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QUARTERLY OF APPLIED MATHEMATICS

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BOOK REVIEWS

SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ($'$), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp\,[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2+b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(\pi x/2b)}{\cos(\pi a/2b)} \text{ is preferable to } \frac{\cos\dfrac{\pi x}{2b}}{\cos\dfrac{\pi a}{2b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1}\sin u\,du \text{ is preferable to } \int \frac{\sin u}{u}\,du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx)\cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n]\cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n)\cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*—S. Timoshenko, *Strength of materials,* vol. 2, Macmillan and Co., London, 1931, p. 237; *for periodicals*—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions,* Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids,* but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten.*

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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—BOOKS RECEIVED—

Contributions to universal algebra. (Colloquia Mathematica Societatis János Bolyai, Vol. 17) Edited by B. Csákány and J. Schmidt. North-Holland Publishing Company, 1977. 607 pp. $86.95.

This volume consists of 41 papers, most of which are the expanded written version of lectures delivered at the colloquium on universal algebra held in Szeged, Hungary. A number of invited contributions are also included.

The majority of the papers are concerned with results in the general theory of algebraic systems, while the rest discuss such topics as problems on groupoids, semigroups, groups and lattices, all of these concerning or suggested by universal algebraic notions. Finally, a list of unsolved problems compiled by the colloquium participants is appended.

Guide to the study of animal populations. By James T. Tanner. University of Tennessee Press, 1978. 186 pp. $8.95.

The purpose of this book is to describe the characteristics of animal populations, the methods of their measurement, and their interactions. The concepts and terms of demography are used where appropriate—which is not always, since most data on animal populations cannot be obtained as in demography: most animals will not stand still and be counted, and individuals have no official identity. Mathematical proofs are rarely given. A basic course in calculus and in statistics is assumed.

Chapter headings are: 1. Density and dispersion measured by sampling; 2. Estimating density by specialized methods; 3. Mark-recapture methods of measuring population characteristics; 4. Sex and age composition; 5. Introduction to population rates; 6. Mortality and survival; 7. Reproduction, migration, and recruitment; 8. Measurement and analysis of population growth rate; 9. The mathematical relations between population characteristics; 10. Prediction of population changes.

Recent developments in statistics. Edited by J. R. Barra, F. Brodeau, G. Romier, and B. van Cutsem. Elsevier North-Holland, New York, 1977. 808 pp.

These are the proceedings of the 1976 European Meeting of Statisticians, held at the University of Grenoble. It features an opening lecture by Harald Cramér entitled "L'Heritage de Bernoulli" and includes papers on the various subjects of classical statistics: theory of estimation and testing hypotheses, non-parametric problems, statistical inference in the classical sense and for stochastic processes, experimental designs, multivariate analysis, stochastic approximation, asymptotic expansions, time series, linear models, etc. Also, numerous applications are presented concerning the medical and social sciences, demography, genetics, physics, etc. There is a considerable number of papers devoted to data analysis and to the relation between computer science and statistics.

Analytical dynamics of discrete systems. By Reinhardt M. Rosenberg. Plenum Press, New York and London, 1977. xvii + 424 pp. $32.50.

This advanced textbook is directed to seniors and beginning graduate students in engineering and the natural sciences and examines the development of classical particle mechanics from Newton to Lagrange. Most of the chapters contain a large number of worked examples, as well as a set of suggested exercises. The author adopts a geometrical approach to the study of dynamics. He provides a logical transition from Newtonian to Lagrangean mechanics by demonstrating the need for a basically different classification of forces in these two theories and the necessity of replacing Newton's third law by d'Alembert's principle. In the first seven chapters, he includes detailed review of: Newtonian mechanics, with attention paid to the historical setting in which it developed; the representation of motion as a trajectory in configuration space, event space, and other spaces; constraints; rigid body kinematics and kinetics. The major portion of the book deals with the theory and application of Lagrangean mechanics, beginning with precise definitions of "virtual displacements," "virtual velocity," and "virtual work." The author then discusses the principles of Hamilton and of least action, the theory of contemporaneous and non-contemporaneous variations, the theory of generalized coordinates and forces, and derivations of Lagrange's equations. Special chapters on celestial problems, gyrodynamics, and impulsive motion are also included.

