QUARTERLY

OF

APPLIED MATHEMATICS

This issue is dedicated to the memory of

Rohn Truell

1913-1968

Member of the faculty of Brown University
1946-1968

Chairman, Division of Applied Mathematics
1963-1968

PRELIMINARY ANNOUNCEMENT OF THE
SIXTH NATIONAL CONGRESS OF APPLIED MECHANICS

The Sixth U. S. National Congress of Applied Mechanics will be held at Harvard University during the week of June 15th through June 19th inclusive, in the year 1970. This Congress, held under the auspices of the National Committee for Theoretical and Applied Mechanics, is held once every four years, intermediate between the International Congresses. Further details of the organization of the meeting and a call for papers will be issued at a later date. In the meantime, any correspondence relative to the Sixth National Congress should be addressed to the General Chairman, Howard W. Emmons, Pierce Hall, Harvard University, Cambridge, Massachusetts, 02138.

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in industry or practical science. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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—BOOK REVIEW SECTION—

Dynamic stability of structures. Edited by George Herrmann. Pergamon Press, Oxford, New York, London, Toronto, Sydney, Paris, Edinburgh, Braunschweig, 1967. ix + 313 pp. $16.50.

This book is a collection of 18 papers resulting from a conference at Northwestern University in 1965. These papers cover the several different points of view currently prevalent in dynamic stability theory.

The first paper by N. J. Hoff presents a comprehensive review of the important engineering work in dynamic structural stability.

Four papers (J. M. Hedgepeth, G. F. McDonough, S. R. Heller, Jr. and J. T. Kammerer, and R. L. Goldman) present case histories of structures in which dynamic stability considerations played a major role in design.

A number of the papers are devoted to investigating appropriate analytical criteria for dynamic stability. Papers by J. J. Stoker and J. P. LaSalle represent the stability problem as a control system problem and discuss some of the techniques which are appropriate to structures. B. Budiansky's comprehensive paper compares a number of estimating procedures which are currently in use. E. Mettler approaches the problem as a non-linear vibration problem with harmonic excitation while V. V. Bolotin investigates the statistical aspects of the problem. J. N. Goodier presents and discusses a number of results obtained in dynamic plastic buckling. J. M. T. Thompson presents his views on dynamic buckling in the framework of response under step loading. G. Herrmann and S. Nemat-Nasser cover energy considerations for dynamic stability of nonconservative systems.

The book concludes with several papers in which applications of various techniques to dynamic stability problems are presented. Three of these involve cylindrical shells. D. A. Evensen and R. E. Fulton study nonlinear dynamic response, Y. C. Fung studies the interaction of mechanical and aeroelastic instability and T. H. H. Pian, H. A. Balmer and L. L. Bucciarelli, Jr., investigate a constrained cylinder. The final problem presented by S. T. Ariaratnam is for a column subjected to random loading.

The collection gives a clear picture of the various activities in this rapidly developing field. The papers which include considerable review (Hoff; Stoker; Budiansky; Goodier; Thompson; Fung; and Herrmann and Nemat-Nasser) are the most valuable as they are helpful in providing some perspective relative to this field. Anyone working in either static or dynamic stability of mechanical systems will find this book a valuable addition to his library.

P. R. PASLAY (*Providence, R. I.*)

The theory of jets in an ideal fluid. By M. I. Gurevich. Translated by R. E. Hunt. Pergamon Press, Oxford, London, Edinburgh, New York, Toronto, Paris, Braunschweig, 1966. viii + 412 pp. $14.50.

This book is a general exposition of the theory of flows with free boundaries. It is devoted in larger part to a systematic account of what may be called the classical problems of the theory, namely, the plane flow problems involving polygonal fixed boundaries and constant speed free streamlines. As is well known, these can be handled directly by complex variable methods. They still comprise a major portion of the literature on free boundaries and are clearly presented here with many examples, tabular results, and comparison of theory with observation. Among the broader topics are: jet flows from vessels, cavity flows with zero and non-zero cavitation number (including various models of cavity flow), flows past submerged hydrofoils, and the theory of planing.

The remainder of the work (more than one third of the whole) is devoted to more specialized and less fully developed aspects of the theory, much of it relatively recent. Although the presentation here is often (necessarily) brief, the material is less familiar and is therefore perhaps the more interesting part

of the book. Illustrative topics are: special exact and approximate solutions of the unsteady flow problem; the Chaplygin theory of jets for compressible fluids; particular solutions involving gravity; problems of surface impact. In addition there are numerous special topics, such as the theory of hollow charges. The chapter on axially symmetric flows contains the author's own interesting derivation of the asymptotic shape of the infinite axisymmetric cavity.

