

QUARTERLY
OF
APPLIED MATHEMATICS

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QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

Manuscripts (two copies) submitted for publication in the QUARTERLY OF APPLIED MATHEMATICS should be sent to the Editorial Office, Box F, Brown University, Providence, R.I. 02912, either directly or through any one of the Editors or Collaborators. In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proofs only. The authors' institution will be requested to pay a publication charge of \$30.00 per page which, if honored, entitles them to 100 free reprints. Instructions will be sent with galley proofs.

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SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

Manuscripts: Papers should be submitted in original typewriting on one side only of white paper sheets and be double or triple spaced with wide margins. Marginal instructions to the printer should be written in pencil to distinguish them clearly from the body of the text.

The papers should be submitted in final form. Only typographical errors may be corrected in proofs; composition charges for all major deviations from the manuscript will be passed on to the author.

Titles: The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line to follow his name.

Mathematical Work: As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the typewriter should be carefully inserted in ink. Manuscripts containing pencilled material other than marginal instructions to the printer will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter *O*, between the numeral one (1), the letter *l* and the prime ([']), between alpha and *a*, kappa and *k*, mu and *u*, nu and *v*, eta and *n*.

The level of subscripts, exponents, subscripts to subscripts and exponents in exponents should be clearly indicated.

Dots, bars, and other markings to be set *above* letters should be strictly avoided because they require costly hand-composition; in their stead markings (such as primes or indices) which *follow* the letter should be used.

Square roots should be written with the exponent $\frac{1}{2}$ rather than with the sign $\sqrt{\quad}$.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol *exp* should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp [(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{(a^2 + b^2)^{1/2}}$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos (\pi x / 2 b)}{\cos (\pi a / 2 b)} \text{ is preferable to } \frac{\cos \frac{\pi x}{2 b}}{\cos \frac{\pi a}{2 b}}$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in printed formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

In handwritten formulas the size of parentheses, brackets and braces can vary more widely than in print. Particular attention should therefore be paid to the proper use of parentheses, brackets and braces. Thus,

$$\{[a + (b + cx)^n] \cos ky\}^2 \text{ is preferable to } ((a + (b + cx)^n) \cos ky)^2.$$

Cuts: Drawings should be made with black India ink on white paper or tracing cloth. It is recommended to submit drawings of at least double the desired size of the cut. The width of the lines of such drawings and the size of the lettering must allow for the necessary reduction. Drawings which are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying the drawings should be written on a separate sheet.

Bibliography: References should be grouped together in a Bibliography at the end of the manuscript. References to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (*for books*)—S. Timoshenko, *Strength of materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; (*for periodicals*)—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5)36, 354–372(1893). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow it.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details like ed., vol., no., chap., p.

Footnotes: As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

Abbreviations: Much space can be saved by the use of standard abbreviations like Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable, but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c.," even if this special abbreviation is defined somewhere in the text.

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Controlled Markov processes. By E. B. Dynkin and A. A. Yushkevich. Springer-Verlag, New York, 1979. (Translation from the Russian text, Nauka, Moscow, 1975.) xvii + 289 pp.

The book is very well written and well motivated. The topic is optimal control for Markov processes with discrete time parameter but general state and action spaces. The first several chapters develop the basic dynamic programming equations for the finite time horizon problem. Wellposedness, existence of optimal and ε -optimal policies and optimality within the pure Markov class is developed for a sequence of models, the most complicated being that with "Borel structure". The illustrative examples are well chosen and well developed. Similar results are then given for the problem over an infinite time interval, but where the total value is bounded. The average cost per unit time problem is discussed and related to the discounted problem for small discount factor. The "partial" observation case is treated by using the conditional probability as the state. Finally, a class of problems drawn from economics is treated, where the data have special convexity properties. Appendices on measurable selections and analytic sets complete the book.

The book is highly recommended. It provides one of the best ways available to learn the basic ideas of the discrete time parameter case.

H. J. KUSHNER (*Providence*)

Computational methods in nonlinear mechanics. Proceedings of the TICOM Second International Conference, Austin, Texas, U.S.A. 26–30 March 1979. Edited by J. T. Oden. North-Holland Publishing Co., Amsterdam and New York, 1980. viii + 540 pp. \$68.25.