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Markov chain models—rarity and exponentiality. (Applied Mathematical Sciences Series, Vol. 28.) By Julian Keilson. Springer-Verlag, New York, 1979. xiii + 184 pages. $12.00.

This work builds on ideas which have been motivated by questions arising in the study of neural firing, human metabolism, congestion, inventories and system reliability. The object of the effort is the working tools needed to quantify the ergoidc and transient behavior of systems of many degrees of freedom such as arise in the study of those applications. The structure of passage time densities and exit time densities to and from subsets of the state space is discussed at length, with special emphasis on the exponentiality and related structural properties present in these densities when the sets visited are seen infrequently. The underlying themes of reversibility in time and complete monotonicity are of particular importance. It is assumed that the reader is familiar with the elements of probability theory and stochastic processes as found, for example, in Feller, Vol. I, and part of Feller, Vol. II. The chapter headings are: 1. Discrete-time Markov chains; reversibility in time; 2. Markov chains in continuous time; uniformization; reversibility; 3. More on time-reversibility; potential coefficients; process modification; 4. Potential theory, replacement, and compensation; 5. Passage time densities in birth-death processes; distribution structure; 6. Passage times and exit times for more general chains; 7. The fundamental matrix, and applied topics; 8. Rarity and exponentiality; 9. Stochastic monotonicity.

Basic applied statistics. (Volume 27: Statistics: Textbooks and Monographs) By B. L. Raktoe and J. J. Hubert. Marcel Dekker, Inc., New York, 1979. x + 426 pages. $14.50.

This volume is based on lecture notes for an introductory course in statistics for students in the social sciences.

Fluid mechanics: A concise introduction to the theory. By Chia-Shun Yih. West River Press, Michigan, 1978. xviii + 622 pages. $16.50.

This is a new, corrected edition of the work first published in 1969. There are several corrections and improvements in the text. The book is intended primarily for first- or second-year graduate students, and no previous acquaintance with fluid mechanics is assumed. There are ten chapters, covering: fundamentals, the basic equations, general theorems for inviscid fluids, irrotational flows of an inviscid fluid, waves in an incompressible fluid, the dynamics of inviscid compressible fluids, effects of viscosity, heat transfer and boundary layers of a gas, hydrodynamic stability, turbulence.

Advances in applied mechanics, vol. 18. Edited by Chia-Shun Yih. Academic Press, New York, 1978. vii + 353 pages. $0.00.

This volume contains the following articles: 1. Aspects of invariance in solid mechanics (Rodney Hill); 2. The optimum theory of turbulence (F. M. Busse); 3. Computational modeling of turbulent flows (John L. Lumley); 4. Unsteady separation according to the boundary-layer equation (Shan-Fu Shen); 5. The theory of ship motions (J. N. Newman); 6. Numerical methods in fluid dynamics (C. K. Chu).

Calculus: an historical approach. By W. M. Priestley. Springer-Verlag, New York, 1979. xvii + 441 pages. $14.80.

"This book is for students being introduced to calculus, and it covers the usual topics, but its spirit is different from what might be expected. Though the approach is basically historical in nature, emphasis is put upon ideas and their place—not upon events and their dates. Its purpose is to have students to learn calculus first, and to learn incidentally something about the nature of mathematics." It aspires to aid a students interested in either (1) receiving an elementary introduction to the basic ideas of calculus; or (2) learning "about" calculus, as a significant element in the history of thought. The chapter headings are: 1. Tokens from the Gods—variables, functions and limits; 2. The spirit of Greece—pre-calculus mathematics; 3. Sherlock Holmes meets Pierre de Fermat—derivatives; 4. Optimistic steps—techniques of optimization; 5. Chains and change—instantaneous rates; 6. The integrity of ancient and modern mathematics—integrals and antiderivatives; 7. A circle of ideas—calculus of trigonometric functions; 8. House of integrals—fundamental principles revisited; 9. The central height—logarithmic and exponential functions; 10. Romance in reason—17th-century mathematics.

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Lie-Bäcklund transformations in applications. By Robert L. Anderson and Nail H. Ibragimov. SIAM, Philadelphia, 1979. x + 124 pages. $14.50.

