—BOOK REVIEW SECTION—

The Monte Carlo method—the method of statistical trials. By N. P. Buslenko, D. I. Golenko, Yu. A. Shreider, I. M. Sobol' and V. G. Sragovich. Edited by Yu. A. Shreider. Translated from the Russian by G. J. Tee and translation edited by D. M. Parkyn. Pergamon Press, Oxford, London, Edinburgh, New York, Paris, Frankfurt, 1966. xii + 381 pp. $12.50.

The Monte Carlo method began as the numerical simulation of sampling from given populations in order to estimate their statistical parameters. As it is now understood, it consists of representing the solution of a given problem as a statistical parameter of a hypothetical population, and using a sequence of numbers, which can, for the purpose of the computation, be considered random, to construct a sample of the population, from which statistical estimates of the parameter are then obtained in a prescribed manner. The method came into extensive use during the Second World War, when scientists faced with urgent physical problems, whose solution by classical methods was prohibitively laborious, were able to use the newly-constructed digital computing machines to simulate the stochastic processes involved. Early ideas were given by Lord Rayleigh (1899), 'Student' (1908), Courant, Friedrichs, and Lewy (1928), Kolmogorov (1931), and Polyà (1938); but the method was given a name and a firm place in the arsenal of numerical methods by John von Neumann, Stanislaw Ulam, Nicholas Metropolis, Herman Kahn, and Enrico Fermi in the 1940's. Since that time, the method has seen ever-widening applications, and a number of new techniques and refinements have been introduced. Much of this work was done in government laboratories and was communicated only by way of reports with limited circulation, or informally. Nevertheless, some 2000 papers and unclassified reports have now been published on various aspects of the method.

There are only very few books on the subject, and every addition to the literature is therefore welcome. A number of conferences have been held, to discuss the development of techniques and applications of the method; notably by the National Bureau of Standards, the I. B. M. Corporation, the RAND Corporation, and the Royal Statistical Society. Papers and a bibliography presented at a Symposium held at the University of Florida, in Gainesville, in 1954, were edited by H. A. Meyer and published by John Wiley & Sons in 1956, as the first generally available book, [1], on the Monto Carlo method. The next notable publications on the subject were both in Russian: Buslenko and Shreider published a text, [2], in 1961, and this was shortly followed by a more comprehensive book, [3], edited by Shreider and written by Buslenko, Golenko, Shreider, Sobol', and Sragovich. This latter text, published in 1962, was twice translated into English: the first translation, [5], by Scripta Technica, was published by the Elsevier Publishing Company in 1964; the second, [6], by G. J. Tee and D. M. Parkyn, was published by the Pergamon Press in 1966. Meanwhile, J. M. Hammersley and D. C. Handscomb published their Methuen Monograph [4] in 1964. As far as the reviewer is aware, this completes the list of books available at present to students of the Monte Carlo method; though at least three other books are now in preparation.

In spite of this paucity of literature (amounting essentially to the three books [1, 3, 4] mentioned above), the student of the Monte Carlo method is very fortunate. Much of the Florida Symposium is still relevant, though now a little old-fashioned; and both Shreider's book and Hammersley–Handscomb are valuable introductions to the subject. Both books cover essentially the same material; though such topics as regression methods, the orthonormal-function method of Ermakov, Zolotukhin, and Handscomb, conditional Monte Carlo, sequential Monte Carlo, integral equations, eigenvalue and eigenfunction determination, interpolation, and the solution of problems in statistical mechanics, long polymers, and percolation appear in [4] but not in [3]; while Wiener integrals, nuclear cascades, queuing and service problems, information theory, and the construction of electronic random-signal generators are treated in [3] but not in [4]. The authors of both books naturally tend to emphasise the subjects in which they and their collaborators are most active, and to approach the subject from their own points of view: in [3] the reader will find considerably more detail of a practical nature (physics, computer programming, electronic engineering: how to implement the methods in practice), while [4] is more concerned with clearly defining the underlying mathematical ideas—though both books give consideration to both of these aspects of the Monte Carlo method. Shreider's book is nearly three times as long as Hammersley–Handscomb; but much of the difference is made up by the latter's greater economy and elegance of language, and omission of relatively unimportant detail, without loss of clarity.

Turning to the relative merits of the two translations [5] and [6], the reviewer did not find much difference of style, though it is hard to find a sentence which both translators handled in the same way. Neither is a literary joy; but both books are fairly easy to follow, and the reviewer did not detect any misleading statements, in a quick perusal; though both books are liberally peppered with misprints. The later version [6] has two distinct advantages over its earlier rival [5]. The paper and typefaces are clearer and more pleasing, and—most important—the authors were given the opportunity to revise their text, and made several additions, which are enumerated in a new Preface. The sections on the solution of linear systems of equations and on the solution of boundary-value problems by random-walk processes have been expanded; and material on eigenvalue problems, and on the practical aspects of simulation of Monte Carlo processes on electronic computers, has been incorporated in the text from the earlier book [2]; while an entirely new section on the simulation of certain classes of random processes has been added to the chapter on the generation of random variables with arbitrary probability-distributions.