This work contains a collection of 26 papers prepared by special and general lecturers at the conference. The papers represent new research results from leading specialists in the field of numerical analysis, computational methods, and theoretical and applied mechanics. The scope ranges from purely computational techniques for the numerical solution of partial differential equations, to practical considerations in the analysis of engineering systems by computational methods. Applications range from transonic flow problems to heat conduction problems, miscible displacement problems, and flow through porous media.

Table of integrals, series and products: corrected and enlarged edition. By I. S. Gradshteyn and I. M. Ryzhik. Academic Press, New York, 1980. xlv + 1160 pp. \$19.50.

From the preface: "When this book was originally planned by Gradshteyn, it was designed to assist mathematicians, scientists, and engineers, who at that time relied almost entirely on analytical solutions to their problems. These were usually obtained in complicated series form involving special functions, and for this purpose a comprehensive table of integrals was necessary. The success of this basic plan was amply demonstrated by the fact that by the time the English language translation was published in 1965, the original Russian version had gained Ryzhik as a coauthor and evolved through four separate editions to become a book of 1080 pages. . . . With the present-day ready availability of powerful computers, the necessity for cumbersome series type solutions has all but gone. In their place have come other types of analytical problems of equivalent complexity, some of a traditional nature, and some quite new and often arising from numerical analysis. They have in common the fact that they also make a demand on tables of this type, showing yet again the fundamental soundness of the original plan."

The chapter headings are: 0. Introduction. 1. Elementary functions. 2. Indefinite integrals of elementary functions. 3.–4. Definite integrals of elementary functions. 5. Indefinite integrals of special functions. 6.–7. Definite integrals of special functions. 8.–9. Special functions. 10. Vector field theory. 11. Algebraic inequalities. 12. Integral inequalities. 13. Matrices and related results. 14. Determinants. 15. Norms. 16. Ordinary differential equations. 17. Fourier and Laplace transforms.

Stochastic integration. By Michel Metivier and J. Pellaumail. Academic Press Inc., New York, 1980. xii + 196 pp. \$25.00.

This is a volume in the series Probability and Mathematical Statistics. The exposition is self-contained and requires as background only a basic knowledge of probability theory. The main idea of the book is that the construction and properties of the stochastic integral need very little from the machinery of the general theory of stochastic processes. The chapter headings are as follows: 1. Stochastic integral with respect to π -processes. 2. The Itô formula. 3. Stochastic integral equations. 4. Martingales and semimartingales. 5. Stochastic measures. 6. Special features of infinite-dimensional stochastic integration.

Network flow programming. By Paul A. Jensen and J. Wesley Barnes. John Wiley & Sons, New York, 1980. xv + 408 pp. \$25.55.

This book presents a synthesis of the more important techniques, both recent and traditional, that are related to network flow programming, at a level of first-year graduate or advanced undergraduate students. The chapter headings are: 1. Network flow models. 2. Modeling applications of network programming. 3. Formalization of network models. 4. Network manipulation algorithms. 5. The shortest path problem. 6. The maximum flow problem. 7. Pure minimum cost flow problems. 8. The out-of-kilter algorithm. 9. Network manipulation algorithms for the generalized network. 10. Generalized minimum cost flow problems. 11. The convex minimum cost flow problem. 12. Concave costs.

Continued from Page 42

Partial differential equations and Hilbert space methods. By Karl E. Gustafson. John Wiley & Sons, Inc., New York, 1980. xv + 270 pp. \$19.95.

The three aims of this book are (i) to provide an introduction at the undergraduate level, (ii) to expose the rudiments of the subject to graduate students in mathematics and to other interested scientists, and (iii) to avoid the pitfall of becoming too involved mathematically in related matters, such as complex or functional analysis. The prerequisite is calculus and elementary systems of differential equations. It includes sections on the elements of bifurcation theory and scattering theory.

Operator inequalities. By Johann Schröder. Academic Press, New York, 1980. xvi + 367 pp. \$39.50.

This book is concerned with inequalities that are described by operators. These operators may be matrices, differential operators, integral operators, etc. Special emphasis is given to differential operators related to boundary-value problems.

