THE CORRESPONDENCE PRINCIPLE OF LINEAR VISCOELASTICITY THEORY FOR MIXED BOUNDARY VALUE PROBLEMS INVOLVING TIME-DEPENDENT BOUNDARY REGIONS*

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1. Introduction. The classical method of solving boundary value problems in the linear quasi-static theory of viscoelasticity is to apply an integral transform (with respect to time) to the time-dependent field equations and boundary conditions. The transformed field equations then have the same form as the field equations of elasticity theory and if a solution to these, which is compatible with the transformed boundary conditions, can be found then the solution to the original problem is reduced to transform inversion. This method of solving viscoelastic stress analysis problems is referred to as the "correspondence principle".

The correspondence principle is clearly applicable whenever the type of boundary condition prescribed is the same at all points of the boundary. For mixed boundary value problems (i.e., problems for which different field quantities are prescribed over separate parts of the boundary) the method is still applicable provided the regions over which different types of boundary conditions are given do not vary with time. (We are, of course, assuming that the region occupied by the body does not vary with time.) There remain those viscoelastic mixed boundary value problems where the regions, over which different types of boundary conditions are given, do vary with time. Particular examples are indentation and crack propagation problems. For problems of this type there will be points of the boundary at which only partial histories of some field quantities will be prescribed. When this is the case the transforms of these quantities are not directly obtainable and the classical correspondence principle is not applicable.

Following a statement of the fundamental field equations of linear thermo-viscoelasticity theory and the correspondence principle in Sec. 2, we give in Sec. 3 of this paper a method of calculating the transforms of the solutions to a fairly wide class of linear thermo-viscoelastic mixed boundary value problems. In Sec. 4 the method is used to derive the solution to a viscoelastic contact problem whose solution was previously obtained by a different method.

2. Formulation of boundary value problems, the correspondence principle. Consider a fixed region $\mathcal{R}$ with boundary $\mathcal{B}$, which is occupied by a homogeneous and isotropic linear viscoelastic material. Let u_i , ϵ_{ij} and σ_{ij} , each of which is to be regarded as a function of position $\mathbf{x}$ and nonnegative time t, denote the Cartesian components of displacement, strain and stress respectively. Employing the usual indicial notation the strain displacement relations and the stress equations of equilibrium for zero body

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force[1] take the form

$$2\epsilon_{ij}(\mathbf{x}, t) = u_{i,j}(\mathbf{x}, t) + u_{j,i}(\mathbf{x}, t), \tag{1}$$

$$\sigma_{ij,j}(\mathbf{x}, t) = 0, \qquad \sigma_{ij}(\mathbf{x}, t) = \sigma_{ji}(\mathbf{x}, t). \tag{2}$$

In stating the accompanying constitutive equations we confine our attention to the relaxation integral law. To this end, let T be the temperature and define the "pseudo-temperature" θ through

$$\theta(\mathbf{x}, t) = \frac{1}{\alpha_0} \int_{T_0}^{T(\mathbf{x},t)} \alpha(T')\, dT', \qquad \alpha_0 = \alpha(T_0). \tag{3}$$

Here T_0 is the uniform "base temperature" of the body and α is the temperature dependent coefficient of thermal expansion. We denote by G_1 and G_2 the relaxation functions in shear and isotropic compression respectively, measured at T_0. With these agreements the stress-strain relations take the form

$$s_{ij}(\mathbf{x}, t) = G_1(t)e_{ij}(\mathbf{x}, 0) + \int_0^t G_1(t - t') \frac{\partial}{\partial t'} e_{ij}(\mathbf{x}, t')\, dt', \quad \text{(a)}$$

$$\sigma_{kk}(\mathbf{x}, t) = G_2(t)[\epsilon_{kk}(\mathbf{x}, 0) - 3\alpha_0\theta(\mathbf{x}, 0)] + \int_0^t G_2(t - t') \frac{\partial}{\partial t'} [\epsilon_{kk}(\mathbf{x}, t') - 3\alpha_0\theta(\mathbf{x}, t')]\, dt', \quad \text{(b)} \tag{4}$$

where e_{ij} and s_{ij} denote the deviatoric components of strain and stress which are defined through