The book in its later version [6] has the following contents: Chapter I—*Principles of the Monte Carlo Method* (definition, simple examples, accuracy, generation of random sequences, solution of linear systems, random walks and boundary-value problems, simulation of Markov processes, eigenvalue problems, specialized machines for Monte Carlo); Chapter II—*Computation of Definite Integrals* (simple examples, variance-reducing techniques, computation of multi-dimensional and Wiener integrals, quasi-random point sequences for integration); Chapter III—*Application of the Monte Carlo Method to Neutron Physics* (elementary-particle problems and cascades, diffusion of neutrons through matter with reactions, transmission through plates, criticality of nuclear reactors); Chapter IV—*Application of the Monte Carlo Method to the Investigation of Servicing Processes* (introduction, the arrival flow of requests, servicing systems, simulation of random arrival flow, algorithm for solving servicing (queuing) problems); Chapter V—*Applications of the Monte Carlo Method to Communication Theory* (statistical properties of signals and noise, the fundamental problem in the theory of detection, its Monte Carlo solution, and other problems); Chapter VI—*Generation of Uniformly Distributed Random Variables on Electronic Computers* (comparison of various methods, generation of pseudo-random numbers on various machines, tests of the quality of uniform pseudo-random sequences, physical generation of random sequences, tests of the correct behaviour of such generators); and Chapter VII—*Transformation of Random Numbers* (properties of quasi-uniform variables, simulation of independent random events, behaviour when random numbers of few digits are used, techniques for generating random variables with arbitrary distributions, simulation of random vectors and random functions, simulation of certain multi-dimensional variables, simulation of certain random processes.)

In conclusion, the reviewer would say that, while the book by Hammersley and Handscomb [4] is an excellent text, and while the earlier translation [5] of Shreider's book [3] is quite acceptable, the present new translation [6], with added material, is preferable to [5] and is a valuable addition to the rather small literature of the Monte Carlo method. It gives a sound and rather extensive survey of the method and of its applications to a variety of problems, giving adequate mathematical formulation and a great deal of practical detail for the potential user of this important and relatively new computation tool.

Bibliography

[1] H. A. Meyer, editor. *Symposium on Monte Carlo Methods* (Wiley, New York, 1956)

[2] N. P. Buslenko, Yu. A. Shreider. *The Method of Statistical Trials (Monte Carlo) and Its Realization on Digital Machines* (Fizmatgiz, Moscow, 1961) [In Russian]

[3] Yu. A. Shreider, editor: N. P. Buslenko, D. I. Golenko, Yu. A. Shreider, I. M. Sobol', V. G. Sragovich. *The Method of Statistical Trials—the Monte Carlo Method* (Mathematical Reference Library, No. 5) (Fizmatgiz, Moscow, 1962) [In Russian]

[4] J. M. Hammersley, D. C. Handscomb. *Monte Carlo Methods* (Methuen Monographs on Applied Probability and Statistics) (Methuen, London, 1964)

[5] Yu. A. Shreider, editor. *Method of Statistical Testing—Monte Carlo Method.* Translated from [3] by SCRIPTA TECHNICA, Inc., Washington, D. C. (Elsevier, Amsterdam, 1964)

[6] Yu. A. Shreider, editor. *The Monte Carlo Method—The Method of Statistical Trials.* Translated from [3] by G. J. Tee; translation edited by D. M. Parkyn (Pergamon International Series of Monographs in Pure and Applied Mathematics, Vol. 87) (Pergamon Press, Oxford, 1966) (see heading of this review article).

John H. Halton (*Madison, Wis.*)

The statistical analysis of series of events. By D. R. Cox and P. A. W. Lewis. Methuen & Co., Ltd., London, and John Wiley & Sons, Inc., New York, 1966. viii + 285 pp. $7.75.

The study of random series of point events is rapidly emerging as a branch of stochastic processes with a theory and style of thought of its own, and applications in a remarkable variety of fields. As the first book specifically to treat such "point processes", this monograph (a larger-than-usual member of the Methuen series of Monographs on Applied Probability and Statistics) is particularly timely and important.

In it, the authors set themselves the difficult task of both describing the outlines of the theory of stochastic point processes, and embedding it in a discussion of appropriate statistical tests and procedures. Chapter headings are: introduction; the Poisson process; analysis of trends; stationary point processes; estimation of second order properties; renewal processes; superposition of processes; comparison of rates of occurrence; some generalizations.

This list gives an indication of the wide range of material covered. The authors have been successful throughout in avoiding unnecessary elaboration and concentrating the reader's attention on real problems and points of interest. Besides giving an unusually vivid insight into the way basic statistical problems are born, their book is a valuable reminder of just how much work remains to be done even after the development of a mathematical model. It is probably no exaggeration to say that every separate chapter leaves one with the feeling that here is a rich and interesting field crammed full with tempting unsolved problems.