The main interest is in results that allow one to derive properties of an unknown element (vector, function, ...) from operator inequalities that involve this element. These properties usually are formulated as estimates in terms of order relations, norms, or other means. The estimation methods can, for instance, be employed to investigate operator equations (boundary value problems, linear algebraic systems of equations, etc.). In particular, these methods can be used to derive statements on the qualitative and quantitative behavior of solutions and to prove uniqueness statements. Moreover, the methods are helpful as a tool in proving the existence of solutions that satisfy certain estimates. The chapter headings are: 1. Some results on functional analysis. 2. Inverse-positive linear operators. 3. Two-sided bounds for second-order differential equations. 4. An estimation theory for linear and nonlinear operators, range-domain implications. 5. Estimation and existence theory for vector-valued differential operators.

Measure theory. By Donald L. Cohn, Birkhäuser Boston, Inc., Boston, 1980. ix + 373 pp. \$19.95.

This book is intended as a straightforward treatment of the parts of measure theory necessary for analysis and probability. The first five or six chapters form an introduction to measure and integration, while the last three chapters should provide the reader with some tools necessary for study and research in a number of directions. The chapter headings are: 1. Measures. 2. Functions and integrals. 3. Convergence. 4. Singed and complex measures. 5. Product measures. 6. Differentiation. 7. Measures on locally compact spaces. 8. Polish spaces and analytic sets. 9. Haar measure. Appendices.

Waves in layered media. 2nd edition. By L. M. Brekhovskikh. Academic Press, New York, 1980. xiii + 503 pp. \$58.00.

This second edition is volume 16 of the series Applied Mathematics and Mechanics; the first edition was published as volume 6 in 1960. The corresponding Russian originals were published in 1973 and 1956, respectively. The second edition represents an extensive revision of the first, with considerable new material, such as the theory of a waveguide in a medium in which the velocity of propagation depends on two coordinates, the theory of a surface waveguide and the waves of whispering galleries, reference (etalon) equations and reference integrals, diffraction rays, etc. The theory of caustics, amongst other subjects, has been greatly expanded and some material has been omitted or compressed. The bibliography has been greatly expanded. Chapter headings are: 1. Plane waves in discretely layered media. 2. Some applications of the theory of wave propagation in discretely layered media. 3. Waves in continuously layered media. 4. Reflection and refraction of spherical waves. 5. The field of a point source of waves in layered inhomogeneous media. 6. The field of a point source in a continuously layered medium. Ray treatment. 7. Exact theory of waveguide propagation in continuously layered media. Normal modes. 8. Propagation of waves in a waveguide that is inhomogeneous along its route. 9. Antiwaveguide propagation of waves in continuously layered media.

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Isovector methods for equations of balance. By Dominic G. B. Edelen. Sijthoff & Noordhoff, The Netherlands, 1980. xxi + 513 pp. \$45.00.

In this book, the author studies systems of second-order partial differential equations by realization of their solutions as surfaces in an appropriately structured space. Preferred systems of vectors (isovectors) are shown to lead to mappings whereby any solution surface is imbedded in a local Lie group of solution surfaces. Special cases and extensions include similarity solutions, pseudopotential solutions, Backlund maps, and a multi-variable Hamilton-Jacobi procedure of the Carathéodory type.

Explicit finite computational methods are given. These are illustrated by worked-out examples from various disciplines. Although no set of methods is universally applicable, isovector methods provide access and insights into problems that are not available through other techniques. The text is selfcontained through the inclusion of an extensive appendix that provides an operations-oriented exposition of the methods of the exterior calculus.

Fitting equations to data: computer analysis of multifactor data. 2nd edition. By Cuthbert Daniel and Fred S. Wood. John Wiley & Sons, New York, 1980. xviii + 451 pp. \$21.95.

