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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\ }$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(\pi x/2b)}{\cos(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials,* vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions,* Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids,* but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

CONTENTS

QUARTERLY OF APPLIED MATHEMATICS
OCTOBER 1978

—BOOK REVIEW SECTION—

Foundations of inference in survey sampling. By Claes-Magnus Cassel, Carl-Erik Särndal and Jan Håkan Wretman. John Wiley & Sons, New York, London, Sydney, Toronto, 1977. xi + 192 pp. $18.95.

In the last twenty years, there has been an often very spirited, on occasions heated debate about the "foundations of survey sampling." The credit for triggering off this debate is largely due to V. P. Godambe, who published, in 1955, a seminal paper on this topic.

In a nutshell, the debate has concerned the "proper" inference procedure when sampling from a finite population, and accordingly (but to a lesser extent) the "proper" design for selecting a sample. Now there are two kinds of such populations:

i The units which make up the first kind of population are indistinguishable among themselves; there is, for example, no list identifying the individual units of the population. The fish in a lake is an example in kind.

ii. The units which make up the second kind of population are distinguishable. Thus, with every unit u_k ($k = 1, \cdots, j, \cdots, N$) in the population an identifying "label" is associated, which is this index k.

Consider now the problem of estimating the population mean $\bar{Y}$ on the basis of a simple random sample s (selected without replacement). Let the observations be $y_1, \cdots, y_i, \cdots, y_n$.

When sampling from a population of the first kind, it is natural to use the sample mean $\bar{y}(s)$ as the estimate. This estimate has some attractive properties; it is, for example, the BLUE.

When sampling from a population of the second kind, the survey statistician has in fact access to some "additional" information: he knows which units in the population are in the sample. The question he has to answer is: can this information be used to advantage, and if so, should it be so used? An example which has become a classic in the literature serves to illuminate the issue. Consider a population with $N = 3$ units and the associated y-values y_1, y_2 and y_3. We select a sample of $n = 2$ units. There are three possible outcomes:

$$s = s_1 : y_1 \quad \text{and} \quad y_2$$

$$s = s_2 : y_1 \quad \text{and} \quad y_3$$

$$s = s_3 : y_2 \quad \text{and} \quad y_3$$

Again it may appear natural to estimate $\bar{Y}$ by

$$\bar{y}(s) = \frac{1}{n} \sum_{i=1}^{2} y_i$$

the mean of the two units selected for the sample. Consider, however, the following estimate:

$$\begin{aligned}
\hat{y}(s) = \hat{y}(s_1) &= (y_1 + y_2)/2, && \text{if units 1 and 2 are selected} \\
= \hat{y}(s_2) &= \left(y_1 + \frac{4}{3} y_3\right) \Big/ 2, && \text{if units 1 and 3 are selected} \\
= \hat{y}(s_3) &= \left(y_2 + \frac{2}{3} y_3\right) \Big/ 2, && \text{if units 2 and 3 are selected.}
\end{aligned}$$

Clearly $y(s)$ is *unbiased.* It is a matter of terminology whether we call it linear or nonlinear. The important point is that for certain sets of y-values in the population, Var $\hat{y}(s) <$ Var $\bar{y}(s)$. In other words, the conventional estimate $\bar{y}(s)$ does not possess the conventional optimality property.

The example just given deals with only one issue in the debate about the foundations. I shall restrict myself here to pointing out two other issues:

i. Some survey statisticians are disturbed by the fact that an estimate which reflects the sampling design used (and especially the probabilities of selection associated with the units in the population) may be in conflict with the likelihood principle. This conflict calls for an answer to the question: *should* the estimate reflect the sampling design? Some survey statisticians go on to challenge the use of designs such as sampling with probabilities proportional to size ("PPS sampling").

ii. Some survey statisticians argue that one *should* consider the finite population as a random sample from an infinite "superpopulation", and that the estimate of $\bar{Y}$ should be constructed accordingly. Other statisticians argue that one should not "arbitrarily" introduce an element of randomness into the inference situation.

The debate referred to above is well documented in a very sizable number of journal papers and also in some proceedings from symposia devoted to the foundation issues. The book by Cassel, Särndal and Wretman aims at providing a "reasonably complete account of what might be called the statistical inference outlook on survey sampling" (from the authors' foreword). That the account is indeed "reasonably complete" is clear from reading the contents: 1. Basic model of sampling from a population with identifiable units; 2. Inference under the fixed population model: the concepts of sufficiency and likelihood; 3. Inference under the fixed population model: criteria for judging estimators and strategies; 4. Inference under superpopulation models: design-unbiased estimation; 5. Inference under superpopulation models: prediction approach using tools of classical inference; 6. Inference under superpopulation models: using tools of Bayesian inference; 7. Efficiency robust estimation of the finite population mean.