This monograph has several purposes. It provides an introduction to the classical treatment of invariance properties of differential equations under a type of surface transformation known as a Bäcklund transformation. The classical treatment, as established primarily in the papers of S. Lie and A. V. Bäcklund is discussed as well as recent examples of Bäcklund transformations as applied to nonlinear optics (sine-Gordon), nonlinear waves (Korteweg-de Vries and Liouville), a turbulence model (Burgers) and quantum mechanics (nonlinear Schrödinger). A second purpose is to present recent results which establish the group theoretical content of a generalization of Lie's contact transformation groups and its application to differential equations. These "Lie-Bäcklund transformation groups" have application to equations which describe the time evolution of systems encountered in engineering, hydrodynamics, mechanics, physics, control mechanisms, ecology, economics and biochemistry. The book discusses examples of these transformations to mechanics, gas dynamics, hydrodynamics, relativity and quantum mechanics.

Stochastic analysis. Edited by Avner Friedman and Mark Pinsky. Academic Press, New York, 1978. xii + 340 pages. $18.00.

These are the proceedings of an international conference on stochastic analysis, held in April 1978 at Northwestern University, the purpose of which was to bring together leading workers in stochastic differential equations and related areas, specifically in foundations, stochastic optimization, stochastic differential geometry, limit theorems and martingale methods. The following authors are represented by the twenty papers: Robert F. Anderson, Alain Bensoussan, M. Donsker and S. R. S. Varadhan, E. B. Dynkin, D. Elworthy, W. H. Fleming and M. Viot, A. Friedman, B. Fristedt and S. Orey. C. J. Holland, R. Holley and D. W. Stroock, N. Ikeda and S. Manabe, K. Ito, P. Malliavin, G. Papanicolaou, E. Pardoux, M. Pinsky, M. Rober, H. Tanaka, S. Watanabe, and P. Malliavin.

Theoretical numerical analysis. An introduction to advanced techniques. By Peter Linz. John Wiley and Sons, New York, 1979. xi + 228 pages. $17.95.

This is a volume in the Wiley series Pure and Applied Mathematics. It assumes no mathematical background beyond advanced calculus and develops the ideas of functional analysis where needed. The book strives to close the communications gap between books emphasizing special techniques, clever tricks and ad hoc procedures on the one hand, and highly abstract treatments having little apparent connection with actual problems on the other. Table of Contents: Part I, Basic concepts: 1. Review of functional analysis; 2. Approximation theory; Part II, Theoretical aspects of computational mathematics: 3. Numerical integration; 4. The approximate solution of linear operator equations; 5. The nonlinear inverse problem; Part III. Special topics: 6. The approximate solution of linear operator equations of the second kind; 7. The finite element method; 8. The solution of nonlinear operator equations by discretization; 9. The solution of improperly posed problems.

Mathematical modelling techniques. By R. Aris. Fearon-Pitman Publishers, California, 1979. x + 191 pages. $15.00.

This is volume 24 of the series "Research Notes in Mathematics". The chapter headings of this attempt to present the essence of the art and science of mathematical modelling are as follows: 1. What is a model? 2. The different types of model; 3. How to formulate a model; 4. How should a model be manipulated into its most responsive form; 5. How should a model be evaluated? There are three appendices each containing a detailed treatment of an example chosen to illustrate the points made in the text: longitudinal diffusion in a packed bed; the coated tube chromatograph and Taylor diffusion; the stirred tank reactor. These examples and those introduced in the text are often connected with the mathematical theory of chemical reactors. No background in chemistry or engineering is assumed.

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Integral transforms in science and engineering. By Kurt B. Wolf. Plenum Press, New York, 1979. xiii + 489 pp. $32.50.

The purpose of this volume is to present a systematic expansion of the theory and some of the applications of the finite and integral transforms associated with the names of Fourier, Bessel, Laplace, Hankel, Gauss, Bargmann, and others. Its scope is best indicated by its table of contents. Part I. Finite-dimensional vector spaces and the Fourier transform: 1. Concepts from complex vector analysis and the Fourier transform; 2. The application of Fourier analysis to the uncoupling of lattices; 3. Further developments and applications of the finite Fourier transform. Part II. Fourier and Bessel series; 4. Function vector spaces and Fourier series; 5. Fourier series in diffusion and wave phenomena; 6. Normal mode expansion and Bessel series. Part III. Fourier and related integral transforms; 7. Fourier transforms; 8. Integral transforms related to the Fourier transform. Part IV. Canonical transforms: 9. Construction and properties of canonical transforms; 10. Applications to the study of differential equations. Three appendices.