From this point of view, the book is a most original and successful attempt to treat an area lying between probability and statistics which is all too often overlooked. As a survey of a new field in stochastic process theory, however, I feel it is less successful. An almost inevitable consequence of the authors' approach is that the theoretical material has to be broken up between chapters, so that one is left with a final impression of an (albeit fascinating) compendium of bits and pieces of stochastic process theory, mathematical statistics, statistical tests, practical dodges, unsolved problems and worked examples. Even apart from this there are some notable omissions—for example there is no attempt to define precisely what is meant by a stochastic point process; characteristic functionals are not mentioned; nor is the work of Khinchin and his successors on the theoretical structure of input and output streams. At the same time there is probably enough theory to deter the non-mathematician who wants a layman's guide to new statistical procedures. The level of treatment is also very uneven, although this is largely a reflection of the uneven development of different parts of the subject. To cite one example, the discussion of trends is limited essentially to exponential trends for a Poisson process and linear trends for renewal processes, and appears to be one area where more work is urgently required.

In sum, its shortcomings do not prevent this from being a valuable, original and stimulating book; it is recommended to anyone interested in stochastic processes and their applications.

D. Vere-Jones (*Canberra, Australia*)

Random process simulation and measurement. By Granino A. Korn. McGraw-Hill Book Co., New York, 1966. 234 pp. $12.50.

The publication of this book marks a significant and important event in the area of analog and hybrid computations. As digital computers have become more rapid and more economic in recent years, more and more application areas heretofore reserved for treatment by analog computers have been taken over by digital computers and simulators. Although in the foreseeable future analog and hybrid computers will continue to play an important role in real-time simulations and in computer studies involving inter-connections of computers and system hardware, it is clear that their role in "classical" system studies will continue to become more and more circumscribed. The present text opens up an entirely new and vitally important application area, an area in which the unique characteristics of high-speed analog and hybrid devices assure them of a dominant role for a long, long time. The appearance of this text can therefore be expected to breathe new life into the analog computer field, and at the same time to introduce engineers to a novel and useful technique of system analysis.

The importance of statistical methods in such engineering areas as control system design, communication and information theory, the treatment of partial differential equations, etc., has become well-established in recent years. Formidable difficulties are involved in the determination of distribution parameters and in dealing with random excitations and parameter fluctuations; and neither conventional analog or conventional digital techniques are capable of coping with them in an effective manner. It is the purpose of the present text to survey modern techniques for utilizing analog, digital and hybrid computers for random process studies. Since much pioneering work in the hybrid area was done by Professor Korn and his students at the University of Arizona, this volume also represents a collection of material previously scattered over many publications and research reports. In addition much previously unpublished material is included in the text.

The book opens with a brief review of the mathematical description of random processes, followed by a concise survey of the role of digital, analog and hybrid computers in system simulation. Computer techniques for implementing statistical input-output relations are then described in some detail with particular emphasis on time-varying linear systems. Chapter IV, one of the most interesting chapters in the book, covers the direct simulation of random phenomena and includes an introduction to Monte Carlo techniques and random noise generators. The measurement of time and sample averages, correlation functions, and amplitude distribution is next described. A separate chapter is devoted to statistical measurements of quantized data, including a discussion of Widrow's quantizing theorem and an interesting evaluation of the use of either. In Chapter VII the measurement of spectral densities, Fourier components and system response is considered. The final chapter of the text is devoted to a discussion of special techniques for implementing hybrid concepts. Here Professor Korn's ASTRAC hybrid system is described, and an impressive variety of important applications is enumerated.

The mathematical level of the text as well as the remarkable clarity of the exposition make it well suited for a senior-level or a first year graduate-level course in an electrical engineering curriculum. Such a course will provide students with an excellent perspective on an important area of analog, digital and hybrid computations. Professor Korn whose text "Electronic Analog Computers" in 1952 proved to be the first and most important text in the analog computer field is to be congratulated in having scored another "first."

W. J. KARPLUS (*Los Angeles, Calif.*)

Introduction to numerical analysis. By Gerard P. Weeg and Georgia B. Reed. Blaisdell Publishing Co. (A Division of Ginn and Co.), Waltham, Mass., Toronto, London, 1966. vii + 184 pp. $7.50.

The intention of the authors in writing this book was to present a few important topics in numerical approximations, at a level of difficulty such that the material could be taught to second-term sophomore or first-term junior students in science or engineering. Prerequisite courses include calculus and an introduction to differential equations. Also, some familiarity with matrix algebra is assumed.

In a small book such as this, it is, of course, possible to cover only a part of the very broad field of numerical analysis and to discuss in detail only a few of the many techniques that have been developed within even a single topic. The authors have made their selection of topics with the idea of emphasizing those that are widely used and that are also representative of large classes of techniques. Thus, although the selection is necessarily limited, the book should serve as a good introduction to numerical methods.

The book begins with a chapter on errors in numerical computation, which is followed by a chapter on methods for calculating real roots of an equation, with the discussion here limited primarily to the case of real roots of polynomials. The next sections are devoted to polynomial approximations and finite differences, numerical integration, and numerical solution of ordinary differential equations. Only the standard methods of Milne, Adams, Bashforth, and Runge–Kutta are covered. The book concludes with chapters on linear algebraic equations, least-squares polynomial approximation, and Gaussian quadrature. In the final chapter there is a good discussion of the use of Legendre, Laguerre, Hermite, and Chebyshev polynomials in the Gaussian quadrature.