Let me add that the authors have, in addition, provided a very *systematic* account of the statistical inference outlook on survey sampling. All survey statisticians—irrespective of their affiliation with one or the other school of thought—should be thankful to the authors for this book.

The authors state in the preface that "This is not a text on the pragmatics of how to run a survey . . .". I hope this statement will not serve as an excuse for practitioners not to familiarize themselves with the problems of inference in survey sampling and with the alternative approaches available to cope with them.

The reviewer will take this opportunity of making a suggestion about the future of the debate. Now that the theoretical basis is provided, we should put the various theories to practical tests. It seems conceivable that one could design a large-scale computerized experiment within the framework of which the performances of the various inference theories (and survey statisticians!) could be compared in a situation which resembles that of the practicing survey statistician.

TORE DALENIUS (*Providence*)

The stability of dynamical systems. By J. P. LaSalle. With an appendix: *Limiting equations and stability of nonautonomous ordinary differential equations,* by Z. Artstein. SIAM, Philadelphia, 1976.

It has been some sixteen years since J. P. LaSalle discovered a simple and very useful relationship between Liapunov functions and the limit sets of G. D. Birkhoff; the resulting invariance principle may be the most important single contribution to stability theory since A. M. Liapunov proposed his direct method. This compact and well-written monograph is an exposition of Liapunov stability theory, as extended by the invariance principle, and an Appendix A by Z. Artstein dealing with limiting equations of nonautonomous ordinary differential equations. The first three chapters are concerned with autonomous difference equations, differential equations, and functional differential equations in $\mathcal{R}^n$. Chapter 4 discusses invariance properties and limiting equations of nonautonomous difference equations in $\mathcal{R}^n$, utilizing the idea of a dynamical system on an abstract Fréchet space. Appendix A contains similar results for nonautonomous differential equations in $\mathcal{R}^n$, although here attention is centered on some very recent and interesting results concerning the form of the associated limiting equations. Extensions tosother types of equations are mentioned, and appropriate references are provided.

In summary, a previously scattered body of information has been brought together and presented in a very nice form, complete with examples and exercises; the unity of approach is apparent. There are several new results as well as simplifications of previous results.

J. A. WALKER (*Evanston*)

The theory of stochastic processes II. By I. I. Gikhman and A. V. Skorokhod. Springer-Verlag, Berlin, Heidelberg, New York, 1975. 441 pp. $48.20.

This volume is devoted to the theory of Markov processes. Chapters 1 and 2 deal with Markov and strong Markov processes, generators, resolvent kernels, Feller processes and multiplicative and additive functionals.

The ideas for the most part are an elaboration of parts of Dynkin's *Markov processes.* On the topics which are treated, the book often has much more detail than has Dynkin's book, and the reading is somewhat easier. There are more calculations and examples, but the coverage is not as comprehensive. Although these chapters contain few significant results that are not in Dynkin, they are well worth reading owing to the added detail and background, particularly concerning the use of semigroups.

Chapters 3 to 5 are devoted to processes with independent increments (with values in separable Banach spaces), jump and semi-Markov and branching processes, and constitute an elaboration of similar material in the authors' *Introduction to the theory of random processes,* Saunders, 1969 (translation). The role of semigroups is emphasized.

HAROLD J. KUSHNER (*Providence*)

Integral geometry and geometric probability. By Louis A. Santaló. *Encyclopedia of mathematics and its applications,* Vol. 1. Addison-Wesley Publishing Co., 1976. 404 pp. $19.50.

Integral geometry is an elegant subject with a long history and with its home in the intersection of measure theory with those groups that naturally belong to geometry. One of the leaders in this field, L. A. Santaló, is the author of a new comprehensive exposition of integral geometry and geometric probability.

It is a beautiful book. It is full of special, but non-trivial, results, many of them in the plane, but a good deal of the book is devoted to integral geometry in a general setting. The last part deals with the case of spaces of constant curvature. The coverage seems to be as close to complete as can be expected and there is an excellent bibliography at the end.

This is the first in a series called *Encyclopedia of mathematics and its applications* edited by Gian-Carlo Rota. The series has the goal of presenting the factual body of all mathematics, an ambitious task to say the least. The present volume augurs well for the Encyclopedia.