New trends in dynamic system theory and economics. Ed. by Masanao Aoki and Angelo Marzollo. Academic Press, New York, 1979. xii + 412 pp. $21.00.

These are the proceedings of a seminar held at the International Center for Mechanical Sciences at Udine, Italy, in September 1977. It is arranged in three parts. Part I consists of paper dealing with information patterns and uncertainty. Decision problems of agents with different or imperfect information or under uncertainty are discussed by Basar, Aoki, Leitman and Wan, Liu and Sutinen, Auloge, and Rustem and Velupillai. In part II, recent advances in optimal control theory and application of control theory are presented by Blaquiere, Haurie, Schiavoni et al., Wiese, Rempala, Idzik, and Wieczorek. Contributions in Part III are on various aspects of disequilibrium analysis. Disequilibrium analysis of a macroeconomic model is represented by Honkapohja, disequilibrium analysis of a different sort (reflecting the influence of the approach to the analysis of dynamic processes and global analysis by Smale) is discussed in contributions by Cornet, Marzollo et al., St. Pierre, Galeotti, and Birchenhall. The last three papers may be said to be on "modern" approaches to tâtonnement processes.

Decision networks. By N. A. J. Hastings and J. M. C. Mello. John Wiley & Sons, Inc., 1978. ix + 196 pp. $25.50.

The decision network method presented in this book is a scientific aid to management decision making. It relies on the use of diagrams and a calculation procedure which are closely akin to those used in the critical-path method.

A decision network is a planning model for a system. It consists of circles which represent states of the system and arrows which represent actions available to management. A path thrugh the network represents a feasible operating plan or policy. The applications cover manpower planning, budget allocation, product development and marketing, replacement policies for plant, process planning and routing, production scheduling and stock control, etc.

General lattice theory. By G. Grätzer. Academic Press, New York, 1978. xiii + 381 pages. $35.00.

It is the goal of this work to discuss in depth the basics of general lattice theory and thus aims to include the most important results and research methods of the entire field. Chapter 1 gives a concise development of the basic concepts, emphasizing the use of diagrams. Chapter 2 develops distributive lattices, including representation theorems, congruences, congruence lattices of general lattices, Boolean algebras, and topological representations. Chapter 3 discusses congruences and ideals of general lattices, and chapter 4 investigates in detail the connection between lattice theory and geometry. Chapters 5 and 6 deal with two new areas of investigation: equational classes of lattices and free products of lattices, respectively. The almost 900 examples form an integral part of the book. There is a bibliography of over 750 entries, and a section with 193 "research problems."

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A course in differential geometry. By Wilhelm Klingenberg. Springer-Verlag, New York, 1978. xii + 178 pages. 14.80.

This book (volume 51 of Graduate Texts in Mathematics) has its origins in a one-semester course in differential geometry given at Göttingen, Mainz and Bonn. The lectures offer an introduction to the classical differential geometry of curves and surfaces, and assume nothing more than a knowledge of basic analysis, real linear algebra and Euclidean geometry (for the last chapter familiarity with the topology of compact surfaces would be useful). The table of contents reads: 0. Calculus in Euclidean space; 1. Curves; 2. Plane curves: Global theory; 3. Surfaces: Local theory; 4. Intrinsic geometry of surfaces: Local theory; 5. Two-dimensional Riemannian geometry; 6. The global geometry of surfaces.

Dynamic programming and its applications. Ed. Martin L. Puterman. Academic Press, New York, San Francisco, London, 1978. xv + 410 pp. $

This volume presents the proceedings of an International Conference on Dynamic Programming and its Applications held at the University of British Columbia, April 14-16, 1977. It consists of four parts: Surveys (six papers), applications (six papers), theory (eight papers) and reports of a panel discussion.

Collected papers of G. H. Hardy. Ed. by a Committee appointed by the London Mathematical Society. Oxford University Press, Oxford, England, 1979. xviii + 897 pp. $78.00

This is the last volume of Hardy's papers (including joint papers with J. E. Littlewood and others). It is divided into six sections: 1. Papers on integral equations and integral transforms (34 papers from 1904 to 1947); 2. Miscellaneous papers (on set theory, differential equations, the Hardy-Weinberg law, and elementary and expository notes); 3. Questions from the "Educational Times"; 4. Obituary notices by Hardy (Ramanujan, Jordan, Mittag-Leffler, Glaisher, Bromwich, Paley, Hobson, Landau, W. H. Young, Hilbert, Lebesgue, Wilton); 5. Book reviews by Hardy; 6. Obituary notices of Hardy, and other writings concerning his life and work.

