HYDROMAGNETIC FLOW AND HEAT TRANSFER OVER A STRETCHING SHEET*

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1. Introduction. Rott [1] studied the nonsimilar solution corresponding to the viscous flow at a stagnation point on a wall moving with constant velocity. A counterpart to this problem in which the free stream velocity is constant and the wall is being stretched with a velocity proportional to x (x being the distance along the wall) was recently investigated by Danberg and Fansler [2].

In the present paper we extend a specialized case of [2] to consider an electrically conducting fluid permeated by a uniform transverse magnetic field, the motion being caused solely by the stretching of the wall. A similarity solution for the velocity and heat transfer characteristics in the flow with uniform suction at the wall is obtained. This problem may have applications to polymer technology (where one deals with stretching plastic sheets) and metallurgy where hydromagnetic techniques have recently been used. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing, these strips are sometimes stretched. Mention may be made of drawing, annealing and tinning of copper wires. In all these cases the properties of the final product depend to a great extent on the rate of cooling. By drawing such strips in an electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final products of desired characteristics might be achieved. Another interesting application of hydromagnetics to metallurgy lies in the purification of molten metals from non-metallic inclusions by the application of a magnetic field [3].

2. Flow analysis. We consider the flow of an electrically conducting incompressible fluid (with electrical conductivity σ) past a porous wall coinciding with the plane $y = 0$, the flow being confined to $y > 0$. Two equal and opposite forces are introduced along the x-axis (Fig. 1) so that the wall is stretched keeping the origin fixed, and a uniform magnetic field B_0 is imposed along y-axis. The basic equations for the steady flow are, in the usual notation,

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} u, \tag{1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

where the induced magnetic field is neglected (which is justified for flow at small magnetic Reynolds number [4]). It is also assumed that the external electric field is zero and the electric field due to polarization of charges is negligible.

Let us introduce

$$u = Cx\, f'(\eta), \qquad v = -(\nu C)^{1/2} f(\eta), \qquad \eta = (C/\nu)^{1/2} y, \tag{3}$$

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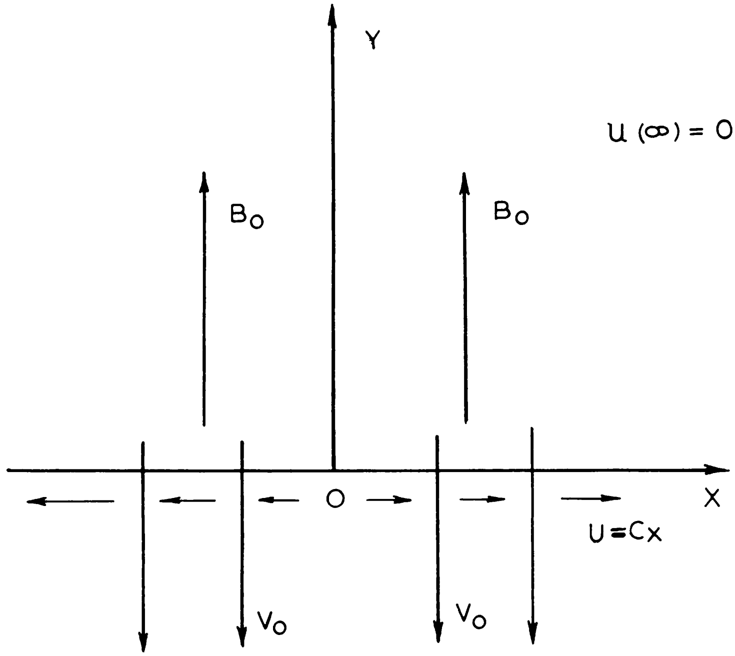

FIG. 1. Sketch of the physical problem.

where $u = Cx$ represents the velocity of the wall with $C > 0$ and a prime denotes differentiation. Clearly u and v in (3) satisfy (2). Substitution of (3) in (1) gives

$$f'^2 - ff'' = f''' - (\sigma B_0^2/\rho C)f' \tag{4}$$

subject to

$$f'(0) = 1, \qquad f(0) = v_0/(\nu C)^{1/2}, \qquad f'(\infty) = 0, \tag{5}$$

where $v_0(>0)$ denotes the suction velocity at the wall, the fluid being at rest at infinity.

A solution of (4) satisfying $f'(\infty) = 0$ is sought in the form

$$f(\eta) = A + B \exp(-\alpha\eta), \tag{6}$$

where A, B and α are constants with $\alpha > 0$. Substitution of (6) in (4) and the use of the remaining boundary conditions of (5) give

$$\alpha = \tfrac{1}{2}\{A + [A^2 + (4M)]^{1/2}\}, \tag{7}$$

$$A = \{R(1 + 2M) + [R^2 + 4(1 + M)]^{1/2}\}/2(1 + M), \tag{8}$$

$$B = -2/\{R + [R^2 + 4(1 + M)]^{1/2}\}, \tag{9}$$

where

$$R = v_0/(\nu C)^{1/2}, \qquad M = \sigma B_0^2/\rho C. \tag{10}$$

Fig. 2 shows that $f'(\eta)$ decreases with increase in R for a fixed value of the magnetic parameter M. As would be expected, the dimensionless transverse velocity $f(\eta)$ at a given position increases with R. Fig. 3 shows that for fixed R, $f'(\eta)$ decreases with increase in M. As M increases, the Lorentz force which opposes the flow also increases and leads to