On the whole the book is concerned with more concrete, although not necessarily applied, aspects of the mathematical theory. Thus it is to be expected that the more theoretical problems such as existence and uniqueness are discussed only briefly, and the extensive literature is summarized with little detail. Less expected is the absence of the linearized theory of cavity flow, which has been very successful in applications. Numerical methods are represented by some of the older calculations, such as those of Brodetsky on cavity flow past a circular cylinder and of Trefftz on the axially symmetric vena contracta. In light of the increasing importance of numerical methods in these problems and improvements on the older work, a discussion of recent contributions would have been useful.

This book meets successfully the author's stated goal of presenting the general concepts of jet theory and of introducing the reader to more specialized problems of the subject. Of interest to all readers, including the specialist, is the extensive bibliography of Russian literature in this field (up to 1959).

<div align="right">DAVID GILBARG (New York, N. Y.)</div>

Introduction to the mechanics of a continuous medium. By L. I. Sedov. Translated from the Russian by Scripta Technica, Inc., Addison-Wesley Publishing Co., Inc., Reading, Mass., Palo Alto, London, 1965. xvi + 270 pp. $12.59, and *Foundations of the nonlinear mechanics of continua.* By L. I. Sedov. Pergamon Press, Oxford, 1966. 268 pp. $12.50.

These two titles are independent translations of the same Russian book by Professor Sedov, published at Moscow in 1962 under the title closely approximated by the first translation. Both volumes are produced well, and the translations are roughly equivalent, although the reviewer, possibly through his imperfect knowledge of English, finds the second translation slightly easier to read. Anyway it is a pity that so much effort has been wasted by two nearly simultaneous translations having been prepared independently.

Professor Sedov's work is an important contribution to the growing number of advanced texts on continuum mechanics. The first chapter gives a readable account of the elements of tensor analysis in Euclidean space. The second chapter on kinematics is perhaps the most lucid account of this aspect of the theory to be found anywhere. The third chapter on dynamic and thermodynamic equations is necessarily more controversial. Universal agreement on the incorporation of irreversible thermodynamics in continuum mechanics has as yet by no means been achieved. Professor Sedov's exposition nevertheless excels by its clarity, and the emphasis placed on physical concepts. A thorough study of this book is, in reviewer's opinion, indispensible to all research workers in continuum mechanics.

<div align="right">W. T. KOITER (Delft, Holland)</div>

Applications of distributions in mathematical physics. By E. M. DeJager. Mathematisch Centrum, Amsterdam, 1964. 184 pp. $5.00.

This is a useful introduction to the way in which distribution theory may be used in parts of mathematical physics. The first Chapter gives a concise review of the general theory of distributions. The other four Chapters demonstrate its application in four related topics of mathematical physics.

The first two concern the wave equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial t^2}.$$

They deal with the diffraction of a cylindrical pulse by a reflecting half-plane and with the linear theory of supersonic flow around wings. The difficulties which in the classical literature were dealt with by Hadamard's theory of the 'finite part' of an infinite integral are here handled by the more systematic theory of distributions.

Chapter IV refers to the Klein–Gordon equation

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = \frac{\partial^2 f}{\partial t^2} + m^2 f,$$

and deals with the problem of obtaining its Lorentz-invariant solutions. It uses the distribution theory of Fourier transforms to establish these. This is, perhaps, a part of the book which gets bogged down in rather too cumbersome rotation. Finally, Chapter V indicates the way in which the ideas of distribution theory can be used to obtain finite values for many of the divergent convolution integrals occurring in quantum electrodynamics.

<div align="right">M. J. LIGHTHILL (<i>London, England</i>)</div>

Time and space, weight and inertia—a chronogeometrical introduction to Einstein's theory. By A. D. Fokker. Translated by D. Bijl and D. Field. Pergamon Press, Oxford, London, Edinburgh, New York, Paris, Frankfurt, 1965. xv + 188 pp. $10.00.

The most surprising thing about the theory of relativity is its ability to generate discussion and controversy even now, fifty years after its discovery. Most branches of mathematics and physics settle down to a graceful middle age after such a time; only in relativity does one find a distinguished physicist proposing to change even the name of the subject! *Time and Space, Weight and Inertia* is a book on the theory of relativity by A. D. Fokker, but the word "relativity" is mentioned only in the preface. Fokker calls his subject "chrono-geometry" and emphasizes quite correctly that its principal business is the search for invariants. As befits the name, the treatment is uncompromisingly geometric throughout.