ULF GRENANDER (*Providence*)

—BOOKS RECEIVED—

Digital image processing. By R. C. Gonzales and P. Wintz. Addison-Wesley, 1977. xvi + 431 pages. Hard binding, $29.50. Paper binding, $19.50.

This is volume 13 in the series Applied Mathematics and Computation. It consists of seven chapters: introduction, digital image fundamentals, image transforms, image enhancement, image restoration, image encoding, image segmentation and description. Appendix A contains a Fortran subroutine for displaying gray-tone images on an ordinary line printer, and appendix B contains a set of coded images suitable for experimenting with the methods discussed in the text.

Computational methods for data analysis. By John M. Chambers. John Wiley & Sons, 1977. xi + 268 pages. $15.75.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It describes the major computational methods which are important for data analysis. The essential results on each topic are presented, including an appraisal of currently competitive methods and references to selected algorithms. An appendix gathers together these references. There is quick reference in the inside back cover, designed to be consulted first by readers looking for a specific topic. Most chapters conclude with a set of problems, intended to be of practical value. There are also many unsolved or partially solved problems, with refences to more detailed treatments. The chapters, reasonably self-contained, are headed: 1. Introduction; 2. Programming (e.g., design and structures, portability, comparison of languages; 3. Data management and manipulation (e.g., data structures, order statistics, sorting, searching); 4. Numerical computations (e.g., error analysis, approximation, integration, Fourier transforms); 5. Linear models (e.g., regression, orthogonal bases, principal components, ANOVA); 6. Nonlinear models (e.g., optimization, quasi-Newton methods, nonlinear least-squares, nonlinear equations); 7. Simulation of random processes (e.g., the concept of randomness, pseudorandom uniforms, generators, derived distributions, special distributions, multivariate distributions, Monte Carlo); 8. Computational graphics (e.g., graphical devices, geometry of plotting—2 and more dimensions, plotting curves and surfaces—hidden lines, contour plotting, scaling, scatter plots, histograms.

There are over 250 items in the list of references. The available algorithms are collected in an appendix, in the form of tables giving references where they are described, the language in which they are written and the source from which they can be obtained.

This book will be a valuable reference and handbook for professionals in statistics, science and engineering who wish to use computers in the analysis of data. It is probably the most modern, complete, authoritative and concise source available.

Decomposition and dimension in module categories (Lecture Notes in Pure and Applied Mathematics Series, Volume 33). By Jonathan S. Golan. Marcel Dekker, 1977. xii + 185 pages. $18.75.

The concepts of primary decomposition and Krull dimension were originally defined in the category of commutative rings, and in recent years have been extended to the category of noncommutative rings. In the process, each concept has developed into several variants designed to suit particular applications and theories. This monograph presents a systematic and unified framework which includes most of these variants and which emphasizes the similarities between the notions of decomposition and dimension in their most general forms. Throughout the book, emphasis is on module categories rather than on categories of rings.

Applied modern algebra. By Larry L. Dornhoff and Franz E. Hohn. Macmillan Publishing Co., 1978. xi + 500 pages. $15.95.

This book presents an introduction to a variety of algebraic structures useful to students of pure and applied mathematics, electrical engineering and computer science. The first six chapters provide an introduction to the more elementary aspects of sets, functions, relations, graphs, semigroups, groups, Polyá counting theory, rings, finite fields, posets, lattices and Boolean algebras. The seventh chapter is a presentation of the linear algebra and field theory needed in the remaining two chapters, on linear machines and algebraic coding theory, respectively. Many illustrative examples are included and the flavor of the book is strongly algorithmic.

(continued on p. 254)

The Hopf bifurcation and its applications. By J. E. Marsden and M. McCracken. Springer-Verlag, New York, 1976. xiii + 408 pages. $14.80.

The goal of these notes (volume 19 in the series Applied Mathematical Sciences) is to give a reasonably complete, although not exhaustive, discussion of what is commonly referred to as Hopf bifurcation (or "Poincaré-Andronov-Hopf bifurcation") with applications to specific problems, including stability calculations. The principal technique employed is that of invariant manifolds. A translation (by L. N. Howard and N. Kopell) of Hopf's original (and generally unavailable) paper is included. The book also includes contributions by: P. Chernoff, G. Childs, S. Chow, J. R. Dorroh, J. Guckenheimer, L. Howard, N. Kopell, O. Lanford, J. Mallet-Paret, G. Oster, O. Ruiz, S. Schecter, D. Schmidt and S. Smale; those by Smale, Guckenheimer and Oster indicate applications to the biological sciences and that by Schmidt to Hamiltonian systems. A chapter is devoted to applications in fluid dynamics and turbulence.