$$e_{ij}(\mathbf{x}, t) = \epsilon_{ij}(\mathbf{x}, t) - \tfrac{1}{3}\, \delta_{ij}\, \epsilon_{kk}(\mathbf{x}, t), \quad \text{(a)}$$

$$s_{ij}(\mathbf{x}, t) = \sigma_{ij}(\mathbf{x}, t) - \tfrac{1}{3}\, \delta_{ij}\sigma_{kk}(\mathbf{x}, t), \quad \text{(b)} \tag{5}$$

in which δ_{ij} is Kronecker's delta. We shall use the notation

$$f^*(\mathbf{x}, p) = \mathfrak{L}\{f(\mathbf{x}, t); t \to p\} = \int_0^\infty f(\mathbf{x}, t)e^{-pt}\, dt \tag{6}$$

for the Laplace transform with respect to time of a function[2] $f(\mathbf{x}, t)$. Assuming that all the pertinent functions possess Laplace transforms we find by invoking the appropriate convolution and differentiation theorems (e.g., see Sneddon [1]) and by applying the Laplace transform to Eqs. (1), (2), (4) that

$$2\epsilon^*_{ij}(\mathbf{x}, p) = u^*_{i,j}(\mathbf{x}, p) + u^*_{j,i}(\mathbf{x}, p), \tag{7}$$

$$\sigma^*_{ij,j}(\mathbf{x}, p) = 0, \qquad \sigma^*_{ij}(\mathbf{x}, p) = \sigma^*_{ji}(\mathbf{x}, p), \tag{8}$$

$$s^*_{ij}(\mathbf{x}, p) = pG^*_1(p)e^*_{ij}(\mathbf{x}, p), \quad \text{(a)}$$

$$\sigma^*_{kk}(\mathbf{x}, p) = pG^*_2(p)[\epsilon^*_{kk}(\mathbf{x}, p) - 3\alpha_0\theta^*(\mathbf{x}, p)]. \quad \text{(b)} \tag{9}$$

We will denote by u_n and u_s (σ_n and σ_s) the vector components of the displacement

[1]The considerations of this and the next section may be generalized in a straightforward manner to accommodate nonzero body force.

[2]The same notation will be used to denote the transform of a vector valued function, with the agreement that it represents the vector whose components are the transforms of the components of the original vector.

vector (traction vector) normal and tangential to $\mathcal{B}$ respectively. Thus u_n, u_s, σ_n, σ_s are vector valued functions of both $\mathbf{x}$ and t which are defined for all $(\mathbf{x}, t)$ on $\mathcal{B} \times [0, \infty)$. Suppose that a, b, c are the elements of any column of the matrix

$$\begin{bmatrix} \sigma_s & \sigma_s & \sigma_n & \sigma_n & u_s & u_s & u_n & u_n \\ u_n & \sigma_n & u_s & \sigma_s & \sigma_n & u_n & \sigma_s & u_s \\ \sigma_n & u_n & \sigma_s & u_s & u_n & \sigma_n & u_s & \sigma_s \end{bmatrix}. \tag{10}$$

Consider the problem of finding a solution to the system of Eqs. (1), (2), (4) which meets the boundary conditions[3]

$$\begin{aligned} a(\mathbf{x}, t) &= A(\mathbf{x}, t), \quad \mathbf{x} \text{ on } \mathcal{B}, \qquad \text{(a)} \\ b(\mathbf{x}, t) &= B(\mathbf{x}, t), \quad \mathbf{x} \text{ on } \mathcal{B}. \qquad \text{(b)} \end{aligned} \tag{11}$$

Here A and B are vector valued functions which are given for all nonnegative times at each point $\mathbf{x}$ of $\mathcal{B}$. Thus we may apply the Laplace transform to (11) and obtain

$$\begin{aligned} a^*(\mathbf{x}, p) &= A^*(\mathbf{x}, p), \quad \mathbf{x} \text{ on } \mathcal{B}, \qquad \text{(a)} \\ b^*(\mathbf{x}, p) &= B^*(\mathbf{x}, p), \quad \mathbf{x} \text{ on } \mathcal{B}. \qquad \text{(b)} \end{aligned} \tag{12}$$

Equations (7)–(9) together with (12) now determine an elastic stress analysis problem in which the elastic constants and boundary values are functions of the parameter p. If this can be solved, $[u_i^*(\mathbf{x}, p), \epsilon_{ij}^*(\mathbf{x}, p), \sigma_{ij}^*(\mathbf{x}, p)]$ is the transform of the solution to the original viscoelastic problem and inversion to give $[u_i(\mathbf{x}, t), \epsilon_{ij}(\mathbf{x}, t), \sigma_{ij}(\mathbf{x}, t)]$ provides the desired viscoelastic solution. This problem solving technique is known as the "correspondence principle" and is described in Sternberg's survey article [2], to which reference should be made for a comprehensive treatment of the material of this section.

