta methods for first order equations, both with and without the Gill modification. To exemplify the numerical solution of boundary-value problems, Eugene L. Wachspress chooses a Sturm-Liouville problem; the tridiagonal system of linear equations arising from the finite difference scheme is solved by Gaussian elimination; round-off error propagation is discussed, convergence and trunction errors are not. Next, J. Certaine is concerned with the solution of ordinary differential equations with large time constants; these are well known to present numerical

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difficulties because standard methods require very small time steps indeed. Alternatively, it is proposed to convert the differential equation into a Volterra integral equation and to use numerical quadrature.

Part IV contains four papers on the numerical solution of partial differential equations. Linear parabolic equations are discussed by Herbert B. Keller; the usual finite difference formulation is given, a maximum principle is derived, solvability established and convergence proved. J. W. Sheldon expounds iterative methods for resolving difference schemes arising in the solution of elliptic equations. Acceleration procedures appropriate to some of the iterative processes in current use are described and the successive overrelaxation method is discussed. Carl N. Klahr next gives a Monte Carlo algorithm for elliptic equations first suggested by J. H. Curtiss. There are two papers on hyperbolic equations, one on each of the two possible approaches: Mary Lister on that via characteristics, P. Fox on that via finite differences. In the first, the mesh is taken in the characteristic, in the second in the original coordinate directions—the latter thus requiring careful stability discussions, based on the work of Lax and Richtmyer.

Part V deals with statistical programs. M. A. Elfroysom presents a most useful step-wise multiple regression program in which each independent variable is added or deleted at each step, in accordance with its contribution to the variance of the dependent variable and with two F values given to the program. This procedure will not lead to a unique model, but is insensitive to linear dependence amongst the data vectors. Harry H. Harman gives a brief account of factor analysis, with emphasis on a modified Jacobi method applied to a reduced correlation matrix. Raymond W. Southworth gives his well-known autocorrelation and spectral analysis program (available through SHARE); the autocovariance function is computed and raw estimates of the spectral density are found which are then smoothed following the ideas of Tukey and Hamming. There is no discussion of aliasing, pre-whitening or trend-removal, nor of alternative spectral windows. Lastly, H. O. Hartley presents his method of programming a general purpose analysis of variance, which he first described in Biometrika in 1956.

The final part is entitled "Miscellaneous Methods" and consists of the following six papers. Herbert S. Wilf treats polynominal equations by a modified Bernoulli method to find a first approximation for the root which is then improved by Bairstow's method. The grave problem engendered by near-multiple roots, requiring multiple precision arithmetic, is mentioned and left there. A good discussion of the theoretical background and a proof of Sturm's theorem are included. Anthony Ralston discusses Gaussian and Newton-Coates methods for numerical quadrature with a careful consideration of the choice of weight-factors; there is no mention of recent work on integration in higher dimensions, but it is unfair to criticise individual contributors for omissions since each was clearly not expected to write an essay on the current state of his subject but to present a working, and so necessarily limited, computer program—and it is this aspect, indeed, which makes the book so exceedingly useful and gives it its place as "first of a kind". Multiple quadrature by Monte Carlo methods is discussed by Herman Kahn, with methods for generating random numbers obeying given probability distributions. In "Fourier Analysis," G. Goertzel evaluates the Fourier coefficients by a simple iterative procedure based on the Euler-Maclaurin expansion. This neat method has the advantage that values of only two trigonometric functions need be stored or calculated. Dean N. Arden, in a concise but comprehensive and lucid exposition of linear programming problems, presents the algorithms of the simplex and of the dual simplex methods, with ample theoretical background. The last chapter, by T. R. Bashkow, entitled Network Analysis describes the steady-state analysis on a computer of a specific class of ladder networks.

A book of this sort should be the first of a series: with each article supplemented by a program in a recognised algebraic language, such a series would form the backbone of every computer center library. The less so can any computer user afford being without the book under review: much programming and problem analysis experience has gone into the making of it and this experience is presented here in a most palatable and attractive form.

WALTER FREIBERGER

BOOK REVIEWS

(Continued from p. 252)

Ludwig Prandtl. Gesammelte Abhandlungen zur angewandten Mechanik, hydro- und Aerodynamik. Edited by W. Tollmien, H. Schlichting, H. Görtler, and F. W. Riegels. Vol. I, xix + 574 pp., Vol. II, xiii + 495 pp., Vol. III, xiii + 550 pp. Springer-Verlag, Berlin, 1961. $68.96 (paperbound), $72.75 (clothbound).

This publication of the collected works of Ludwig Prandtl has been sponsored by the Max-Planck-Gesellschaft zur Förderung der Wissenschaften, the Wissenchaftliche Gesellschaft für Luftfahrt, and the Gesellschaft für Angewandte Mathematik und Mechanik. In addition to an excellent portrait of Prandtl, a Preface by A. Betz and Th. v. Kármán, an Introduction by W. Tollmien, and a chronological list of publications, the first volume contains contributions to elasticity, plasticity, and rheology, as well as wing and propeller theory. The second volume is devoted to papers on boundary layers and drag, turbulence, creation of vorticity, and gasdynamics. The third volume contains papers on meteorological applications, model experiments, and miscellaneous subjects, as well as lists of publications that have not been reproduced in the present work, of dissertations written with Prandtl's guidance, and of biographical data and awards.

Computing methods and the phase problem in X-ray crystal analysis. Edited by Ray Pepinsky, J. M. Robertson and J. C. Speakman. Pergamon Press, New York, Oxford, London, Paris, 1961. viii + 326 pp. $9.00.

This is a collection of 28 papers and ensuing discussion presented at a conference held at Glasgow in August, 1960. A similar volume under the same title was issued in 1952 by the first-named editor; the present work is, in a sense, volume II. The international group of authors of the papers include a large fraction of those active in more extensive crystallographic computations and in general attacks on the phase problem.

Somewhat more than half of the book is devoted to the strategy of crystallographic calculations on a variety of computers. The calculations include evaluation of three-dimensional Fourier summations, calculation of structure factors, and refinement of trial structures by least-squares and other methods. Computer programs are described in general terms so that the methods may be widely applied. This, plus description of much practical experience, should make this book extremely valuable to someone who wishes to program similar calculations on a new computer.

What is called the "phase problem" in crystal analysis arises from the fact that the experimental method gives the magnitudes, but not the phases, of Fourier coefficients needed to work out the structure of a crystal. Methods of solution range from trial-and-error to highly sophisticated statistical approaches. This volume constitutes a progress report on new developments in this field, which seem to be less numerous and less optimistic than those of five years ago.

This up-to-date and authoritative book is recommended to crystallographers and to computer experts who deal with them.

G. B. CARPENTER

Elements of mathematical statistics. By Howard W. Alexander. John Wiley & Sons, Inc., New York, London, 1961. xi + 367 pp. $7.95.

This book is intended as an introduction to probability and mathematical statistics for students who have completed a year of calculus. A feature of the presentation is that even at this elementary level probability theory is erected on a foundation of set theory. This gives the student the proper orientation for more advanced work.

WALTER FREIBERGER