Proving programs correct. By Robert B. Anderson. John Wiley & Sons, Inc., New York, 1979. viii + 184 pp. $8.95 paperback.

The emphasis in this book is on rather informal correctness proofs of the type programmers can employ in trying to convince themselves systemmatically of their program's correctness. Chapter headings: I. Mathematical induction. II. Proving the correctness of flowchart programs. III. Proving the correctness of programs written in a standard programming language. IV. Proving the correctness of recursive programs. V. Current research related to proving program correctness.

Numerical solution of differential equations. By I. Fried. Academic Press, New York, 1979. xiii + 261 pp. $23.50.

This is a volume in the series Computer Science and Applied Mathematics. It covers initial, boundary and eigenvalue problems, with emphasis on the finite elements method. The chapter headings are: 1. Finite differences; 2. Two-point boundary value problems; 3. Variational formulations; 4. Finite elements; 5. Discretization accuracy; 6. Eigenproblems; 7. Algebraic properties of the global matrices; 8. Equation of heat transfer; 9. Equation of motion; 10. Wave propagation.

Nonlinear mappings of monotone type. By Dan Pascali and Silviu Sburlan. Sijthoff & Noordhoff International Publishers, Netherlands, 1979. x + 341 pp. $43.00.

This book is a self-contained presentation of concepts and results in the recent theory of nonlinear mappings which generalize monotype operators. The text follows essentially four interrelated directions: nonlinear mappings of monotone type, Hammerstein equations, odd operators and variational problems. The applications of these topics are referred to the existence theory of solutions to nonlinear elliptic differential equations. The text is divided into six sections: 1. Background in functional analysis (3 chapters); 2. Methods of compactness (4 chapters); 3. Nonlinear mappings of monotone type; 4. Hammerstein equations; 5. Homotopy arguments; 6. Variational problems and inequalities.

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Bifurcation of maps and applications. By G. Iooss. North-Holland Publishing Co., Amsterdam and New York, 1979. x + 232 pages. $31.75.

These notes cover and extend a course given by the author at the University of Minnesota during the fall of 1977. Their spirit is, as far as possible, to give explicit formulae for the bifurcated objects and the simplest possible way to use them. The presentation is mainly analytic and no global results are given. The chapter headings are: 1. Stability or instability of a fixed point of a map in a Banach space; 2. Bifurcation of fixed points in IR; 3. Hopf bifurcation in IR^2; 4. Subharmonic bifurcations of fixed points in IR^2-strong resonance; 5. Invariant manifolds and applications; 6. Bifurcation of an invariant circle into an invariant 2-torus for a one-parameter family of maps.

Alexander Weinstein Selecta. Ed. by J. B. Diaz. Fearon-Pitman Publishers, Belmont, California, 1979. xxi + 629 pages. $36.75.

This volume contains a brief biographical sketch of Weinstein and reprints of 109 of his papers, dating from 1923 to 1975. Among the subjects covered are the following: representation of the linear group of matrices, Helmholtz's problem for jets; boundary problems in an infinite strip; intermediate problems for eigenvalues ("Weinstein's method"); singular partial differential equations.

Branching processes (Advances in Probability and Related Topics Series, Volume 5). Edited by Anatole Joffe and Peter Ney. Marcel Dekker, New York, 1979. x + 322 pages. $34.50.

This volume presents a current survey of various aspects of the theory of branching processes, based on lectures given at a conference held August 11 to 20, 1976. There are eleven papers on many of the major lines of work of the past decade or so.

Practical finite mathematics. By Gareth Williams. Allyn and Bacon, Inc., Boston, 1979. viii + 408 pages. $14.95.

This text has been designed for courses in finite mathematics to students in the social and behavioral sciences, business, economics and liberal arts. It treats matrices, graph theory, linear equations, linear programming, probability, Markov chains, statistics, game theory and computer programming, all on an elementary level.