3. Extension of the correspondence principle. Consider now the boundary value problem governed by Eqs. (1), (2), (4) together with the boundary conditions[4]

$$\begin{aligned} a(\mathbf{x}, t) &= A(\mathbf{x}, t), \quad \mathbf{x} \text{ on } \mathcal{B}, \qquad \text{(a)} \\ b(\mathbf{x}, t) &= B(\mathbf{x}, t), \quad \mathbf{x} \text{ on } \mathcal{B}_1(t), \qquad \text{(b)} \\ c(\mathbf{x}, t) &= 0, \quad \mathbf{x} \text{ on } \mathcal{B}_2(t), \qquad \text{(c)} \end{aligned} \tag{13}$$

where A and B are prescribed vector valued functions[5]. Here $\mathcal{B}_1$ and $\mathcal{B}_2$ designate time-dependent complementary subregions of the boundary $\mathcal{B}$ so that $\mathcal{B}_1(t) \cup \mathcal{B}_2(t) = \mathcal{B}$ at each nonnegative time. When $\mathcal{B}_1$ and $\mathcal{B}_2$ vary with time it is easy to see that there will be points $\mathbf{x}$ of $\mathcal{B}$ for which neither $b(\mathbf{x}, t)$ nor $c(\mathbf{x}, t)$ are known for all nonnegative times. In this case the correspondence principle is not applicable as a method of solution since the required Laplace transforms of the boundary conditions are not obtainable.

Consider, however, the one parameter family of static thermoelastic boundary value problems[6] governed by the boundary conditions (13) together with the field

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[3]We consider the "pseudo-temperature" θ to be given.

[4]*Ibid.* 3.

[5]It is to be emphasized that the vector valued function B appearing here is only defined for those $(\mathbf{x}, t)$ which belong to the set $\{(\mathbf{x}, t) \mid 0 \le t < \infty, \mathbf{x} \in \mathcal{B}_1(t)\}$.

[6]The parameter is t.

equations (1), (2) and

$$s_{ij}(\mathbf{x}, t) = 2\mu e_{ij}(\mathbf{x}, t), \quad \text{(a)}$$
$$\sigma_{kk}(\mathbf{x}, t) = 3\kappa[\epsilon_{kk}(\mathbf{x}, t) - 3\alpha_0\theta(\mathbf{x}, t)], \quad \text{(b)} \tag{14}$$

where μ and κ are constants standing for shear and bulk modulus respectively. Suppose that the solutions to these problems, which we denote by $[u_i^e(\mathbf{x}, t), \epsilon_{ij}^e(\mathbf{x}, t), \sigma_{ij}^e(\mathbf{x}, t)]$, are such that

$$b^e(\mathbf{x}, t) = B^e(\mathbf{x}, t), \qquad \mathbf{x} \text{ on } \mathcal{B}, \tag{15}$$

$$c^e(\mathbf{x}, t) = k(\mu, \kappa)C^e(\mathbf{x}, t), \qquad \mathbf{x} \text{ on } \mathcal{B}, \tag{16}$$

where the vector functions B^e and C^e are each independent of the elastic constants μ and κ while k is a function of these constants alone[7]. Since the boundary conditions (13) are satisfied we have

$$B^e(\mathbf{x}, t) = B(\mathbf{x}, t), \qquad \mathbf{x} \text{ on } \mathcal{B}_1(t), \tag{17}$$

$$C_\epsilon(\mathbf{x}, t) = 0, \qquad \mathbf{x} \text{ on } \mathcal{B}_2(t). \tag{18}$$

Consider now the thermo-viscoelastic boundary value problem governed by the Eqs. (1), (2), (4) together with the boundary conditions (13a) and

$$b(\mathbf{x}, t) = B^e(\mathbf{x}, t), \qquad \mathbf{x} \text{ on } \mathcal{B}. \tag{19}$$