In summary, the authors have not developed new mathematics or new methods in this book; instead they have chosen to discuss certain important topics in numerical analysis, and they have included a large number of illustrative examples and problems to be assigned. The presentations are clear, and the book should prove useful in the kind of course for which it was intended.

RAYMOND W. SOUTHWORTH (*Williamsburg, Va.*)

Techniques de Calcul Numérique. By Henri Mineur. Dunod, Paris, 1966. XVII + 605 pp. $19.86.

This text is primarily concerned with classical methods of numerical analysis in the field of functions of one independent variable. There is no discussion of the relative merits of these methods in automatic computation, and several methods (e.g. Romberg quadrature) that have proved particularly efficient in automatic computation are not presented at all. Except for these shortcomings the coverage is adequate, and the presentation is clear and easy to follow. There are six chapters that are primarily concerned with interpolation, three on the general theory, and one each on inverse interpolation, operations in numerical tables, and interpolation of functions of two variables. There are four chapters on numerical integration and one on numerical differentiation, two on the solution of algebraic equations, two on special polynomials (Legendre, Bernoulli), one on the numerical integration of ordinary differential equations, and one on smoothing.

W. PRAGER (*La Jolla, Calif.*)

Real and complex analysis. By Walter Rudin. McGraw-Hill Book Co., New York, St. Louis, San Francisco, Toronto, London, Sydney, 1966. xi + 412 pp. $14.00.

This is a very rich book. Starting with material which is normally covered in the two introductory graduate courses "Real Variables" and "Complex Variables," the author within 400 pages reaches many important and beautiful results in classical analysis. Thus, for instance, he develops the basic properties of the Fourier Transform on the line, including the Plancherel theorem, and later on he presents the Paley-Wiener theorems on holomorphic L^2-functions as well as the Denjoy-Carleman theorem on quasi-analytic classes. He proves the Müntz-Szasz theorem as an application of results on the zeros of analytic functions and of the Hahn-Banach theorem. Having proved a convexity property of the maximum modulus of an analytic function in a strip, he applies this result to obtain the Hausdorff-Young generalization of Bessel's inequality to L^p. He develops the basic theory of commutative Banach algebras, (itself an application of elementary complex analysis), and uses it to deduce a number of consequences in classical analysis, including Wiener's Lemma on the reciprocal of an absolutely convergent Fourier series. The theory of H^p-spaces for the disk is treated in detail.

Approximately the first half of the book is devoted to the fundamental results on measure and integration, including the study of complex measures (needed for later applications), the Riesz representation theorems for positive and bounded linear functionals, and the differentiation of Borel measures on R^k. A brief discussion of Hilbert and Banach spaces is included here, with application to Fourier series.

It appears to this reviewer that the author has succeeded in selecting the most essential results (from the point of view of application to analysis) of each of the general theories he discusses. He presents these results simply and directly and refrains from drowning the reader in a flood of definitions and theorems, and he usually shows the relevance of a general result to interesting concrete problems.

Above all, he treats analysis as a connected whole in which many different techniques occur in combination. Happily, we do not find here a separation of mathematics into watertight compartments in each of which sits a specialist glaring suspiciously at the specialists in the other compartments.

The standard elementary topics in complex analysis are covered, including the Riemann mapping theorem, with a discussion of the boundary behavior of the Riemann map. Conformal mapping of multiply connected domains is discussed only for the annulus and Riemann surfaces are represented only by the Riemann sphere. Clearly you cannot have everything. On the other hand, Runge's approximation theorems are established, and used to prove the general Cauchy integral theorem. In the last chapter Mergelyan's theorem on polynomial approximation on compact sets in the complex plane is proved by a variant of Mergelyan's original method.

The book has a wealth of good exercises and it is printed in a very pleasant way. In sum, this book is a goldmine and will richly reward the student who is able and willing to do the necessary mining.

JOHN WERMER (*Providence, R. I.*)

Algèbres de Lie semi-simples complexes. By Jean-Pierre Serre. W. A. Benjamin, Inc., New York, Amsterdam, 1966. 134 pp. $8.00.

These are notes of a short course given by the author at Algeria in May, 1965. They give an exposition of the structure and representation theory of the semisimple complex Lie algebras, with emphasis on its significance for the theory of Lie groups. Although the author starts with initial conditions involving no specialized knowledge in this area, he is evidently addressing students of considerable mathematical maturity. In particular, much of the content of this little book, and many features of the presentation, cannot be fully appreciated without a previous understanding of the fundamental relations between Lie algebras and Lie groups. Remedies are offered in the form of references to a short bibliography, which are carefully positioned at the strategic points of the text.

The first two chapters give the most important notions and results from general Lie algebra theory. No proofs are given in these short introductory chapters, but the arrangement strongly suggests the proper order of events for an efficient construction of the foundations of the theory. The level attained here is exemplified by the Cartan-Killing criterion for the solvability of a Lie algebra, and by the result that the finite-dimensional representations of the semisimple Lie algebras are fully reducible.