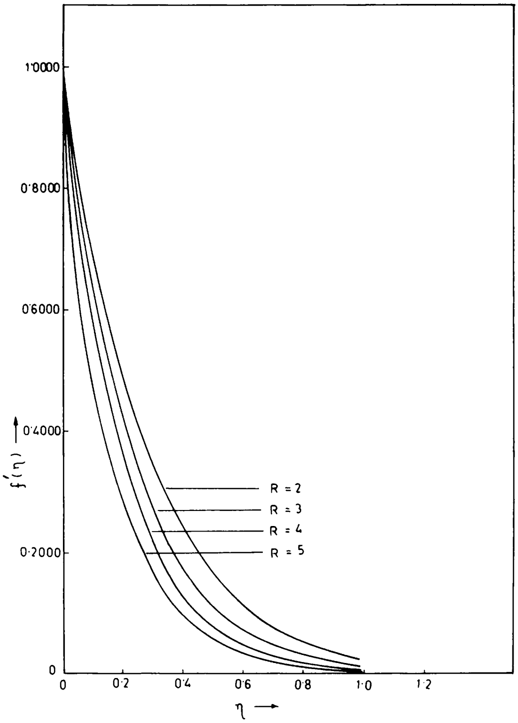

FIG. 2. Variation of $f'(\eta)$ for several values of R with $M = 5$.

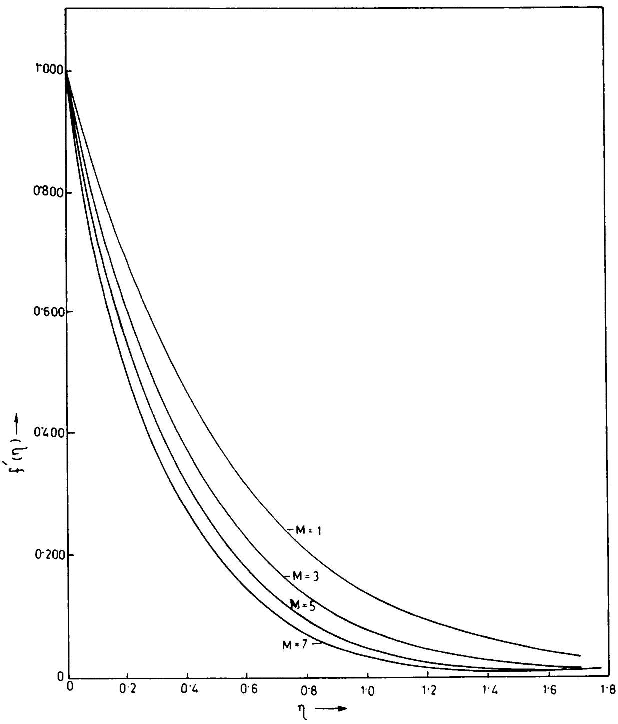

FIG. 3. Variation of $f'(\eta)$ for several values of M with $R = 1$.

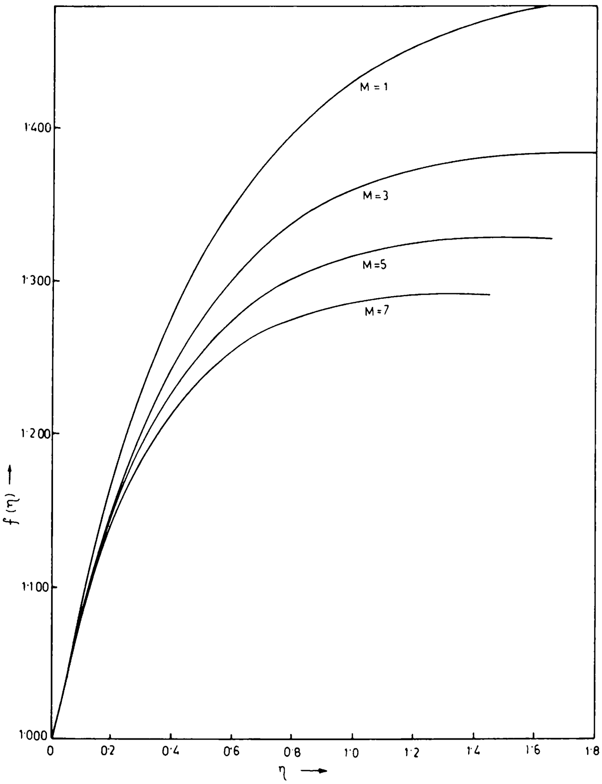

FIG. 4. Variation of $f(\eta)$ for several values of M with $R = 1$.

enhanced deceleration of the flow. It may be seen from Fig. 4 that for fixed R, the transverse velocity decreases with increase in M due to the inhibiting influence of the Lorentz forces.