This is the second edition of a highly regarded book in the Wiley Series in Probability and Mathematical Statistics, first published in 1971. It differs from the first principally in its extended use of "component and component-plus-residual plots," the invention of one of the authors (F.S.W.). There are several other important revisions of the programs and user's manual. Chapter headings are: 1. Introduction. 2. Assumptions and methods of fitting equations. 3. One independent variable. 4. Two or more independent variables. 5. Fitting an equation in three independent variables. 6. Selection of independent variables. 7. Some consequences of the disposition of the data points. 8. Selection of variables in nested data. 9. Nonlinear least squares, a complex example.

Contact problems in the classical theory of elasticity. By G. M. L. Gladwell. Sijthoff & Noordhoff, The Netherlands, 1980. xv + 716 pp. \$60.00. \$35.00 paper.

This book is volume no. 1 in the series Mechanics of Elastic and Viscoelastic Solids. Problems concerning the contact between elastic bodies have provided a challenge to applied mathematicians ever since the work of Heinrich Hertz in the 1860's. This book is concerned with idealized versions of these problems as they appear in the classical theory of elasticity. During the last hundred years this theory has been the cradle in which a number of powerful mathematical methods have grown. Amongst these are the complex variable methods developed by N. I. Muskhelishvili and his coworkers and the integral transform techniques pioneered by I. N. Sneddon and others. These methods, and many others, are here developed from fundamental concepts.

The aim of the book is to present a fairly complete account of the subject which can be read without extensive reference to other books. Thus the classical theory of elasticity, complex variable theory, integral transforms and elliptic functions are developed from fundamentals. The book has an extensive bibliography of about 700 entries and there are examples at the ends of most sections; answers are provided for many of these examples. The book covers the whole range of classical studies of Shtaerman, Galin and Lur'e in the Soviet Union and Mindlin, Reissner, Koiter and Sternberg in the West, to modern work undertaken in America, Britain, Europe, Japan and the Soviet Union.

Minimal submanifolds and geodesics. Proceedings of the Japan-United States Seminar on Minimal Submanifolds, including Geodesics, Tokyo, 1977. Edited by Morio Obata. North-Holland Publishing Co. Amsterdam and New York, 1979. xiii + 292 pp. \$58.50.

These proceedings contain extended or corrected versions of the lectures delivered at the seminar and also contain several papers related to the subjects of the seminar which were contributed later by participants.

Continued from Page 118

Theory of shells. Proceedings of the Third IUTAM Symposium on Shell Theory, Tbilisi, U.S.S.R., 22–28 August, 1978. Edited by W. T. Koiter and G. K. Mikhailov. North-Holland Publishing Co., Amsterdam, New York, 1980. xii + 702 pp. \$117.00.

The general lectures printed in this volume are: E. K. Kharadze—Life and activities of Ilia N. Vekua; M. S. Anderson—Practical experience in analysis of aerospace shell structures; A. Sawczuk—On plastic analysis of shells; R. C. Tennyson—Interaction of cylindrical shell buckling experiments with theory; I. I. Vorovich—Resonance properties of semiinfinite shells. There are also the texts of 32 contributed papers and four brief communications.

Linear operators in Hilbert spaces. By Joachim Weidmann. Springer-Verlag, New York, 1980. xiii + 402 pp. \$34.00.

This is volume 68 of Graduate Texts in Mathematics; the original German edition was published in 1976. The aim of the book is to give an introduction to the theory of linear operators of Hilbert spaces and then to proceed to the interesting applications of differential operators to mathematical physics. Only a fundamental knowledge of complex analysis and of ordinary differential equations is assumed. Lebesgue integration, as needed, is summarized in an appendix. Chapter headings: 1. Vector spaces with a scalar product, pre-Hilbert spaces; 2. Hilbert spaces; 3. Orthogonality; 4. Linear operators and their adjoints; 5. Closed linear operators; 6. Special classes of linear operators; 7. The spectral theory of self-adjoint and normal operators; 8. Self-adjoint extensions of symmetric operators; 9. Perturbation theory for self-adjoint operators; 10. Differential operators on $L_2(\mathbb{R}^m)$; 11. Scattering theory; Appendices.

Advances in geometric programming. Edited by Mordecai Avriel. Plenum Press, New York, 1980. x + 460 pp. \$39.50.