Since both A and B^e are given for all nonnegative times t at each point $\mathbf{x}$ of $\mathcal{B}$ we may apply the Laplace transform to (13a) and (19) and find

$$a^*(\mathbf{x}, p) = A^*(\mathbf{x}, p), \qquad \mathbf{x} \text{ on } \mathcal{B}, \quad \text{(a)}$$
$$b^*(\mathbf{x}, p) = B^{e*}(\mathbf{x}, p), \qquad \mathbf{x} \text{ on } \mathcal{B}. \quad \text{(b)} \tag{20}$$

Comparing the system of Eqs. (20) and (7)–(9) with (20) and the Laplace transforms of (1), (2) and (14) we see that for the problem now under consideration[8]

$$[u_i^*(\mathbf{x}, p), \epsilon_{ij}^*(\mathbf{x}, p), \sigma_{ij}^*(\mathbf{x}, p)]$$
$$= \left\{[u_i^{e*}(\mathbf{x}, p), \epsilon_{ij}^{e*}(\mathbf{x}, p), \sigma_{ij}^{e*}(\mathbf{x}, p)];\ \mu = \frac{p}{2} G_1^*(p), \kappa = \frac{p}{3} G_2^*(p)\right\}. \tag{21}$$

By taking the Laplace transform of (16) we see that

$$c^{e*}(\mathbf{x}, p) = k(\mu, \kappa)C^{e*}(\mathbf{x}, p), \qquad \mathbf{x} \text{ on } \mathcal{B}, \tag{22}$$

which by virtue of (21) implies that for the thermo-viscoelastic displacement-stress field meeting the boundary conditions (13a) and (19)

$$c^*(\mathbf{x}, p) = k\left(\frac{p}{2} G_1^*(p), \frac{p}{3} G_2^*(p)\right)C^{e*}(\mathbf{x}, p), \qquad \mathbf{x} \text{ on } \mathcal{B}. \tag{23}$$

Inverting (23) we find that[9]

[7]These conditions must be met for the extended correspondence principle given here to be valid. An example where they are met is given in the next section. For other examples see [3].

[8]This is a direct application of the correspondence principle.

[9]Equation (24) remains valid if the roles of K and C^e are interchanged.

$$c(\mathbf{x}, t) = K(t)C^e(\mathbf{x}, 0) + \int_0^t K(t - t') \frac{\partial}{\partial t'} [C^e(\mathbf{x}, t')] \, dt', \qquad \mathbf{x} \text{ on } \mathcal{B}, \tag{24}$$

where[10]

$$K(t) = \mathcal{L}^{-1}\left[\frac{1}{p} k\left(\frac{p}{2} G_1^*(p), \frac{p}{3} G_2^*(p)\right); p \to t\right]. \tag{25}$$

We will now turn our attention to the particular circumstances that $\mathcal{B}_1(t)$ is monotonic increasing with time. Thus if t_1 and t_2 are any two nonnegative times such that $t_1 \le t_2$ then $\mathcal{B}_1(t_1) \subseteq \mathcal{B}_1(t_2)$[11]. On substituting from (18) into (24) it is found that in these circumstances $c(\mathbf{x}, t)$ as it is defined through (24) satisfies the condition[12].

$$c(\mathbf{x}, t) = 0, \qquad \mathbf{x} \text{ on } \mathcal{B}_2(t). \tag{26}$$

Thus provided $\mathcal{B}_1(t)$ is monotonic increasing with t and the requirements of (15) and (16) are satisfied we have by virtue of (17) and (26) that the Laplace transform of the required viscoelastic field meeting the boundary conditions (13) is given by (21). It is worth emphasizing, and may be seen from an examination of Eq. (24), that if in (13) the vector function c is prescribed to be nonzero on $\mathcal{B}_2(t)$ then the considerations of this section do not, in general, supply us with the Laplace transform of the required viscoelastic solution.

The correspondence principle described in the previous section admits straightforward generalization to anisotropic and inhomogeneous materials (for a list of references see [2]). It is a simple matter to verify that this generalized correspondence principle is capable of an extension, in complete analogy to that given in this section, to the correspondence principle for homogeneous and isotropic bodies[13].