3. Heat transfer. By using boundary layer approximations and neglecting viscous and ohmic dissipation, the equation for temperature T is given by

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_1 \frac{\partial^2 T}{\partial y^2}, \tag{11}$$

where α_1 is the thermal diffusivity. The boundary conditions are

$$T = T_\omega \quad \text{at } y = 0, \qquad T \to T_\infty \quad \text{as } y \to \infty, \tag{12}$$

where T_ω and T_∞ are constants. Introducing $\theta(\eta) = (T - T_\infty)/(T_\omega - T_\infty)$ and the Prandtl number $\Pr = \nu/\alpha_1$, Eq. (11) gives, on using (3),

$$-f\,\theta' = (1/\Pr)\,\theta'' \tag{13}$$

TABLE 1. $R = 1$, $\Pr = 0.73$ (air).

M	5	10	15	20
$-\theta'(0)$	0.9154	0.8837	0.8645	0.8522

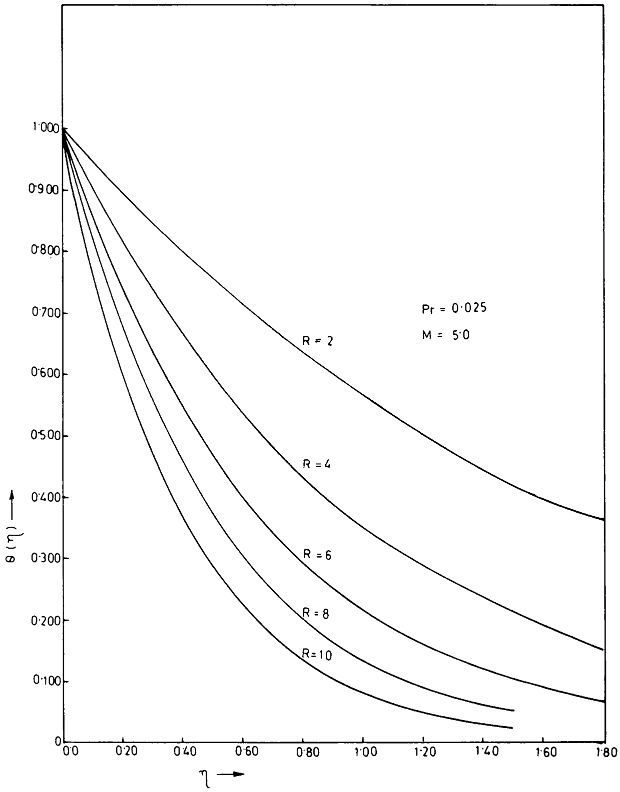

FIG. 5. Variation of $\theta(\eta)$ for several values of R.

subject to

$$\theta(0) = 1, \qquad \theta(\infty) = 0. \tag{14}$$

Substitution from (6) in (13) gives, on integration, the solution for $\theta(\eta)$ satisfying (14) as

$$\theta(\eta) = \int_0^{-(\mathrm{Pr}B/\alpha)\exp(-\alpha\eta)} t^{(\mathrm{Pr}A/\alpha)-1} \exp(-t)\,dt \Big/ \int_0^{-(\mathrm{Pr}B/\alpha)} t^{(\mathrm{Pr}A/\alpha)-1} \exp(-t)\,dt. \tag{15}$$

The dimensionless heat transfer coefficient at the wall given by $-\theta'(0)$ is now computed from (15) for several values of M by using an algorithm due to Bhattacharjee [5] for calculating the incomplete gamma functions appearing in (15). The results are shown in Table 1.

TABLE 2. Values of $\theta(\eta)$.

η \ M	2.0	4.0	6.0	8.0
1.0	0.95543	0.95548	0.95552	0.95555
2.0	0.91285	0.91294	0.91301	0.91308
3.0	0.87217	0.87229	0.87240	0.87249
4.0	0.83330	0.83346	0.83359	0.83371

For application of this problem to liquid metals, we must take $\Pr \ll 1$. Expanding exp $(-t)$ under the integral signs in (15) in series form and assuming A, B, α of order 1, we get by term-by-term integration

$$\theta(\eta) = \left[\frac{(X_1 \exp(-\alpha\eta))^P}{P} + \sum_{n=1}^{\infty} (-1)^n \frac{(X_1 \exp(-\alpha\eta))^{P+n}}{n!\,(P+n)}\right] \Bigg/ \left[\frac{X_1^{\,P}}{P} + \sum_{n=1}^{\infty} (-1)^n \frac{X_1^{\,P+n}}{n!\,(P+n)}\right], \tag{16}$$

where

$$X_1 = -\frac{\Pr\cdot B}{\alpha}, \qquad P = \frac{\Pr\cdot A}{\alpha}. \tag{17}$$

Using the above expressions, calculations were performed on an EC 1030 computer for mercury with $\Pr = 0.025$. Fig. 5 shows that for $M = 5$, temperature decreases with increase in suction. Table 2 gives the temperature distribution for liquid sodium with $\Pr = 0.00741$ and $R = 6$. Thus at a given position temperature rises with increase in M.

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