The topics covered are the standard ones in special and general relativity, but the point of view is more geometrical, even in those parts of special relativity which are usually done in a completely algebraic manner. Instead of using either real or thought-experiments to derive the Lorentz transformations, Fokker uses a purely geometrical argument, and only later goes on to discuss the Michelson–Morley experiment and the propagation of a spherical light pulse. Then, after covering in great detail the geometric interpretation of the Lorentz transformation, he introduces some of the formalism of tensor analysis, and applies it to the Lorentz electron theory and to accelerated coordinate systems. This last topic leads quite naturally into the problem of geodesics in Riemannian space-time, and finally, in the last chapter, to the Einstein field equations and a discussion of the three classical tests of general relativity. The only problem which is worked out is the Schwarzschild exterior solution, and there is no mention of any developments in the field since 1920! (Except for an appendix on the Pound's measurements of the gravitational red-shift.) There is no mention of such topics as gravitational radiation, the linearized Einstein equations or cosmology which usually appear in texts on this subject.

This reviewer has found Fokker's book to be often eccentric, sometimes confusing, and occasionally absolutely incomprehensible. The notation and terminology adopted is so unusual that a student who attempted to learn the subject from this book would be hard-put to read any of the more conventional literature. For example, Fokker habitually uses the term "mass vector" for "energy-momentum vector", a practice sure to confuse the student who reads in all other books that mass is a scalar. In addition, he has coined several new words of Greek origin to replace the English words used for most of the concepts. An example is "telethigma" (the plural of which, naturally, is "telethigmata") which means "null-ray". This may sound good in Dutch, but it is barbarous in English. Although this is intended as a textbook, it contains no problems.

The date of publication of the book is 1966, but I get the distinct impression that it is a translation of a not very carefully revised work dating from the middle thirties. It contains incredible statements like "We cannot as yet guess that reason for the appearance of two particles instead of one single one" (in electron-position pair production). The reason is now well known and it is ironical that it is a consequence of Lorentz invariance, which is the subject of most of this work.

<div align="right">ROBERT A. SCHWARTZ (<i>Providence, R. I.</i>)</div>

Group theory and its physical applications. By L. M. Falicov. The University of Chicago Press, Chicago and London, 1966. 224 pp. $2.00.

This book consists of the notes from a course given by the author, and provides a well-formulated introduction to applications of group representation theory in quantum mechanics. Except for a brief final discussion of the two- and three-dimensional rotation groups, the author restricts himself to finite groups and develops their basic representation theory entirely in terms of matrices, thereby making this material available in a more elementary, if less elegant, form. Rather, he emphasizes conceptual understanding of how group theory simplifies problems in molecular and solid-state physics, and formulates the general principles used in finding selection rules and the splitting of degeneracies. In particular, he derives the thirty-two crystal point groups, discusses the corresponding space groups, describes the use of Brillouin zones, and gives illustrative applications to crystals, imposing periodic boundary conditions to reduce the translation symmetries to a finite group. Also he introduces and determines the time-reversal operator, and formulates the contribution of time-reversal symmetry to finding degeneracies. The book assumes familiarity with basic quantum mechanics and its terminology, hence is primarily intended for physicists although capable of wider circulation.

JOHN S. LEW *(Providence, R. I.)*

Mathematics in the social sciences, and other essays. By Richard Stone. The Massachusetts Institute of Technology Press, Cambridge, 1966. xiii + 291 pp. $12.00.

Professor Stone, a distinguished econometrician, has gathered together in this volume seventeen of his papers on topics in econometrics and mathematical economics. American colleagues will welcome the volume especially because the papers appeared originally in a large number of different places, some of them obscure, and most of them European.

In economics as in other scientific disciplines, the phrase "applied mathematics" has a whole spectrum of meanings. At the one extreme, it means mathematical invention whose original motivation stems—directly or at several removes—from some problem in a natural science. This is the meaning professional mathematicians usually attach to the term; natural scientists in the field of application do not always accept the relevance (as distinct from elegance) of such invention, sometimes referring to it as "cleanup" or prettying-up" work.

At the other extreme, applied mathematics means invention and discovery in a natural science that employs mathematical reasoning—often of the kind called "quick and dirty." Such work does not usually create new mathematical knowledge; professional mathematicians sometimes do question the appropriateness of the name. Thus, "applied mathematics" may mean a host of things, depending on the relative emphasis on noun or adjective, respectively.