Chapter III establishes the main facts concerning the most important tool for what follows, namely the Cartan subalgebras of the semisimple complex Lie algebras. To an extent sufficient for most purposes, the real case is included here, via complexification.

Chapter IV deals with the unique semisimple complex Lie algebra of the lowest possible dimension three. This Lie algebra, $sl(2, C)$ is the building block for all the semisimple complex Lie algebras. Its finite-dimensional representations are here determined completely in a few pages, thus paving the way to the general results of the later chapters.

Chapter V contains the geometric theory of root systems and describes their classification completely. The detailed calculations needed for this are not given, but the principles of the classification procedure are indicated. The correspondence between the semisimple Lie algebras and the root systems is established in chapter VI, so that the classification of the former is achieved. This includes, in an appendix, the explicit construction (due to Chevalley and Harish-Chandra) of the Lie algebra belonging to a given root system.

The representations of the semisimple complex Lie algebras are dealt with in chapter VII. It is shown that the isomorphism classes of the "primitively generated" modules are in bijective correspondence with the linear functions on a given Cartan subalgebra of the Lie algebra. In particular, the isomorphism classes of the finite-dimensional irreducible modules correspond to the linear functions that take non-negative integral values at the dual roots. The chapter then goes on to describe, without proofs, the character theory of H. Weyl.

The final chapter VIII, which contains no proofs, gives a survey of some of the most important results concerning the semisimple complex analytic groups and the compact real analytic groups, such as the main properties of the Cartan and the Borel subgroups of the semisimple complex analytic groups, and the determination of the centers of the simply connected simple complex analytic groups.

This short text of casual appearance goes far beyond the standards one would normally impose on a reproduction of lecture notes. The exposition is perfectly balanced in coverage and emphasis, and it proceeds on the highest level of technical precision and elegance. It provides inspiring guidance for the student who wishes to enter into Lie group theory, and it illuminates a beautiful mathematical landscape for the expert.

G. HOCHSCHILD (*Berkeley, Calif.*)

Dynamics of nonhomogeneous fluids. By Chia-Shun Yih. The Macmillan Co., New York, and Collier-Macmillan Ltd., London, 1965. xiii + 306 pp. $11.95.

Before entering upon the inevitable discussion of a book's weaknesses some emphatic words of praise are due in this case. It is all too easy to criticise the choice of material and emphasis given to a subject which different people come upon in very different ways; but there is no best way for topics currently discussed by researchers which do not form part of a recognised syllabus but are only, at long last, working in that direction, unless it be the author's own way, for then at least we savour his manner of thought, his motives, his interests. This is a very likeable book, and it will certainly be very useful and stimulating.

Nonhomogeneous fluids do not naturally form a section of fluid mechanics on their own. Pride of place after an introductory chapter naturally goes in Professor Yih's selection to the calculation of large amplitude disturbances in stratified fluids: this alone could form a useful monograph and it is where the author's own contribution is considerable.

In chapter 4, which (alone) has quotations (in French and Chinese— the latter being very long) to prove that even literary gentlemen have noticed the wind ruffling the water, hydrodynamic stability is discussed through an odd collection of problems treated with varying degrees of thoroughness. One cannot require completeness and I think that an author is absolutely justified in writing about what he finds interesting: the result is not in the least dull and there will be something new for almost everyone. Such obscurities as there are must stem from a difference of approach: quite evidently it is all plain to Professor Yih.

Chapter 5 concerns porous media. This is probably one of the most useful and thorough treatments available for mathematicians because it is almost rigorous and not spoiled by engineering jargon and reference to rules of practice. However, many of the results are really much more obvious than they are made to seem.

Typically Professor Yih treats the mathematics as a kind of magic which reveals beautiful and mysterious results; but it can equally be regarded as a means to establish with quantitative rigour what has been already perceived. Thus he is content with Howard's very neat proof of Miles's result that a horizontally stratified inviscid fluid stream is not unstable if the Richardson number is nowhere less than $\frac{1}{4}$. But this is a useless result; it is a local criterion whose derivation is dressed up in the language of eigenvalues, and it adds to the illusion among many practical workers that there ought to be a unique local Richardson number critical for instability. In practical problems other factors always intervene to produce more non-dimensional numbers.

The author's contentment with the verdict of such mathematics as is complete makes his references to geophysical problems often rather naive. In meteorology at any rate the obtaining of a neat solution is usually only one third of the job and one has to discuss the phenomena in the light also of all the mechanisms not properly represented. In discussing density currents (which he fussily calls 'gravity' currents on the grounds that density differences alone are not enough, and in reply to which pedantry it can be remarked that the currents might occur in a spin drier) he refers to the dust picked up by a cold squall and notes that it might make the cold air even heavier: but equally the absorption of sunshine by the dust makes the air appreciably warmer. His treatment of gravity waves is almost entirely two dimensional and he makes no remarks about the limitations of such treatment and harps a little too much on the fact that the Boussinesq approximation is not valid for airstreams of great depth (due to the omission of the factor $\exp \frac{1}{2} \beta z$), without offering more than a few of the many thoughts which arise from these issues.