The objective of this book is to bring to the attention of interested readers some of the advances made in recent years in geometric programming and related fields which reflect the greatly widened scope of this branch of nonlinear optimization. The advances are in three major categories: analysis, computations, and applications. The papers appearing in the book can be also classified accordingly. There is a complete bibliography at the end of each article, as well as a special bibliography covering the last twenty years.

An introduction to variational inequalities and their applications. By David Kinderlehrer and Guido Stampacchia. Academic Press, New York, 1980. xiv + 313 pp. \$35.00.

This work presents the elements of variational inequalities and free boundary problems with several examples of their applications. It is a textbook, and the first few sections of each chapter are written in such a manner as to make the material accessible to economists, engineers and other scientists. The perspective of the book makes it of particular appeal to readers acquainted with partial differential equations, and the study of that subject is, indeed, part of its motivation. Chapter headings: 1. Variational inequalities in \mathbb{R}^n ; 2. Variational inequalities in Hilbert space; 3. Variational inequalities for monotone operators; 4. Problems of regularity; 5. Free boundary problems and the coincidence set of the solution; 6. Free boundary problems governed by elliptic equations and systems; 7. Applications of variational inequalities; 8. A one-phase Stefan problem.

Graph algorithms. By Shimon Even. Computer Science Press, 1979. iv + 249 pp.

This is a volume in the Computer Software Engineering Series. It is meant to be a textbook for an upper-level undergraduate or first-year graduate course. The mathematical background needed for understanding chapters 1 to 8 is some knowledge of set theory, combinatorics and algebra; to follow chapters 9 and 10, background in the theory of computation (Turing machines, Church's thesis) is also needed. Chapter headings: 1. Paths in graphs; 2. Trees; 3. Depth-first search; 4. Ordered trees; 5. Maximum flow in a network; 6. Applications of network flow techniques; 7. Planar graphs; 8. Testing graph planarity; 9. The theory of NP-completeness; 10. NP-complete graph problems.

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Base change for $GL(2)$. By Robert P. Langlands. Annals of Mathematics Studies. Princeton University Press, Princeton, N.J., 1980. vii + 236 pp. \$17.50. \$7.00 paper.

This is Annals of Mathematics Study Number 96. Chapter headings: 1. Introduction; 2. Properties of base change; 3. Applications to Artin L -functions; 4. σ -conjugacy; 5. Spherical functions; 6. Orbital integrals; 7. Characters and local lifting; 8. Convolution; 9. The primitive state of our subject revealed; 10. The trace formula; 11. The comparison.

The symmetric eigenvalue problem. By Beresford N. Parlett. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1980. xix + 348 pp. \$25.00.

The scope of this comprehensive introduction to the theory and practice of the title problem is indicated by the chapter headings: 1. Basic facts about self-adjoint matrices; 2. Tasks, obstacles, and aids; 3. Counting eigenvalues; 4. Simple vector iterations; 5. Deflation; 6. Useful orthogonal matrices. (tools of the trade); 7. Tridiagonal form; 8. The QL and QR algorithms; 9. Jacobi methods; 10. Eigenvalue bounds; 11. Approximations from a subspace; 12. Krylov subspaces; 13. Lanczos algorithms; 14. Subspace iteration; 15. The general linear eigenvalue problem; Appendices.

Nonlinear partial differential equations in engineering and applied science (Lecture Notes in Pure and Applied Mathematics Series, Volume 54). Edited by Robert L. Sternberg, Anthony J. Kalinowski, and John S. Papadakis. Marcel Dekker, Inc., New York, 1980. xvi + 480 pp. \$55.00.

This volume contains twenty-eight papers from the Conference on Nonlinear Partial Differential Equations in Engineering and Applied Science, held at the University of Rhode Island in June, 1979. The book includes purely mathematical topics, such as numerical analysis and bifurcation theory, as well as many practical applications, such as fluid dynamics, nonlinear waves, elasticity, viscoelasticity, hyperelasticity, solitons, metallurgy, shockless airfoil design, quantum fields, and Darcy's law on flows in porous media.

Elements of applicable functional analysis. By Charles W. Groetsch. Marcel Dekker, Inc., New York, 1980. x + 300 pp. \$24.75.