Positively ordered semigroups. (Lecture Notes in Pure and Applied Mathematics, Volume 42). By Motupalli Satyanarayana. Marcel Dekker, Inc., New York, 1979. v + 102 pages. $12.50.

The study of positively totally ordered semigroups drew the attention of Hölder in connection with the problem of determining when a totally ordered semigroup is embeddable in the additive semigroup of positive real numbers. This monograph is the first systematic study of the topics and consolidates the essential results.

Semigroups and combinatorial applications. By Gerard Lallement. John Wiley & Sons, New York. xi + 376 pages. $27.50.

The purpose of this book—a volume in the Wiley series on Pure and Applied Mathematics—is to present those parts of the theory of semigroups that are directly related to automata theory, algebraic linguistics, and combinatorics. The book is intended as a textbook for graduate students in mathematics and computer science and as a reference book for researchers interested in associative structures. The chapter headings are: 1. Elementary definitions and examples; 2. Green's relations; 3. Simple semigroups; Rees-Suschkewitsch theorem; 4. The prime decomposition theorem for finite monoids, 5. Free monoids. Languages, and modes; 6. Automata, national languages, and their syntactic monoids; 7. Star problems for rational languages; 8. Rational prefix codes; 9. Algebraic languages; 10. The Burnside Problems and related questions; 11. Rearrangements, factorizations, and MacMahon's master theorem.

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(Mathematical foundations for a general theory) Mechanizing hypothesis formation. By P. Hájek, T. Havránek. Springer-Verlag, Berlin, 1978. xv + 396 pages. $24.00.

The question to which this book addresses itself is "Can computers formulate and justify scientific hypotheses? Can they comprehend empirical data and process them rationally, using the apparatus of modern mathematical logic and statistics to try to produce a rational image of the observed empirical world?"

There is an introductory chapter which explains the subject of the authors' theory of hypothesis formation or "logic of discovery". The remainder is divided into two parts: A—the "logic of induction", and B—the "logic of suggestion", the two divisions of the "logic of discovery". In part A the authors define and investigate formal calculi appropriate for formalizing observational and theoretical languages of scientific theories based on empirical data. The definitions are motivated by statistical considerations, usually neglected in Artificial Intelligence. In part B the authors give a formal definition of "General Unary Hypotheses Automaton (GUHA)" methods of automated formation of hypotheses, and construct and study sufficiently complex particular GUHA methods capable of machine realization.

Some knowledge of the very basic parts of recursion theory, predicate logic and probability theory is assumed.

Mathematical logic: Proceedings of the First Brazilian Conference. (Lecture Notes in Pure and Applied Mathematics, vol. 39) Edited by A. I. Arruda, N. C. A. da Costa and R. Chuaqui. Marcel Dekker, Inc., New York, 1978. xii + 303 pages. $29.50.

These are the proceedings of the First Brazilian Conference on Mathematical Logic, held at the State University of Campinas in July 1977. They provide a review of the work being done by Latin American logicians. The papers range over the spectrum of work being done in the field, but the topics covered can be classified into three basic subject areas: non-classical logics, foundations of set theory, and algebra and logic.

Infinitely divisible point processes. By K. Matthes, J. Kerstan and J. Mecke. John Wiley & Sons, Inc., 1978. xii + 532 pages. $42.50.

This volume in the Wiley Series in Probability and Mathematical Statistics is an English translation (by B. Simon) based on a revised German edition of the book "Unbegrenzt teilbare Punktprozesse", first published by Akademie-Verlag, Berlin, in 1974. The translation was produced on the initiative of Professor D. G. Kendall.

The theory of of point processes has developed both from an examination of models in revewal theory and similar situations and also from an attempt to formulate and answer some theoretical questions about special classes of stochastic processes.

This book gives a self-contained presentation of the fundamental concepts and results of this branch of probability theory which has been developed over the last ten years by the authors and their collaborators. A particularly important topic developed fully is the definition and establishment of the properties of the Campbell measure of a point process on an arbitrary complete separable metric space. The characterization of infinitely divisible point processes by means of functional equations is also treated in detail. Chapter headings: 1. Basic concepts. 2. The canonical representation of infinitely divisible probability distributions, 3. Weak convergence of distributions, 4. Cluster fields, 5. The Campbell measure, 6. Stationary distributions, 7. Cox distributions, 8. The Palm measure, 9. The Palm distribution, 10. The generalized Palm-Khinchin theorem, 11. Homogeneous cluster fields, 12. Spatially homogeneous branching processes.