Professor Stone's essays are easy to locate on this continuum, for they have no pretense of contributing to mathematical knowledge. They are essays on economics that happen to use mathematics freely as the language of thought. Most of the mathematics is elementary—calculus, matrix algebra, some simple stochastic processes, linear programming—because elementary mathematics suffices to handle the economic problems under consideration. Other mathematical economists have applied heavier mathematical artillery to some of the topics the author considers—particularly economic development—or have dotted some *i*'s and crossed some *t*'s. It is not clear that they have added very much to our answers to questions of economic policy.

Professor Stone's essays can be classified along several other dimensions. Their concern is with macroeconomics—the operation of a whole economy and its major parts—rather than with the microeconomic detail of the behavior of individual economic agents. His main orientation is toward policy rather than description: policies for economic development, national economic planning, international trade policy. He is concerned also, however, with providing the descriptive numerical base for policy prescription, as evidenced in this volume by papers on social accounts, consumption patterns, and inter-sector flows. Thus, his mathematical models are strongly influenced by his ultimate intention to use them in addressing questions of policy, and to employ statistics to estimate the value of parameters that occur in them. In these directions his work, sampled in this volume, attains very high standards of theoretical and practical intelligence as well as econometric technique.

To understand how numbers are used in these papers, as well as in most other econometric studies,

we must keep in mind the distinction between the scientist's and the engineer's attitudes toward theory. The scientist needs numbers in order to test theories. He wishes his models to be overdetermined, so that if the world is different from his theory of it, the discrepancies will become visible from contradictory estimates of the parameters.

The engineer usually works with tested and accepted theories. He does not expect his data to disprove Hooke's Law; he wants the data in order to *estimate* by the modulus of elasticity. Professor Stone is primarily a social engineer, who wants to design economic policies that will raise incomes in a developing economy or maintain full employment in a developed one.

Econometric statistical methods, exemplified by those used in this volume, stem largely from the "engineering" rather than "scientific" point of view. They are concerned primarily with estimating parameters rather than testing hypotheses. This is shown, for example, by Professor Stone's treatment of the so-called "identification problem," on pages 26–30. The identification problem is the problem of obtaining the proper number of equations and variables for statistical analysis. But if we are estimating parameters for an accepted theory, we want *just enough* equations to use up the number of degrees of freedom we have; while if we are testing theory, we want the values of the variables to be as *overdetermined* as possible. The theory pertaining to identification that has been developed in econometrics addresses itself almost wholly to the estimation problem, not the hypothesis-testing problem.

Anyone who wishes to learn about the indispensable role that mathematics plays today in analyses and discussions of economic policy will find here a whole series of examples of such applications at their best. My only (small) unhappiness with the volume is that Professor Stone did not take a few extra hours to provide it with a good index.

<div align="right">HERBERT A. SIMON (Pittsburgh, Pa.)</div>

A first course in stochastic processes. By Samuel Karlin. Academic Press, New York and London, 1966. xi + 502 pp. $11.75.

This book contains a wealth of material concerning certain types of stochastic processes and is written in a very lucid and helpful style. The only real reservation which the reviewer has concerns the title and stated purpose of the book. Since virtually the only types of processes considered have Markov properties (or are closely related to such processes), this fact may well have been indicated in the title. Further, even though "advanced topics" are omitted, the subject matter considered is developed in some considerable depth. Thus it seems to us that this book will be more useful to the graduate student planning to later specialize in processes of Markov type, than as a general first course in stochastic processes suitable for the Senior level.

After the first introductory chapter, the basic properties of Markov chains are developed in Chapters 2–4. Chapter 5 discusses ratio theorems for transition probabilities and points the reader to more advanced and expanding work in this area. In Chapters 6, 7 many applications of Markov chains in discrete and continuous time, are discussed. Examples are: sums of independent random variables, counter models and various types of birth and death processes.

Chapter 8 contains an analytical treatment of continuous time chains, and in Chapters 9, 10 there is a discussion of applications such as order statistics, empirical distribution functions and Brownian motion.

Chapter 11 is a long chapter on branching processes. In Chapter 12, compounding of processes is considered with reference to multi-dimensional Poisson processes, population growth models and so on.

Chapter 13 is again a long chapter, dealing with applications in the general area of genetics. Finally, queueing processes are considered in Chapter 14.

