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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unneccessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

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Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\surd$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2 + b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x/2b)}{\cos (\pi a/2b)} \text{ is preferable to } \frac{\cos \dfrac{\pi x}{2b}}{\cos \dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

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Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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—BOOK REVIEW SECTION—

Iterationsverfahren, Numerische Mathematik, Approximationstheorie. Edited by L. Collatz, G. Meinardus, H. Unger and H. Werner. Birkhäuser Verlag, Basel & Stuttgart, 1970. 257 pp. SFR. 36.—.

The book contains some abstracts and several papers read at the international colloquia in Oberwolfach, in November 1968 (Nonlinear Numerical Analysis), June 1969 (Numerical Methods in Approximation Theory) and November 1969 (Iteration Procedures in Numerical Analysis). Although only some of the papers read at the above symposia are accounted for, this collection contains papers of great and even vital interest to any expert working in the field of numerical analysis.

Most of the 24 papers are in German; six of them are in English. While special results cannot be discussed in this review, we give below a list of the titles (in English):

1. Brosowski, B., K.-H. Hoffmann, E. Schäfer and H. Weber, Continuity theorems for metrical projections.
2. Brosowski, B., K.-H. Hoffmann, E. Schäfer and H. Weber, Metric projections on linear subspaces of $C_0[Q, H]$.
3. Frehse, J., On convergence of difference procedures in nonlinear variational problems.
4. Laasonen, P., On some procedures of solution of nonlinear systems.
5. Lancaster, P., Spectral properties of operator functions.
6. Mayer, H., Estimates for the deficiency vector of the solution of a linear system in terms of errors in the initial data, and numerical application of these estimates.
7. Nitsche, J., Convergence of the Ritz-Galerkin procedure for nonlinear operator equations.
8. Nixdorff, K., Nonlinear procedures of direction-finding techniques.
9. Nixdorff, K., Observations on the application of the harmonic balance procedure.
10. Reimer, M., Semi-definite Peano kernels for stable difference forms.
11. Wetterling, W., On minimal conditions and a Newton iteration in non-linear optimization problems.
12. Zeller, K., Approximation methods of Newton and Tschebischeff.
13. Anselone, P. M., Abstract Riemann integrals, monotone approximations and generalizations of Korovkin's theorem.
14. Cheney, E. W. and K. Price, Minimal interpolating projections.
15. Collatz, L., Approximation theory and its applications.
16. Gilbert, R. P., Integral operator methods for approximating solutions of Dirichlet problems.
17. Haussmann, W., Hermite interpolation in several dimensions.
18. Locher, F. and K. Zeller, Approximations in the knots of a net.
19. Lupas, A., On approximation by linear positive operators.
20. Smith, L. B., Using interactive graphical computer systems on approximation problems.
21. Döring, B., A theorem on a class of iteration procedures considered by Grebenjuk.
22. Gekeler, E., Relaxation for a class of nonlinear systems.
23. Niethammer, W., Convergence acceleration for one-step iteration procedures using summation.
24. Wacker, H. J., A procedure for solution of nonlinear boundary value problems.

A. M. Ostrowski (*Basel*)

Markov processes: structure and asymptotic behavior. By Murray Rosenblatt. Springer-Verlag, Berlin, Heidelberg, New York, 1971. xiii + 268 pp. $20.20.

The contents of this interesting book correspond more to the research interests of its author than to a systematic coverage of the titular field. Chapter I discusses some of the basic ideas of discrete-parameter Markov processes with stationary transition probabilities (but the continuous-parameter

Poisson and Brownian-motion processes are mentioned). As special cases sums of independent random variables are discussed, both with summands which are real-valued and summands which take on values in countable commutative groups. Chapter II uses statistical mechanics, learning theory, and a resource flow model to 'suggest or even motivate a number of the questions taken up in this book'. This is the closest the author comes to applications. Chapter III discusses functions of Markov processes, stressing conditions under which the new processes are Markovian, or at least under which the first-order transition probabilities satisfy the Chapman-Kolmogorov equations. Here and later only discrete-parameter processes are considered. Chapter IV discusses the ergodic theory of Markov processes with stationary transition probabilities, in other words the ergodic theory of positive contraction operators. Chapter V is sufficiently characterized by its title: 'Random walks and convolution on groups and semigroups'. Chapter VI discusses the representation of Markov sequences by functions of independent random variables. Such representations, essentially nonlinear analogues of the Wold representation, are important in prediction theory. Chapter VII shows how various mixing conditions make possible the extension to Markov sequences of the central limit and associated theorems for sequences of independent random variables.

Besides these seven chapters there are appendices written to provide background in probability, topological spaces, linear operators, and topological groups. In spite of these appendices, a reader needs a solid background in modern analysis and considerable knowledge of probability to appreciate this book. The book is written at a sophisticated level, and the relation between the 'applications' in Chapter II and the theory in the other chapters is tenuous.

Since the seven chapters cover only 219 pages, it is clear that no chapter can offer more than a few significant results. This does not mean that no deep theorems are proved. For example, Chapter I contains the Berry-Esseen central limit theorem, Chapter IV contains the Chacon-Ornstein-ratio ergodic theorem on positive contractions in L^1, and Chapter VII contains the Kolmogorov theorem on the approximation of successive convolutions of a distribution function by corresponding infinitely-divisible distribution functions. But the author has covered only key results in the directions that interest him (and in fact some of the results are due to him). Readers interested in these topics will find much of value in this book, and there is good browsing for all probabilists.

J. L. Doob (*Urbana*)

Algebraic methods in statistical mechanics and quantum field theory. By Gerard G. Emch. Wiley–Interscience, New York, 1972. vii + 333 pp. $19.95.

In the last decade there have been important developments in quantum mechanics, quantum field theory and statistical mechanics which have used the language of C*, Jordan, and von Neumann algebras. Conversely, some of the significant new ideas in the mathematical theory of these algebras have had their origin in physical problems. The book under review is an introduction to and partial summary of the motivations, results, and problems involved in this cross-fertilization. It is divided into four chapters: 1. General motivation, 2. Global theories, 3. Canonical commutation and anti-commutation relations, and 4. Quasi-local theories.

The first chapter has two parts of rather different character. In the first, entitled "Why Not Stay in Fock Space?", the behavior of several examples of systems of an infinite number of degrees of freedom (two model meson theories and the BCS model in statistical mechanics) is contrasted with that of systems of a finite number of degrees of freedom. In the second part, attempts to characterize the logical structure of general quantum mechanics going back to Jordan, Wigner, von Neumann, and Segal are described and an outlook reflecting the more recently developed point of view of the author and C. Piron is summarized. Chapter 2 is by far the longest; it treats the theory of C* and von Neumann algebras and their symmetries. Chapters 3 and 4 describe the special features arising in the C* algebras associated with the CCR, CAR, and the quasi-local algebras of statistical mechanics and quantum field theory.

The book would be ideal as a basis for a seminar in which there are participants with strong backgrounds in some of the disciplines involved. It devotes much attention to the motivating ideas of the subject but does not pretend to offer a self-contained introduction to all the mathematics or physics needed. (Look at the definition of the tensor product of Hilbert spaces on p. 7, for example.)

The results covered by this book have given a new and richer meaning to the phrase "applied mathematics". The book opens the way for the reader into an exciting and fundamental subject.

A. S. Wightman (*Princeton*)