Intermediate business statistics: analysis of variance, regressions, and time series. By Robert B. Miller and Dean W. Wichern. Holt-Rinehart and Winston, 1977. xiv + 525 pages. $20.00.

This book emphasizes statistical model building and concentrates on fixed effects analysis of variance, multiple regression and time series according to the methods of Box and Jenkins. It assumes a first course in statistics and background in calculus and matrix algebra. There are review chapters on probability and statistical inference and review appendices on calculus matrices.

An introduction to quantitative methods for decision making, 2nd edition. By Richard E. Trueman. Holt-Rinehart and Winston, 1977. xx + 725 pages. $15.00.

An elementary introduction, assuming only a basic knowledge of algebra. It treats: probability distributions; decision making; linear programming; transportation, assignment and network models; dynamic programming; inventory; queueing and stimulation models; Markov analysis.

Statistics and probability in modern life. By Joseph Newmark. Holt-Rinehart and Winston, 1977. xiii + 516 pages. $13.95.

An elementary introduction for students with no mathematical background other than simple arithmatic.

Sampling techniques, 3rd edition. By William G. Cochran. John Wiley & Sons, Inc., New York, 1977. xvi + 429 pages. $17.95.

This volume in the Wiley Series in Probability and Mathematical Statistics is the third edition of the well-known text first published in 1953. The topics in this edition are presented in essentially the same order as before: introduction, simple random sampling, sampling proportions and percentages, the estimation of sample size, stratified random sampling, ratio estimators, regression estimators, systematic sampling, single-stage cluster sampling, subsampling with units of equal and of unequal sizes, double sampling, and sources of errors in surveys. New features include: sections on the randomized response method; description of recent work on measuring the accuracy of complex estimates in surveys of complex structure and on the sampling techniques when two overlapping lists are necessary or economical in covering the population; summaries of the newest methods developed for the study of errors of measurement in surveys; sampling with unequal probabilities of selection of the units.

(continued on p. 268)

Theory of functional differential equations. By Jack K. Hale. Springer-Verlag, New York, 1977. x + 365 pages. $24.80.

This is a completely rewritten version of the author's *Functional differential equations,* published in 1971 as volume 3 in Springer's Applied Mathematical Sciences series, now offered in cloth binding and set in type. It present a more comprehensive theory than the original lecture notes, attempts to consolidate those elements of the theory which have stabilized and to include recent directions of research. The following chapters were not contained in those notes: chapter 1 is an elementary presentation of linear difference equations with constant coefficients of retarded and neutral type; chapter 4 develops the recent theory of dissipative systems; chapter 9 is a new chapter on perturbed systems; chapter 11 is a new presentation incorporating recent results on the existence of periodic solutions of autonomous equations. Chapter 12 is devoted entirely to neutral equations. Chapter 13 gives an introduction to the global and generic theory. There is also an appendix on the location of the zeros of characteristic polynomials.

The remainder of the material has been completely revised and updated with the most significant changes occurring in chapter 3 on the properties of solutions, chapter 5 on stability, and chapter 10 on behavior near a periodic orbit. Of the chapters not mentioned above, 2 is on the basic theory of retarded differential equations, 6 on general linear systems, 7 on linear autonomous equations, 8 on linear periodic systems and 12 on equations of neutral type. There is an appendix on the stability of characteristic equations and an eighteen-page bibliography (containing nearly 500 items).

An introduction to matrices, vectors, and linear programming. By Hugh G. Campbell, Prentice-Hall, Inc., New Jersey, 1977. xiii + 316 pages. $10.95.

For this second edition of the book first published in 1965, several chapters have been rewritten and a section on characteristic values has been added. It assumes elementary algebra and plane geometry and is suitable for a freshman or sophomore course in elementary matrix algebra or linear programming; it would also be useful as a text in various applied disciplines.

A short course in computational probability and statistics. By W. Freiberger and U. Grenander. Springer-Verlag, New York, 1977 (2nd revised printing). xii + 155 pages.

This is a second, revised printing of volume 6 in the series Applied Mathematical Sciences, first published in 1971. Numerous misprints and minor errors have been corrected.

Ordinary and delay differential equations. By R. D. Driver. Springer-Verlag, New York, 1977. ix + 501 pages.

This textbook (volume 20 in the series Applied Mathematical Sciences) is designed for an intermediate-level course on ordinary differential equations. It treats: existence and uniqueness theory, linear systems, stability theory, and introductory phase-plane analysis of autonomous second-order systems, as well as a substantial introduction to delay differential equations.