This introductory graduate-level textbook draws together the modern and classical elements of applicable functional analysis. Whereas traditional books on functional analysis stress axiomatics and abstraction for their own sakes, the object of this book is to present functional analysis as a useful language for applied mathematics. The principal areas covered in this book are numerical analysis and optimization. Throughout the book are modern applications (such as those involving variational methods).

Calcul tensoriel dans un continuum amorphe. By J. Winogradzki. Masson, Paris, New York, Barcelona, Milan, 1979. viii + 219 pp.

This is a publication of the University of Roven, and is the first volume of a projected four-volume series by the authors entitled *Les méthodes tensorielles de la physique*. Chapter headings (translated): 1. Prolegomena; 2. n -dimensional continuum and coordinate systems; 3. Introduction of tensors; 4. Particular tensors; 5. The three elementary algebraic operations; 6. Compound algebraic operations; 7. Tensoriality; 8. Linear algebra; 9. Analysis.

C-algebra extensions and K-homology.* By Ronald G. Douglas. Annals of Mathematics Studies. Princeton University Press, Princeton, New Jersey, 1980. xii + 83 pp. \$12.50; \$4.50 paperback.

This is Annals of Mathematics Study number 95. Chapter headings: 1. An overview; 2. *Ext* as a group; 3. *Ext* as a homotopy function; 4. Generalized homology theory and periodicity; 5. *Ext* as *K*-homology; 6. Index theorems and Novikov's higher signatures.

Cyclotomic fields II. By Serge Lang. Springer-Verlag, New York, 1980. xi + 164 pp. \$19.80.

This is volume 69 of Graduate Texts in Mathematics. Volume I consisted of chapters 1 to 9; this second volume (chapters 10 to 17) incorporates a number of results which the author discovered and/or systematized since the first volume was being written. The author again limits himself to the cyclotomic fields proper without introducing modular functions.

Extensions of linear-quadratic control theory. By D. H. Jacobson, D. H. Martin, M. Pachter, and T. Geveci. Springer-Verlag, Berlin, Heidelberg, New York, 1980. xi + 288 pp. \$19.20.

This is volume 27 of Lecture Notes in Control and Information Sciences. The notes were prepared for presentation as a summer seminar at the National Research Institute for Mathematical Sciences in Pretoria, South Africa, in February 1980. There are fifteen lectures, the first six and last two being comprehensive surveys and lectures 7 to 13 presenting research results.

Linear systems. By Thomas Kailath. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1980. xxi + 682 pp. \$27.95.

The main focus of this book is the emphasis on the structure of finite-dimensional linear systems. The aim is to take a beginning student with some prior exposure to linear system analysis (elementary transform and matrix theory) through a motivated and integrated development of modern perspectives on linear system theory. Chapter headings: 1. Background material; 2. State-space descriptions—some basic concepts; 3. Linear state-variable feedback; 4. Asymptotic observers and compensator design; 5. Some algebraic complements; 6. State-space and matrix-fraction descriptions of multivariable systems; 7. State feedback and compensator design; 8. General differential systems and polynomial matrix descriptions; 9. Some results for time-variant systems; 10. Some further reading; Appendix: some facts from matrix theory.

Optimization techniques. Proceedings of the 9th IFIP Conference on Optimization Techniques, Warsaw, Sept. 4–8, 1979. Edited by K. Iracki, K. Malanowski and S. Walukiewicz. Springer-Verlag, Berlin, Heidelberg, New York, 1980. Part I, 560 pp.; Part II, 621 pp. \$34.90 each.

These are volumes 22 and 23 of Lecture Notes in Control and Information Sciences. The first volume contains the eight plenary lectures by Balakrishnan, Gutkowski, Kantorovich, Krasovskii, Kulikowski, Lions, Powell and Wiezbicki, the panel addresses of the round-table session on systems techniques in economics by Intriligator, Kantorovich and Robinson, as well as contributed papers under the headings stochastic control, differential games, optimal control (ordinary and delay differential equations), optimal control (partial differential equations), multiobjective problems, and applications to biomedical systems. In the second volume there are contributed papers devoted essentially to mathematical programming and various applications.