4. An Example. We will now give an example to illustrate the use of the extended correspondence principle obtained in the previous section. Suppose that, in terms of circular cylindrical co-ordinates (ρ, θ, z), the region $\mathcal{R}$ is the half space $z \ge 0$ with boundary $\mathcal{B}$ given by the plane $z = 0$. We consider the axisymmetric problem governed by the following boundary conditions:

$$\begin{aligned}
&\sigma_{\rho z}(\rho, 0, t) = \sigma_{\theta z}(\rho, 0, t) = 0, && \rho \ge 0, && \text{(a)} \\
&u_z(\rho, 0, t) = D(t) - \beta(\rho), && 0 \le \rho \le a(t), && \text{(b)} \qquad (27)\\
&\sigma_{zz}(\rho, 0, t) = 0 && \rho > a(t), && \text{(c)}
\end{aligned}$$

where the field quantities are independent of θ. These boundary conditions have the same form as (13), corresponding to the special circumstances in which the quantities a, b, c are chosen from the first column of (10). The region $\mathcal{B}_1(t)$ is now that part of

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[10]Here and subsequently the notation $\mathcal{L}^{-1}$ indicates that we take the inverse Laplace transform.

[11]Since $\mathcal{B}_1(t) \cup \mathcal{B}_2(t) = \mathcal{B}$, the requirement that $\mathcal{B}_1(t)$ is monotonic increasing with time is equivalent to the requirement that $\mathcal{B}_2(t)$ is monotonic decreasing with time; i.e. that $0 \le t_1 \le t_2$ implies $\mathcal{B}_2(t_1) \supseteq \mathcal{B}_2(t_2)$.

[12]It is easy to see that if $\mathcal{B}_1(t)$ is not monotonic increasing with time then $c(\mathbf{x}, t)$ as it is defined through (24) will not in general satisfy (26).

[13]The application of this extension of the correspondence principle to the solution of practical problems would of course depend on the availability of a family of solutions $[u_i^e, \epsilon_{ij}^e, \sigma_{ij}^e]$ to the field equations of anisotropic and inhomogeneous elastostatics which met the boundary conditions (13) and satisfied requirements analogous to those of (15), (16).

the plane $z = 0$ for which $0 \leq \rho \leq a(t)$, while $\mathcal{B}_2(t)$ is the part of $z = 0$ for which $\rho > a(t)$. In accordance with the conditions of the extended correspondence principle given in the previous section we will restrict ourselves to the particular circumstances that $\mathcal{B}_1(t)$ is monotone increasing with time so that $a(t)$ is a monotone increasing function of time. We assume that the function $\beta(\rho)$ appearing in (27b) is at least once continuously differentiable and that its derivative is never negative. The boundary conditions (27) then correspond to the physical circumstances of an axisymmetric punch of curved profile being pressed against the surface of a viscoelastic half space in such a way that the radius of the circular area of contact is monotone increasing with time.

If we can find a solution to the system of eqs. (1), (2) and (14) which satisfies the conditions (27) then the corresponding viscoelastic solution meeting (1), (2), (4) and (27) will be given through (21) provided the requirements of (15), (16) are satisfied. From here on we will assume that the pseudo-temperature is identically zero so that[14]

$$\theta(\mathbf{x}, t) = 0, \qquad \mathbf{x} \text{ on } \mathcal{R}, \qquad t \geq 0. \tag{28}$$

In this instance the solution to the elastic problem governed by the field equations (1), (2), (14) and the boundary conditions (27) is given by Sneddon [4]. Adapting that solution to our notation we find in particular that[15]

$$\sigma_{zz}(\rho, 0, t) = \frac{2\mu(\mu + 3\kappa)}{(3\kappa + 4\mu)\rho} \frac{d}{d\rho} \int_\rho^{a(t)} \frac{yg(y, t)\, dy}{\{y^2 - \rho^2\}^{1/2}}, \qquad 0 \leq \rho \leq a(t), \tag{29}$$

where

$$g(y, t) = \frac{2}{\pi} \left\{ D(t) - y \int_0^y \frac{d\beta/d\rho}{\{y^2 - \rho^2\}^{1/2}}\, d\rho \right\}. \tag{30}$$