Operations research: an introduction, 2nd edition. By Hamdy A. Taha. Macmillan Publishing Co., 1976. xiv + 648 pages. Solutions manual available 151 pages.

This is the second edition of a book first published in 1971. It consists of three parts, 1. Linear, dynamic and integer programming (seven chapters); 2. Probabilistic models (six chapters); 3. Nonlinear

(continued on p. 278)

programming (two chapters); one introductory chapter; two review appendices on linear algebra and calculus; an appendix listing a Fortran program for computing Poisson queuing formulas and one with answers to selected problems. A 151-page solution manual for all the 480 problems in the book is also available. The material in part 1, with the exception of a chapter on advanced linear programming, is written at an elementary level. Part 2 includes probability, decision theory and games, PERT-CPM, inventory, queuing and simulation. Part 3 requires a higher level of mathematical maturity.

Explicit a-priori inequalities with applications to boundary value problems. By V. G. Sigillito. Pitman Publishing Ltd., London, 1977. 103 pages. £5.50

This volume presents explicit a-priori inequalities useful in computing approximate solutions, with error bounds, to boundary-value problems. Numerical examples—dealing with second- and fourth-order elliptic problems and second-order parabolic problems—are included to illustrate uses of the inequalities. Also described is recent work on the application of a-priori inequalities in eigenvalue estimation.

Introduction to fibre bundles. By Richard D. Porter. Marcel Dekker, New York, 1977. v + 170 pages. $19.50

Developed from a graduate course at Brown University, the book requires only a standard undergraduate mathematics background. The first section contains definitions of the classical groups and the definition of homogeneous space. In the next two sections, cellular decompositions of the groups are given and Pontryagin rings are calculated. A thorough discussion of fibre bundles then follows; this includes discussions of pullbacks, classifications of bundles in terms of coordinate transformations, fibre bundles with structure groups, and the bundle structure theorem for homogeneous spaces. The final section contains proofs of the covering homotopy theorems, definitions of the homotopy and homology groups of a pair, and the precise homotopy sequence of a bundle. Also included are examples of universal bundles with structure group one of the classical groups, and a sketch of Milgram's construction of the classifying space of a topological group.

Statistics from scratch. By P. Nemenyi, S. K. Dixon, N. B. White, Jr., and M. L. Hedstrom. Holden-Day, Inc., San Francisco, 1977. 629 pages. $11.95

This is an elementary textbook of statistics, requiring as background no more than a year of algebra. It is, however, an unusual book, in that it uses a congenial and painless pace, and yet manages to reach quite difficult material—it introduces tests not usually found in elementary texts. This "pilot edition" is the first of two volumes, but can profitably be used (with cuts) as a self-contained one-semester course. There are sixteen chapters, the last two being on straight line regression and correlation, and on ANOVA, respectively. There is emphasis on the nonparametric approach, on using everyday English rather than technical terminology, and on a vast number of examples from many real-life applications.

Differential games and control theory II. Edited by Emilio O. Roxin, Pan-Tai Liu and Robert L. Sternberg. Marcel Dekker, Inc., New York, 1977. xii + 485 pages. $35.00

This volume presents the proceedings of the Second Kingston Conference on Differential Games and Control Theory. The conference brought together an international groups of mathematicians, scientists, and engineers from a variety of disciplines, all having a common interest in the theme of the conference—stochastic problems and applications. The subject matter of the papers ranges from almost pure-mathematical topics to applications in electrical and aeronautical engineering, fisheries management, birth and death processes, military combat, and Fermat's principle in a stochastic medium.

(continued on p. 322)

Complete normed algebras. By F. F. Bonsall and J. Duncan. Springer-Verlag, New York, 1973. x + 299 pages. $26.20

The aim of this book (Volume 80 of the Ergebnisse series) is to give an account of the principal methods and results in the theory of Banach algebras, both commutative and non-commutative. Chapter headings: concepts and elementary results, commutativity, representation theory, minimal ideals, star algebras, cohomology, miscellany. There is a bibliography of 488 items.

Using computers. By B. L. Meek and S. Fairthorne. Halsted Press, New York, 1977. 208 pages. $16.50

A general introduction to various uses of computers in science and society with some information on their history and architecture.

The Stone-Čech compactification. By Russell C. Walker. Springer-Verlag, New York, 1974. x + 332 pages. $30.40

The goal of this book (volume 83 in the Ergebnisse series) is to make many of the recent results concerning the Stone-Čech compactification easily accessible by collecting them in a single source together with the necessary introductory material. A list of exercises follows each chapter and there is an extensive 20-page bibliography.