The diagrams are on the whole good, and most of the inadequacies arise because he has taken figures unchanged from original papers (mostly his own) without adjusting the notation. Thus in fig. 39 unexplained symbols appear, in fig. 13b the 'dashed line' mentioned in the legend is simply not there, and in fig. 37 the nose of the cold squall is marked as being inclined at $\pi/3$ to the horizontal although this is explained nowhere.

There are a few 'misprints' although their persistence makes one suspect that the printer is not to blame. For example, on p. 130 equations (140), (141), and (142) all lack a factor h^2. The notation is not always happy because it is not economical. Thus on p. 211–2 we have two sets of axes defined where one will do and a set of direction cosines defined and then made redundant by the naming of the components of a vector: 6 symbols quite unneeded. The analysis in the following two pages could be shortened and clarified by the use of vectors or tensors.

There is an interesting feature of the mode of argument at this point: a certain body of fluid is "assumed" to move as a solid, which assumption is "verified" *a posteriori*. Actually, from the equations it is obvious (at least to anyone with experience in examination problems in dielectrics) why the unique solution will be of the form supposed, and it would have been appropriate to draw attention to the applicability of the uniqueness theorem to these problems on porous media. But while Professor Yih likes to leave us gaping in wonder at the mysterious beauty of mathematical analysis I prefer to gape at the wonders of nature and put the mathematics in its proper place with our other technologies. He seems, therefore, to expend his rigour in deriving the obvious and to miss remarking how the mathematics merely helps one to see the obvious in the first place.

The index is very disappointing. It does not contain 'waves,' 'instability,' 'salinity,' and a host of other topics which the book is about, but carefully lists the occurrence of all proper names—good, no doubt, for the egos of those mentioned, but quite unhelpful to the reader seeking new knowledge. Some entries are silly: thus we find, indexed without qualification, "System I" and "System II," which are merely names given for convenience to the differential equations and boundary conditions for disturbances in two arbitrarily chosen examples of stratified streams. We find "Stratified flow into a sink" indexed but no entries covering any of the many other behaviours of stratified currents.

There is a long and pretty comprehensive list of references which contains many more papers than are referred to in the text, which fact, after mention of the above annoying points gives me excuse to say that any book must, nowadays, make a personal selection from a vast collection of topics. Professor Yih's choice is a stimulating one: it is useful: his personal manner of writing is likeable, and the magnitude of his own contribution is considerable. Actually I have always said that a few mistakes can do a student's morale a world of good: which of us has not wilted at some time at the awful correctness of Lamb?

R. S. Scorer (London, England)

Perturbation methods in fluid mechanics. By Milton Van Dyke. Academic Press, New York and London, 1964. x + 229 pp. $7.00.

Two of the most important methods for singular perturbation problems were developed primarily for use in fluid mechanics. They are the method of "matched asymptotic expansions," which is used to obtain asymptotic representations of solutions of differential equations having radically different dependence on a parameter in different regions, and the method of "strained coordinates," which is used to obtain asymptotic solutions near their singularities. This is the first, and so far the only book devoted to these methods. It is mainly a collection of singular perturbation problems of wide variety, many of which led to developments in the methods. It has a clear discursive style and the author's wealth of experience is apparent in the comments and hints that accompany the solutions of the problems, whose common features and contrasts are well described.

Some regular perturbation problems are given, and solutions for flow past thin aerofoils, weak waves in steady supersonic flow, viscous flow past semi-infinite and finite flat plates at high Reynolds' number, and viscous flow at low Reynolds' number past a sphere and a circle each occupy one chapter. One chapter contains assorted examples of singular perturbation problems in inviscid flow.

Asymptotic expansions and limit processes are defined and discussed but there is no attempt to give a sound basis for the "principles" which are stated in large numbers. The emphasis is on technique rather than on general theory. The reader seeking a deep understanding of the methods would need to study the original papers referred to in the book, but the variety of problems and the ingenious exercises given at the end of each chapter would be sufficient for him to be able to make use of the methods.

A. M. Watts (Brisbane, Australia)

Theoretische Strömungslehre. By Karl Wieghardt. B. G. Teubner Verlagsgesellschaft, Stuttgart, 1965. 226 pp. $9.03.

Centuries ago fluid dynamics (which may include gasdynamics, according to present terminology) was considered to be a part of physics. However, the knowledge of fluid dynamics proved to be so vital to the development of engineering and the build-up of our present day civilization that it was considered more and more a part of engineering. The sad consequence was that the physics departments in universities spent very little time on course work in this topic and left it to those departments of the schools of Engineering which were vitally interested in it. In this country this does not cause a serious difficulty for young physicists who want to get a deeper knowledge of fluid and gasdynamics—they just take some courses in Aeronautics, Engineering Mechanics, or Mechanical Engineering. The situation is quite different in Germany where universities, in general, have no School of Engineering, since these schools, with some additional general studies programs, are geographically separated from the Universities. Therefore, in Germany if the young physicist in a general University should desire to become acquainted with the vast field of fluid dynamics, he has a difficult time doing so. This is a definite disadvantage at present, when technical progress crosses long established boundaries. Professor K. Wieghardt is teaching at the University of Hamburg and is lecturing, in addition, at the Technical University of Hannover. He is very familiar with the above described problem. Therefore, he has written an introduction to Fluid Dynamics which should help the young physicist to get acquainted with this field. The book, with its 226 pages, can certainly only pave the way for further study, if wanted.