Since the punch is assumed to have a curved profile, D is related to $a(t)$ through the equation

$$D(t) = a(t) \int_0^{a(t)} \frac{d\beta/d\rho}{\{a^2(t) - \rho^2\}^{1/2}}\, d\rho. \tag{31}$$

Further we have that outside the area of contact the normal displacement of the surface of the half space is given by

$$u_z(\rho, 0, t) = \int_0^{a(t)} \frac{g(y, t)\, dy}{\{\rho^2 - y^2\}^{1/2}}, \qquad \rho > a(t). \tag{32}$$

Since the elastic constants are absent from (32) and appear in a separate factor in (29) the requirements of (15), (16) are satisfied and the extension of the correspondence principle obtained in the previous section is applicable. Thus we find that for the viscoelastic boundary value problem[16] determined by (1), (2), (4) and (27) the normal displacement outside the contact area is given by (32), where g is given by (30) and D is related to $a(t)$ through (31), provided only that $a(t)$ remains a monotonic increasing function of time. By combining (24) and (25) with (27c) and (29) we find that

$$\sigma_{zz}(\rho, 0, t) = K(t)\mathcal{C}(\rho, 0) + \int_0^t K(t - t') \frac{\partial}{\partial t'} [\mathcal{C}(\rho, t')]\, dt', \tag{33}$$

[14] Cf. 3.

[15] Formulas for the stresses and displacements obtaining in the interior of the half space are given in [1].

[16] Recall that θ satisfies (28).

where[17]

$$\mathcal{C}(\rho, t) = \frac{H(a(t) - \rho)}{\rho} \frac{d}{d\rho} \int_\rho^{a(t)} \frac{yg(y, t)\, dy}{\{y^2 - \rho^2\}^{1/2}} \tag{34}$$

and where K is given by

$$K(t) = \mathfrak{L}^{-1}\left[\frac{G_1^*(p)(G_1^*(p) + 2G_2^*(p))}{2(2G_1^*(p) + G_2^*(p))};\ p \to t\right]. \tag{35}$$

The total pressure acting on the punch is given by

$$P(t) = -2\pi \int_0^{a(t)} \rho\sigma_{zz}(\rho, 0, t)\, d\rho. \tag{36}$$

Substituting from (33) and (34) into (36) we find that

$$P(t) = K(t)\mathcal{P}(0) + \int_0^t K(t - t') \frac{\partial}{\partial t'} [\mathcal{P}(t')]\, dt', \tag{37}$$

where

$$\mathcal{P}(t) = 2\pi \int_0^{a(t)} g(y, t)\, dy. \tag{38}$$

If we take $\beta(\rho) = \rho^2/2R$, where R is a constant, the solution given here reduces to that which was originally given by Lee and Radok [5] for the problem of a viscoelastic half space against whose surface is pressed a rigid paraboloid of revolution. Subsequently this solution was rederived, using dual integral equations, by Hunter [6]. In [7] it was shown by the present author that the solution given in [5], [6] is a special case of a relation of the type of (33) between the solution to a viscoelastic contact problem for a punch of arbitrary smooth profile and the solution to the corresponding one parameter family of elastic contact problems.

The general approach of each of these solutions is to find a pressure distribution which vanishes outside the contact area, i.e., satisfies (27c), and which when taken together with (27a) determines a viscoelastic solution for which (27b) is satisfied. In terms of the boundary conditions (13) the approach takes the form of finding $c(\mathbf{x}, t)$, $\mathbf{x}$ on $\mathcal{B}_1(t)$ which, when taken together with (13a) and (13c), determines a viscoelastic solution which satisfies (13b). What we have done in this paper is to derive conditions, in terms of a one parameter family of static elastic solutions meeting (13), which are sufficient to ensure that the viscoelastic problem determined by (13) may be reduced to one to which the correspondence principle may be applied. When these conditions are met it is found that, for the viscoelastic problem determined by the conditions (13), $b(\mathbf{x}, t)$, $\mathbf{x}$ on $\mathcal{B}_1(t)$, has the same values as those computed from the static elastic analysis, while $c(\mathbf{x}, t)$, $\mathbf{x}$ on $\mathcal{B}_1(t)$, is generated by the formula (24).

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[17]Here H stands for the Heaviside unit step function which is defined by $H(\xi) = 0$, $\xi < 0$; $H(\xi) = 1$, $0 \leq \xi < \infty$.

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