Maps and statistics. By Peter Lewis. Halsted Press, New York 1977. xviii + 318 pages. $22.50 cloth; $13.95 paper

This is an elementary introductory text, covering enough material for a one-year course for geography students. Maps are looked upon as measuring scales for the property of location. Maps and measurements provide the bases for particular propositions which are likely or unlikely rather than true or false. The link between map and proposition, proposition and hypothesis, hypothesis and test statistics is expressed in terms of probability distributions of random variables. The book develops the probabilistic and statistical background and applies it to these problems. It assumes only a knowledge of elementary algebra.

Urn models and their applications. By Norman L. Johnson and Samuel Kotz. John Wiley and Sons, 1977. xiii + 391 pages. $21.95

This volume in the Wiley Series in Probability and Mathematical Statistics demonstrates how numerous results in discrete probability theory can be derived from simple urn models. Chapter 1 presents an introduction to concepts of basic probability theory; Chapter 2 develops a range of useful probability distributions and their characteristics via urn models; Chapter 3 contains detailed discussions of occupancy distributions, while Chapter 4 deals with urn models with stochastic replacements involving Polya-Eggenberger and related distributions; Chapter 5 provides a broad range of possible applications—including genetics, learning processes, military, biological, computer, and division-theoretic applications—and also some sampling procedures and testing hypotheses; Chapter 6 presents recent developments in limiting distribution theory of occupancy and urn models, incorporating a large amount of unique western, Russian, and East European research.

Applied and computational complex analysis, II. By Peter Henrici. John Wiley and Sons, New York, 1977. ix + 662 pages. $32.50

This volume in the series Pure and Applied Mathematics offers—as did the first volume of the set—a self-

contained presentation of major complex analysis areas used in applied mathematics and mathematical physics, and emphasizes the algorithmic aspects of the theory. The chapters are: infinite products (particularly those useful in number theory, including Jacobi's triple product identity, Euler's representation of the gamma function, Sterling's formula), ordinary differential equations (emphasizing the systems approach, presenting the full theory of linear and quadratic transforms of the hypergeometric series, and clarifying the distinction between formal and actual solutions and the analytic meaning of formal solutions of asymptotic expansions), integral transforms (including a thorough presentation of Laplace transforms, Fourier integral theory, Polya's theory of functions of exponential type), asymptotic methods (with applications to systems of differential equations and treating Watson's lemma with applications of the Lagrange-Bürmann formula), continued fractions (a novel treatment using Möbius transformations and emphasizing geometric aspects; an algorithmic treatment of the problem of converting power series into corresponding continued fractions; Stieljes' theory; applications to remainders of continued fractions).

Field theory. By Masayoshi Nagata. Pure and Applied Mathematics, Volume 40. Marcel Dekker Inc., New York, 1977. xii + 276 pages. $23.50

This introduction to field theory requires only a basic undergraduate background in mathematics. Theorems are proved in detail and numerous exercises with answers and commentary are provided. The volume opens with a discussion of conventions and basic results on set theory. The remaining chapters cover groups, rings, algebraic extensions of fields, theory of valuations, ordered fields and a discussion of the Galois theory of algebraic extensions of infinite degree.

Les equations aux derivées partielles en physique et en mechanique des milieux continus. By Serge Colombo. Masson, Paris, 1976. viii + 188 pp. 90F

The book is addressed to physicists and engineers. Chapter headings are: 1. General considerations on partial differential equations; 2. Mathematical models in mechanics and physics; 3. Equations of first order; 3. Equations of second order; 5. Hyperbolic equations, wave propagation; 6. Newtonian potentials, Laplace's equations.

Moving boundary problems in heat flow and diffusion. Edited by J. R. Ockerdon and W. R. Hodgkins. Oxford University Press, New York, 1975. x + 300 pages. $19.25

These are the proceedings of a conference held at the University of Oxford, 25–27 March 1974. It is in three parts: applications (9 papers), analytical and numerical methods (8 papers) and thermal explosion (3 papers), each with an introduction. The papers provide an account of the methods for solving those time-dependent heat flow and diffusion problems which involve phase transitions and moving boundaries.

Introductory statistics. 3rd ed. By Thomas H. Wonnacott and Ronald J. Wonnacott. John Wiley and Sons, 1977. xxii + 650 pp. $15.95.

This is the third edition of a volume in the Wiley Series in Probability and Mathematical Statistics, first published in 1969. The text has been substantially revised and rewritten. For instance, the coverage of Bayesian statistics has been expanded and sampling theory is now illustrated by Monte Carlo simulation.