The essentials, theory of incompressible, compressible and viscous flows, are presented and much more use is made of mathematics than in Prandtl's *Führer durch die strömungslehre* (translation available: Essentials of Fluid Dynamics). Sometimes, one cannot avoid seeing how examples are chosen to demonstrate clearly a situation without really handling the problem in full generality. But, if an introduction into a field of science does not develop in a handbook, this probably cannot be avoided. Occasionally, as in the discussion of the Kutta-Joukowski Theorem, the student may wonder about the figure showing the finite wing sitting in a parallel flow while the mathematical treatment is concentrated on the wing of infinite span, thus avoiding the difficulties caused by the trailing vortices (the finite wing is considered 60 pages later!). The reviewer feels that the author's deep knowledge of the fluid mechanics field has created a book which will be a valuable introduction for an outsider to this field.

I. FLÜGGE-LOTZ (*Stanford, Calif.*)

The theory of road traffic flow. By Winifred D. Ashton. Methuen & Co. Ltd., London, and John Wiley & Sons, Inc., New York, 1966. viii + 178 pp. $5.00.

This is a brief introduction to recent developments in traffic flow theory. Compared with the only other text book on this subject, Frank Haight, "Mathematical Theories of Traffic Flow", Academic Press 1963, this book is less mathematical, broader in scope, less detailed, and more up-to-date. It is the only book suitable as an introductory text and therefore partially fills a serious need. It is quite shallow, however. The author seems to have read a large collection of papers, tried to extract the essence of them, and put it all together without much discrimination or real insight. The book does, however, make some comments on nearly all topics of traffic theory except those relating to flow of networks.

GORDON F. NEWELL (*Richmond, Calif.*)

Technological change: its conception and measurement. By Lester B. Lave. Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966. xx + 228 pp. $6.95.

The study and analysis of technological change is quite a recent phenomenon in economics. Models of the firm and of the economy used to assume that the state of technological knowledge and usage remained constant. Technological change (TC) was not one of the variables in the models. Along with this lack of concern about TC went a relative lack of interest in the problem of fostering and sustaining the growth rate of economies.

All this has been changed since the end of World War II. One of the major economic issues of the last twenty years has been economic growth, the interest arising from both the East-West cold war rivalries and the growing disparity between the developed and underdeveloped nations. As the growth process was studied, it became apparent during the middle 1950's that much of the observed growth in the developed countries could not be accounted for using the usual economic analysis. There must have been some forces operating related to TC which were of prime importance in the growth process. From about 1956 on, a fair amount of work has been done to estimate empirically the contribution of something called TC and to incorporate TC in theoretical models representing growing economies.

Lave's book is an attempt to bring together much of the material produced since 1956. He feels that "the body of this work seemed ill-related, ill-understood, at least by part of the profession and in danger of needless duplication." This is probably true. Unfortunately, his book goes only part-way in correcting this deficiency. It contains descriptions of most of the important methods which have been used to measure TC, along with the empirical results obtained. He includes some of his own work in testing the effects of possible data errors in using the TC estimating methods, and discussions of some of the theoretical and conceptual problems involved. However, two major faults are present. First, understanding by the previously uniformed reader would require a great deal better organization of the material than Lave provides. Second, a more useful discussion of the problem of TC needs a somewhat broader perspective of the issues involved than he is willing to assume.

In a book intended to be primarily expository it is most disconcerting to find an organizational structure which seems to interfere with the reader's obtaining a clear picture of the state of research in the field. The derivations of particular TC estimating methods and the interpretations which go with them are spread out in many places throughout the book. Several bits and pieces must be read before a clear idea of the approach of one worker is obtained. For example, one empirical article by Johansen has six page references in the index, most of which have to be consulted before the reader can grasp the full meaning of Johansen's contribution. Every time the same piece of work is discussed in a new place in the book, there is inevitable repetition of background statements as to the man's assumptions, methods, data, etc. One symptom of the disjointed organization is the constant cross-referencing to later chapters. The discussion is carried only to a certain point and then dropped until a later point. The reader is torn between following the thread of the chapter he is reading or resolving the point being discussed by jumping temporarily to a later chapter. Chapter 5, and to a lesser extent Chapter 7, repeat much of the very elementary discussion adequately covered in Chapter 2. These two chapters were evidently originally written as independent papers; they should have been edited to coordinate better with the rest of the book.