As noted above (and as will be seen from the above listing) the book contains a wealth of material concerned with process of Markov type. This material is presented in a very well organized fashion. One extremely minor criticism refers to the somewhat disconcerting "multivalued nature" of some of the figures (e.g. P. 277). The question of typographical style of a book is perhaps largely a personal one. The reader may find this particular topography a little oppressive and tending to detract from the otherwise pleasant style of the book.

To summarize, it is the reviewer's opinion that this is an excellent book which will serve many useful purposes. It will be particularly useful to students of probability who may wish to specialize their research interests to the area of stochastic processes of Markov type.

<div align="right">M. R. LEADBETTER (Chapel Hill, N. C.)</div>

Wahrscheinlichkeits-Theorie. By Hans Richter. Springer-Verlag, Berlin, Heidelberg, New York, 1966. XII + 462 pp. $17.00.

This is the second, only slightly revised, edition of the author's book. It gives a rigorous mathematical treatment of probability theory based on Kolmogorov's measure theoretic approach. In order to make the book self-contained and to increase its usefulness as a text book the author presents an exposition of those parts of the theory of measure and integration which are necessary for an introduction into the mathematical theory of probability. This introductory material is covered in two chapters: chapter 1 deals with the basic concepts of measure theory (fields, σ-fields, point and set functions, construction of measures); chapter 4 is an introduction into the theory of integration, the construction of product measures and Kolmogorov's extension theorem are also treated here. The book places great emphasis on the foundations of probability theory and gives motivations for the axioms which are adopted. This is accomplished by a broad discussion (in chapter 2) of the intuitive background before introducing axioms or giving definitions. The elements of probability theory (addition and multiplication rule, Bayes theorem, random variables) are the content of chapter 3. The discussion is here restricted to finite sample spaces, the last section of this chapter prepares for the transition to the general theory. Chapter 5 deals with general probability fields and random variables (vectors) defined on them. This covers independence, expectations, moments, conditional expectations and conditional probabilities, characteristic functions and their most important properties. Certain special distribution functions are introduced in chapter 6 (Gamma distribution, multinomial distribution, Poisson distribution, normal distribution, chi-square distribution, Student's t-distribution, F-distribution, Hotelling's T^2-distribution). In chapter 7, the author treats the convergence of random variables. The various modes of stochastic convergence are defined and their relationships as well as zero-one laws are studied. As an illustration, the author discusses Bernouilli trials and proves various convergence theorems (weak and strong law of large numbers, normal convergence theorem, law of the iterated logarithm) for this particular case. He then gives conditions for the validity of the weak (strong) law of large numbers for independently and identically distributed random variables with finite expectations (respectively finite variance in the case of the strong law of large numbers). The central limit theorem is proved only for sequences of independent random variables having a finite variance. The modern central limit problem is mentioned in the last few pages of the book but not discussed.

The book is written in a broad style with an extensive discussion of the motivation for the procedures used. It offers a rigorous introduction into the theory of probability and is easy to read. A short bibliography at the end of the book should be useful to the student who wishes to supplement the present volume by reading some of the more specialized monographs dealing with topics which the author omitted in order to keep the size of the volume within reasonable bounds.

EUGENE LUKACS (*Washington, D. C.*)

Stochastic processes—basic theory and its applications. By N. U. Prabhu. The Macmillan Co., New York and Collier-Macmillan Ltd., London, 1965. xii + 233 pp. $7.95.

This is an introductory text in stochastic processes. The choice of topics was dictated by two factors; suitability for students just beyond advanced calculus, with only an introductory course in probability, and, secondly, the value and attractiveness of the material from the point of view of applications. Chapter headings: 1) Regular stochastic processes, 2) Markov chains, 3) Diffusion processes, 4) Discontinuous Markov processes, 5) Renewal theory, 6) Further results from renewal theory. The author's treatment strikes the reviewer as extremely successful in the case of Markov processes with discrete time parameter, but markedly less so for processes in continuous time (Chapters 3 and 4). The reason is that one can do very little in continuous time, even in an appealingly heuristic manner, without getting into a discussion of sample function behavior. Thus the treatment of diffusion is both formal and nonrigorous, neither the ideas nor the name of P. Lévy having found their way into the text. By contrast the chapters on Markov chains, random walk and renewal theory are rigorous, elegant, well motivated, and quite up to date. A variety of attractive applications (to population processes, queues, dams, counters, risk and ruin problems) illustrate the theory, and provide material for well chosen exercises at the end of each chapter.

F. SPITZER (*Ithaca, N. Y.*)