Modern problems in elastic wave propagation. Edited by J. Miklowitz and J. D. Achenbach. John Wiley and Sons, Inc, 1978. x + 561 pages. $32.50.

These are the proceedings of a symposium organized by the International Union of Theoretical and Applied Mechanics and held at Northwestern University, Evanston, Illinois in September 1977. The papers are organized into seven parts: 1. Methods—experimental, numerical, and analytical (six papers); 2. Wave diffraction and scattering (seven papers); 3. Waves in anisotropic and inhomogeneous media (three papers); 4. Waves in waveguides (three papers); 5. Surface waves (three papers); 6. Seismology problems (three papers); 7. Abstracts of short papers. There is also an abstract of a special lecture by C. L. Pekeris on a certain solution of the tidal equations.

Relativistic theories of materials. (Springer Tracts in Natural Philosophy, vol. 29) By A. Bressan. Springer-Verlag, Berlin, 1978. xiv + 290 pp. $49.00.

The purpose of this book is to describe the foundations of the general relativistic theories that include constitutive equations, and to present some applications, mainly to elastic waves, of these theories. The tract is divided into two parts. In the first part only the Eulerian point of view is considered; basic equations of general relativity, other than constitutive equations, are stated in full generality (except for couple stresses which are considered in part 2). Part 1 also thoroughly covers fluids, including constitutive equations. In the second part, the Lagrangian point of view is introduced and some mathematical tools connected with materials coordinates are elucidated; by means of these tools, solids and materials of a very general kind—possibly capable of couple stresses—are dealt with. An introductory chapter reviews the main ideas underlining the origin and development of relativity: among others, the mass-energy equivalence principle, the debated general principle of relativity, its local form, and Fock's privileged harmonic co-ordinates. The remaining nine chapters are divided into two parts: I. Basic equations of gravitation, thermodynamics and electromagnetism, and constitutive equations from the Eulerian point of view (four chapters) and II. Materials from the Lagrangian point of view (five chapters).

Numerical solution of partial differential equations: Finite difference methods. By G. D. Smith. Oxford University Press, 1978. xii + 304 pages. $23.50 cloth; $11.50 paper.

This volume in the Oxford Applied Mathematics and Computing Science Series is the second edition of the book first published in Oxford Mathematical Handbooks in 1965. It requires no background in numerical methods whatever. Substantial additions have been made to most chapters, outmoded material has been eliminated and all the work on iterative methods has been condensed into one chapter. Chapter headings: 1. Introduction and finite-difference formulae, 2. Parabolic equations, 3. Convergence, stability and consistency, 4. Hyperbolic equations and characteristics, 5. Elliptic equations and systematic iterative methods.

Complex analysis, 3rd edition. By Lars V. Ahlfors. McGraw-Hill Book Co., New York, 1978. xiv + 331 pages. $21.50.

This is the third edition of the elementary text first published in 1953. There are no radical innovations in this edition and the geometric approach is retained in the introductory chapters. In addition to corrections and classifications, the main difference between the second (1960) and third editions are as follows: notations and terminology have been modernized; a section on the change of length and area under conformal mapping has been added; there is a new and simple proof of Cauchy's theorem; a section on the Riemann zeta function has been added; large parts of the chapter on global analytic functions have been rewritten.

Path integrals, and their applications in quantum, statistical, and solid state physics. Edited by G. J. Papadopoulos and J. T. Devreese. Plenum Publishing Co., New York, 1978. x + 515 pages. $49.50.

These are the proceedings of a NATO-sponsored advanced study institute held at the University of Antwerp in July 1977. The development of path (or functional) integrals in relation to stochastic problems dates back to the early 1920's when Norbert Wiener obtained the fundamental solution to the diffusion equation using Einstein's joint probability of finding a Brownian particle in a succession of space intervals during a corresponding succession of time intervals. Dirac in the early 1930's inaugurated the path integral formulation of quantum mechanics. The major step, however, was Feynman's work in quantum and statistical physics, and quantum electrodynamics. Recent applications extend to continuum mechanics, the study of polymers and to many other branches of theoretical physics. The path integral approach serves, also, as a unifying entity among apparently disconnected disciplines.