Numerical analysis of spectral methods: theory and applications. By David Gottlieb and Steven A. Orszag. Society for Industrial and Applied Mathematics, 1977. v + 172 pages. $12.25.

This monograph discusses the formulation and analysis of spectral methods—that is, methods seeking the solution to a differential equation in terms of a series of known, smooth functions. They are a viable alternative to finite-difference and finite-element methods for the numerical solution of partial differential equations. The key recent advance making this possible was the development of transform methods for the efficient implementation of spectral equations. Spectral methods have proved particularly useful in numerical fluid dynamics where large spectral hydrodynamics codes are now regularly used to study turbulence and transition, numerical weather prediction and ocean dynamics. Topics treated in this monograph include approximation and convergence theory, algebraic stability, hyperbolic equations, time differencing, advection-diffusion equations, and other.

Functional analysis in modern applied mathematics. By Ruth F. Curtain and A. J. Pritchard. Academic Press Inc., (London) Ltd., 1977. ix + 339 pages. $21.15.

This book aims at providing research workers in the applied sciences with a working knowledge of functional analysis. It consists of three parts: 1. Basic functional analysis (normed linear spaces, integration theory for real-valued functions, linear transformations, Hilbert spaces, probability theory, calculus in Banach spaces, topological spaces); 2. Analysis of abstract equations (differential equations—ordinary, stochastic, delay, partial, abstract, evolution; semigroup theory; spectral theory—in finite dimensions, on a normed linear space, for compact normal operators, for unbounded linear operators); 3. Applications (stability theory, linear systems theory, optimization problems, numerical methods, infinitely dimensional linear systems theory).

Wahrscheinlichkeitstheorie. By P. Gänssler and W. Stute. Springer-Verlag, Berlin-Heidelberg-New York, 1977. xii + 418 pages. $16.60.

This text presupposes a basic knowledge of analysis, linear algebra, and set-theoretic topology. The necessary measure theory is summarized. Previous acquaintance with probability theory and mathematical statistics is, according to the authors, desirable at the level of the text by Krickeberg and Ziezold. The chapter headings are: basic definitions and basic concepts of probability theory, laws of large numbers, empirical distributions, the central limit theorem, conditional expections and distributions, martingales, stochastic processes, random elements in metric spaces, central limit theorems for martingales, invariance principles. There is a bibliography of 154 items.

Game theory. Lectures for economists and systems scientists. By N. N. Vorob'ev. Springer-Verlag, New York, 1977. xi + 178 pages. $16.80.

This is volume 7 of the Springer series "Applications of Mathematics". Its basis is a series of lectures given to third-year students of economics at Leningrad State University who specialize in "economical cybernetics." It has, according to the translator, been found suitable on a text for a one-semester course for senior or beginning graduate students specializing in applied mathematics or operations research. The book's main purpose is to provide students with a relatively simple and easy-to-understand manual containing the basic mathematical machinery utilized in the theory of games. Only elementary tools of linear algebra and mathematical analysis are used. There are four chapters: matrix games, infinite antagonistic games, noncooperative games, cooperative games.

Regression analysis by example. By Samprit Chatterjee and Bertram Price. John Wiley and Sons, 1977. xiv + 228 pages. $16.95.

This volume in the Wiley Series in Applied Probability and Statistics focuses on exploratory data analysis rather than on statistical theory. It describes methods of regression analysis by using realistic examples that emphasize the analysis of data which contain the kind of irregularities met with in practice. It relies heavily on graphical representation of data. There are nine chapters which cover simple linear regression, detection and correction of model violations, multiple regression, qualitative variables as regressors, weighted least squares, correlated errors, collinear data, biased estimation of regression coefficients, and selection of variables in a regression equation.

The concepts and logic of classical thermodynamics as a theory of heat engines. By C. Truesdell and S. Bharatha. Springer-Verlag, New York, 1977. xxii + 154 pages. $24.80.

This work is in three parts: I. Calorimetry, II. Carnot's general axiom, III. Universal efficiency of ordinary Carnot cycles. It aims to provide a simple logical structure for the classical thermodynamics of homogeneous fluid bodies. It takes as primitive "just those quantities which every pioneer took as primitive", namely: time t, volume $V(t)$, temperature $\theta(t)$, pressure $p(t)$, heating $Q(t)$. No mathematical tools not already available in the 1820s are employed, and every thermodynamic concept used was introduced by 1854 at the latest. The scope of the tractate is conceptual, pro-historical and paedagogical.

Stochastische Methoden. By K. Krickeberg and H. Ziezold. Springer-Verlag, Berlin, Heidelberg, New York, 1977. viii + 201 pages $12.90.