Chapter 10 covers the many aspects of economics which must take TC into account, but is really only a catalog of cases. At this point, a real contribution might have been made by trying to construct a framework into which the diverse studies might be fit. This would facilitate obtaining a picture of the present state of our knowledge and the kinds of gaps still existing. In addition, it might have been helpful to pull together in one place some of the many empirical estimates of TC presented so that comparisons and perhaps some conclusions could be made.

Leaving questions of organization, let us turn to more substantive matters concerning the nature of the problem of measuring TC. Briefly, the general method used by most measurers of TC is this: There is an underlying production function relating inputs to outputs, although it may not be necessary to state the specific form of the function. As inputs change, so will outputs. However, outputs will generally increase more over time than can be accounted for by increases in inputs over time. The unaccounted for increase in outputs, usually called the "residual," is considered to be due to some force loosely called TC. The statistical problems involved in isolating the residual and the economic problems of interpreting what it means are, of course, tremendous, with the difficulties only now becoming somewhat more clear. Some of the problems may turn out to be unsolvable.

Lave's position in the face of these difficulties seems to be a little disappointing, perhaps a bit inconsistent, and somewhat misleading as an indication of the direction which present research is taking. He is rightfully horrified by the enormous assumptions which the simple TC estimating methods must make. Many assumptions must be made about the behavior of the firm, the nature of the product and factor markets, and the types of TC which take place. Many of these assumptions are probably not, in general, true. The natural procedure, and the one followed in most recent studies of this problem, is construct more complex TC estimating procedures which allow some of the stringent assumptions to be relaxed and more of the real-world phenomena to enter the model. Although he never says so explicitly, Lave's feeling seems to be that little progress will be made doing this. He gives little attention to the technical means which recent work has developed to allow the elasticity of substitution, returns to scale, and neutrality characteristics of the TC to go free. The results of studies are noted, but no feeling for the methods used to achieve these advances is given.

In contrast to his uneasiness with the economic assumptions underlying the simple estimating methods, Lave adopts some of the simplest techniques for his relatively extensive empirical work. About 35% of the book is devoted to such empirical work, a rather large amount of work done with measuring devices which he implies may have little economic meaning. On page 34, he seems to reject the whole idea of using aggregate production functions. If the use of these functions is disallowed, numerical estimates of some index can certainly be computed, but it is not likely that we will learn anything about the phenomenon we might want to call TC. The economists will be back in the situation which keeps plaguing certain sectors of the field, measurement without theory. The drive towards more complex estimators is not being done just to confuse the non-experts in the field; it is recognized as a necessity if understanding instead of a collection of empty numbers is to be accumulated.

There are three other areas where the recent interest in TC has had a great impact and to which more attention should be paid if the account of the field is to be balanced. First, virtually all economic growth models now include TC in one form or another. These models should have implications for empirical work, and visa versa. Only three pages are devoted to this area. One outstanding omission is the work of Uzawa, whose articles are listed in the bibliography but not discussed in the text. This is perhaps too short a treatment considering the fact that the original impetus for the study of TC came from problems of economic growth.

Second, the discussion of the sources and diffusion of innovations covers one descriptive study of rubber and the numerical results of Mansfield's studies. The much broader question of the production and distribution of knowledge has not been touched, nor have the subareas of research and development or the several studies which have been done using patent statistics. Third, most workers in the field have recognized that a large part of what we want to call TC evidences itself in quality changes in the outputs or inputs. Meaningful measurements of quality have been most difficult to obtain. It is a little jarring to see Lave comment on perhaps the most successful attempt (by Griliches) at allowing for quality changes as "tantamount to tampering with the data." (p. 18) Progress will come when better "tampering" is possible.

In conclusion, this book is certainly useful as a reference to work in the area of measuring TC. Its bibliography is rather complete. It has not, however, synthesized the diverse work in a very satisfactory manner and pointed the way towards fruitful future research. Part of the difficulty may be due to the present furious but rather unconnected activity in many areas related to TC. A clear picture is rather elusive, and subject to rapid change. Ultimate progress will eventually demand some retrospective and prospective considerations such as attempted in Lave's book.

MARK B. SCHUPACK (*Providence, R. I.*)

Topics in communication theory. By David Middleton. McGraw-Hill Book Co., New York, St. Louis, San Francisco, Toronto, London, 1965. x + 126 pp. $2.95.

In this short monograph the author takes the reader on a carefully guided tour through the area of statistical communication theory, an area which is covered in depth in the author's definitive *Introduction to Statistical Communication Theory.* The aim here is to provide a snapshot of the current status and aspirations of statistical communication theory.

First a general philosophical and mathematical setting for the problem of a single channel providing information to a recipient is given. Essentially the recipient is called upon to make optimal decisions based upon the noisy received signal. Then the general theory is specialized to considerations of signal detection and signal extraction. Typical methods and results are given, and numerous references for further reading are provided. This is followed by a discussion of canonically optimal threshold detection, and shows what may be done to overcome some of the analytical difficulties. The last chapter is a look at some of the future problems, emphasizing adaptive systems, and is a fascinating essay in itself. This is an impressive work which is of interest to both students and experts. Its low price, attractive format and lucid style are added benefits.

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