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The QUARTERLY prints original papers in applied mathematics which have an intimate connection with application in engineering. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unneccessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e(a^2 + b^2)^{1/2}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos\,(\pi x/2b)}{\cos\,(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u\, du \text{ is preferable to } \int \frac{\sin u}{u}\, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

—BOOK REVIEW SECTION—

Transformations de Laplace et de Mellin. By S. Colombo and J. Lavoine. Gauthier-Villars, Paris, 1972. xiv + 170 pp.

This is more a collection of rules of operation and formulae, together with a comprehensive set of tables of transforms, than a textbook on the Laplace and Mellin transforms, but within its limits it is an admirable production.

These limits are indicated by the chapter headings: 1. Integral transforms, 2. Unilateral Laplace transforms, 3. Bilateral Laplace transforms, 4. The Mellin transform, 5. Distributions and their Laplace transforms, 6. The Mellin transform of distributions. These six chapters occupy 42 pages of the book; the remainder consists of 104 pages of tables of transforms (both direct and inverse), 5 pages of bibliography and 11 pages listing the notations employed in the text and in the accompanying tables. There is also an index.

The book is lucidly written and clearly printed and will be of invaluable use to anyone whose day-to-day work involves the use of integral transforms. It is a most useful supplement to the Bateman Project volumes of tables of integral transforms.

IAN N. SNEDDON (*Glosgow*)

Theory of partial differential equations. By H. Melvin Lieberstein. Academic Press, New York and London, 1972. xiv + 283 pp. $16.50.

It is extremely difficult to find a text in partial differential equations for a senior-level or first-year-graduate course. One always seems unsatisfied, but the book by Lieberstein will get a chance my next time around. The topics are well chosen, the student may become familiar with very concrete examples before going on to more general theory, and there are specific applications (although mainly from fluids and transmission lines) to indicate the relevance of the topics. Numerical techniques are also interspersed throughout for the explicit computation of solutions, and occasionally for theoretical purposes. By the end of the text, one should be prepared for the modern theory of partial differential equations. More exercises would have been welcome. An outline by chapters is: 1. The theory of characteristics, classification, and the wave equation in E^2; 2. Various boundary-value problems for the homogeneous wave equation in E^2; 3. Various boundary-value problems for the Laplace equation in E^2; 4. Various boundary-value problems for simple equations of parabolic type; 5. Expectations for well-posed problems; 6. Existence and uniqueness considerations for the nonhomogeneous wave equation in E^2; 7. The Riemann method; 8. Classical transmission line theory; 9. The Cauchy-Kovalevski theorem; 10. A sketch of potential theory; 11. Solution of the Cauchy problem for the wave equation in terms of retarded potentials; 12. A priori inequalities; 13. Uniqueness of regular solutions and error bounds in numerical approximation; 14. Some functional analysis; 15. Existence of $\mathcal{L}^p$-weak solutions.

JACK K. HALE (*Providence*)

Celestial mechanics II: perturbation theory. By Yusuke Hagihara. Part 1: xvii + 504 + x pp. $30.00. Part 2: xv + 219 + x pp. $30.00. MIT Press, Cambridge, Mass., 1972.

Celestial mechanics, Volume II, by Y. Hagihara is devoted to perturbation theory. Perturbation theory studies the ways in which successive approximations to solutions of problems in celestial mechanics can be obtained. Successive approximations are expressed in the form of trigonometric series with linear functions of time as arguments. The successive approximations obtained by the theory are not in general

convergent, and error estimates are lacking. Nevertheless, in some cases they can be used to predict results which agree reasonably well with observation. The work of Kolomogorov, Arnold and Moser proves that under certain conditions the successive approximations do converge. Hagihara proposes to discuss this work in Volume IV, which has not yet appeared.

The present volume discusses in full detail the classical perturbation method of Lagrange and its application to the motion of the major planets by Le Verrier. The perturbation theories of Delaunay, Von Zeipel, Hansen, Newcomb, Gyldén, and others are presented and a chapter is devoted to Hill's lunar theory. Extensive references to the literature are given at the end of each chapter. Computations are carried out in detail and the book contains a wealth of formulas. Applications of the theory to many specific problems in celestial mechanics are given. Volume II refers to Volume I in several places and hence is not self-contained. The book is written for the reader with some prior knowledge of celestial mechanics.

ROBERT W. EASTON (*Boulder, Colorado*)

Input-output techniques. Edited by A. Bródy and A. P. Carter. North-Holland Publishing Co., Amsterdam and London, 1972. 600 pp. $35.00.

This volume, the Proceedings of the Fifth International Conference on Input-Output Techniques in Geneva, January, 1971, contains a selection of the papers presented there. The variety of topics, and the broad representation of economic specialties and of contributing countries testify to the vitality of input-output (IO) analysis and to its popularity among economic theorists and practitioners. A new field of application, spearheaded by the father of IO analysis, Wassily Leontief himself, is environmental control, from air pollution to the quality of urban life (Werner Z. Hirsch). Demographic analysis is ably represented by Richard Stone's "matrix of the active sequence," where "active" refers to education or economic activities, and by a sociological flow model for occupational structure (Coleman). More along established lines are foreign trade models and interregional IO systems. Problems of price equilibrium, related to the transpose of the IO matrix, are treated towards very practical ends: price reform, price control and foreign exchange control. The classical material balances appear in a contribution from the Soviet Union (Isayev). Of particular interest to Western readers are applications to decision-making in large corporations (Matuszewski). The effects of changes in the IO coefficients are studied, primarily for practical purposes such as trend analysis and technological forecasting, in several papers (among others, Staeglin and Aujac). Many different studies deal with the application to national planning, notably in the Soviet Union and in Norway. The papers from Eastern countries are invariably presented as theoretical studies and reveal little about actual practice. Much more informative in that respect are reports on sectoral plans, such as the energy sector model for Mexico (Manne), or an oil industry model for the United States (Byrd). The greatest theoretical interest attaches to dynamic IO models. In particular, market dynamics and stability analysis in general are presented. The most ingenious paper is undoubtedly Tsukui's calculation and discussion of an actual turnpike for the Japanese economy computed from IO tables. The editors are to be congratulated for their selection which shows to advantage the great scope of IO analysis. Among the complaints of this reviewer are that the "summary report on standardized IO tables" is insufficiently referenced and that neither abstracts (of the papers printed or of those presented) nor an index are included. The book's usefulness is further diminished by its exorbitant price ($35) which puts it out of reach of all but the experts.

M. J. BECKMANN (*Providence*)