This is an introduction to mathematical statistics, at the same time rigorous and practical. There are nine chapters: 1. discrete probability spaces; 2. three basic methods of mathematical statistics; 3. conditional probability, independence; 4. moments; 5. limit theorems; 7. general probability theory; 8. statistics of normal random variables; 9. regression and analysis of variance.

An introduction to linear control systems (Control and Systems Theory Series, Volume 5). By Thomas E. Fortmann and Konrad L. Hitz. Marcel Dekker, Inc., 1977. xiv + 744 pages. $29.75.

This textbook provides a detailed and integrated introduction to both modern and classical control systems theory. Mathematical foundations are explained, and the different points of view—time domain and transform methods, state-space and input/output models, controllability/observability and pole-zero cancellations—are simultaneously developed. The basic principles underlying linear systems theory are covered in the opening chapters of the text. Later chapters introduce the classical design techniques based on root-locus, Bode and Nyquist diagrams, as well as multi-variable topics such as state feedback, pole-placement, and observers. Time-varying systems, discrete-time systems, and linear optimal control are covered in survey chapters.

Analysis of linear dynamic systems. By John B. Lewis. Matrix Publishers, 1977. xii + 862 pages. $29.95.

This textbook comprises six chapters: introduction, signal models, linear system models, linear system response, system performance, applications. The mathematics employed is elementary, and appendices summarize results on transforms, matrices and probability theory.

The theory of gambling and statistical logic. By Richard A. Epstein. Academic Press, Inc., 1977. xv + 450 pages. $19.50.

This book presents perhaps the only unified and complete account of the mathematical theory of games of chance and skill. It reviews the figures and events associated with the historical development of the mathematics of gambling, presents the elements of probability theory, statistics and game theory, and provides a mathematical foundation for the analysis of games of chance and skill by deriving the basic theorems of gambling. The theory presupposes only an understanding of such elementary mathematical tools as basic calculus. Applications of the theory range from coin tossing and finger matching through dice and blackjack to horse racing and the stock market. The book discusses games of chance and statistical games such as poker, bridge, war games, games with information lag, duets, truels and polyvels. It also covers games of pure skill such as chess, checkers, go and nim-type games. It calculates probabilities of winning, develops optimal strategies, and discusses the application of computers to many of these games. This revised edition contains new, updated material on both the stock market and a winning strategy in blackjack, a strategy that, it is claimed, caused casino managements to change some of their strategies.

Robust statistical procedures. By Peter J. Huber. Society for Industrial and Applied Mathematics, 1977. v + 56 pages. $6.75.

This monograph deals with statistical techniques which are robust (i.e., insensitive) with regard to small deviations from the assumptions; primarily, it is concerned with the case where the underlying distribution differs slightly from the normal one. The author, one of the principal creators of the field, covers the following topics: qualitative and quantitative robustness; M-, L-, and R-estimates; asymptotic minimax theory, multiparameter problems, finite sample minimax theory; adaptive estimates.

Nonlinear ordinary differential equations. By D. W. Jordan and P. Smith. Oxford University Press, 1977. viii + 360 pages. $27.50 cloth; $14.95 paper.

This is a textbook in the qualitative theory of nonlinear differential equations. It is directed towards practical applications in mechanics, biology and electricity. It treats recent developments and subjects such as singular perturbations and phenomena associated with forced nonlinear equations. Topics include the phase plane, linear approximation, nonlinear damping, geometrical and nonlinear aspects, small-parameter expansions and singular perturbations, harmonics, subharmonics, entrainment and jump effects, and the formal treatment of the stability of systems. There are many exercises, with hints and answers given.

Inverse problems in quantum scattering theory. By K. Chadan and P. C. Sabatier. Springer-Verlag, New York, 1977. xxii + 344 pages.

The inverse problem in physics is to infer the forces on particles or the constitution of matter from the particles' motion or the propagation of radiation. Its study may be said to have begun with Lord Rayleigh's work on inferring the density distribution in vibrating strings from their frequencies of vibration, or in Marc Kac's 1966 lecture "Can one hear the shape of a drum?"

This book consists of seventeen chapters, of which the first two are introductory, surveying all useful results of potential scattering. The sixteenth chapter deals with approximate methods, the seventeenth with one-dimensional inverse problems, and the remaining thirteen chapters with spherically symmetric potentials. The latter fall into two parts: in chapters 3–9, the "result" analyzed is a phase shift (E), known as a function of E for all positive energies ("inverse problems at fixed l"); in chapters 10–15, problems in which the "experimental result" is the cross-section are analysed.