The Proceedings are divided into three parts. The first ("Methods of functional integration": six papers) gives a general introduction consisting of concepts and methods of evaluation, interspersed with applications to Brownian motion, quantum mechanics, quantumn field theory and classical mechanics. In the second part ("Applications of the path integral approach": seven papers) more specialized topics from condensed matter and statistical mechanics, together with recent developments in the field, are considered. The third part gives an account of two seminars.

Energy modeling for a uncertain future. National Academy of Science, 1978, xvii + 225 pp., $9.75.

This is the report of the modeling resource group synthesis panel of the Committee on Nuclear and Alternative Energy Systems of the National Research Council.

There are five chapters and six appendices in this report: 1. Options, choices and their probable outcome, 2. The work of the modeling resource group, 3. Findings, 4. An exploratory analysis of research, development and demonstration programs under uncertainty. Appendices: A. Description of energy models used by the modeling resource group. B. Definitions and explanations of assumptions underlying the values of certain driving variables; C. Intermodel comparison of built-in and implicit price and income elasticities of demand for energy; D. An approach to the problem of comparing intermodel differences; E. Respondents to the second-round poll, and R & D costs.

Survival probabilities: the goal of risk theory. By H. L. Seal. John Wiley & Sons, New York, 1978. x + 103 pages. $24.50.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is concerned with numerical probability of ruin for realistic models for nonlife insurance companies and the associated computer programs. It deals with the probability of an insurance company surviving a limited time period during which the annual number of claims is expected to be reasonably constant but could increase significantly. It is a continuation of the author's earlier book, *Stochastic theory of a risk business* (John Wiley, 1969). The chapter headings are: 1. Historical introduction; 2. The choices for $p_n(t)$ and $B(\cdot)$; the simplest model of a nonlife company and the use of queueing techniques; 3. A computational accessory—the Laplace transform; 4. The probability of t-year survival; 5. Approximations and controls. There is an appendix with Fortran programs.

Elementary number theory. 2nd edition. By Underwood Dudley. W. H. Freeman and Company, San Francisco, 1978. ix + 249 pages. $15.00.

This is the second edition of the text first published in 1969. It is designed for a one-semester course, with minimal prerequisites—only elementary algebra and the properties of real numbers are assumed. The proofs are detailed, and numerical examples are included to illustrate ideas and show how these ideas originated. There are many exercises with answers to selected ones. One appendix contains problems which can be programmed for computer solution.

Introduction to mathematical economics. By Murray C. Kemp and Yoshio Kimura. Springer-Verlag, New York, 1978. vi + 249 pages. $19.80.

This book has the following two objectives: to provide a compact exposition of some sub-branches of mathematics which are of interest to economists but are underemphasized in textbooks, and to demonstrate the usefulness of mathematics by providing a systematic account of modern neoclassical economics. Chapter headings: 1. Linear inequalities; 2. Nonnegative matrices; 3. Some special matrices; 4. Stability analysis of some dynamic systems; 5. Neoclassical economies; 6. Neoclassical economies—Dynamics.

Contemporary developments in continuum mechanics and partial differential equations. Edited by Gilherme M. de la Penha and Luis Adauto J. Medeiros. North-Holland Publishing Co., Amsterdam, 1977. viii + 613 pages. $53.25.

These are the proceedings of an International Symposium, held in Rio de Janeiro in August 1977. It had as its aim to acquaint mathematicians, physicists and engineers with the principal lines of modern research and to assess accomplishments in the interrelated fields. Two short courses were given, by M. E. Gurtin and J. L. Lions, and C. Truesdell presented a critical appraisal of continuum thermodynamics and the kinetic theory of gases. These talks, as well as papers by the following are reproduced in this volume: S. S. Antman, G. S. S. Avila and D. G. Costa, J. B. Baillon and J. M. Chadam, T. B. Benjamin, H. Brezis, F. Cardoso and J. Hounie, M. O'Carroll, C. Do, B. D. Coleman, R. L. Fosdick, R. Glowinski and O. Pironneau, P. Podio Guidugli, D. D. Joseph, R. C. MacCamy, W. Noll, P. Nowosad, S. Osher, J. P. Puel, J. Serrin, W. A. Strauss, G. Svetlichny and P. Otterson, L. Tartar, R. Temam, W. O. Williams.