Combinatorial problems and exercises. By Laszlo Lovasz. North-Holland Publishing Co., Amsterdam and New York, 1978. v + 450 pp. \$42.24 hardbound; \$26.75 paperback.

The aim of the book is to introduce some of the methods of combinatorics to those who wish to apply these methods to the solution of theoretical and practical problems. There are fifteen chapters, each of which is divided into three parts: problems, hints and solutions: 1. Basic enumeration; 2. The sieve; 3. Permutations; 4. Two classical enumeration problems in graph theory; 5. Parity and duality; 6. Connectivity; 7. Factors of graphs; 8. Independent sets of points; 9. Chromatic number; 10. Extremal problems for graphs; 11. Spectra of graphs; 12. Automorphisms of graphs; 13. Hypergraphs; 14. Ramsey theory; 15. Reconstruction.

Introduction to the theory of nonparametric statistics. By Ronald H. Randles and Douglas A. Wolfe. John Wiley & Sons, Inc., New York, 1979. xiii + 450 pp. \$24.95.

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is an intermediate graduate-level text, assuming a course on the level of, say, Hogg and Craig. The aim is to impart a basic understanding of concepts and theory without sacrificing the intuitive flavor prevalent in most of the early works in nonparametric statistics. Chapter headings: 1. Preliminaries; 2. Distribution-free statistics; 3. *U*-statistics; 4. Power functions and their properties; 5. Asymptotic relative efficiency of tests; 6. Confidence intervals and bounds; 7. Point estimation; 8. Linear rank statistics under the null hypothesis; 9. Two-sample location and scale problems; 10. The one-sample location problem; 11. Additional methods for constructing distribution-free procedures; 12. Other important problems; Appendix.

Algebra. 2nd edition. By Saunders MacLane and Garrett Birkhoff. Macmillan Publishing Co., New York, 1979. xv + 586 pp. \$18.95.

This is a second edition of the well-known text first published in 1967. It aims to present modern algebra from the beginning, for undergraduates or graduates, by covering the standard materials in a way which combines the use of algebraic manipulations and axiomatic methods with the striking general ideas which have developed in recent decades. These have to some extent reshaped the conceptual organization of mathematics, for example in the emphasis on the (homo-)morphisms of each type of algebra and the consequent use of categories and universal constructions. This book is organized around this continued development. Chapter headings: 1. Sets, functions, and integers; 2. Groups; 3. Rings; 4. Universal constructions; 5. Modules; 6. Vector spaces; 7. Matrices; 8. Special fields; 9. Determinants and tensor products; 10. Bilinear and quadratic forms; 11. Similar matrices and finite abelian groups; 12. Structure of groups; 13. Galois theory; 14. Lattices; 15. Categories and adjoint functors; 16. Multilinear algebra.

Differential equations. Edited by Shair Ahmad, Marvin Keener and A. C. Lazer. Academic Press, 1980. ix + 278 pp. \$20.00.

These are the proceedings of the Eighth Fall Conference on Differential Equations, held at Oklahoma State University in October 1979. There are 22 papers, on partial as well as on ordinary and functional differential equations. Previous conferences in the series were called Midwest Conferences, but this local reference has been dropped.

Analytic function methods in probability theory. By B. Gyires. North-Holland, Amsterdam and New York, 1979. 379 pp. \$63.50.

This book comprises the proceedings of the Colloquium on the Methods of Complex Analysis in the Theory of Probability and Statistics held in the Kossuth University of Debrecen, Hungary, 29 August–2 September, 1977. The papers published also include some submitted after the colloquium. One of the main topics of this colloquium was the problem of characterization, but the participants discussed many other applications, e.g. in the areas of infinite divisibility, asymptotic expansions, and stochastic processes.

C. P. Ramanujam—a tribute. Tata Institute of Fundamental Research. Springer-Verlag, Berlin, 1978.

"C. P. Ramanujam joined the School of Mathematics of the Tata Institute of Fundamental Research, Bombay in 1957. He did brilliant work in Number Theory and Algebraic Geometry and was, without doubt, one of the outstanding young Indian mathematicians of the last twenty years. Gifted with a remarkably deep and wide mathematical culture, he was at home as much with abstract Algebraic Geometry as with Analytic Number Theory. A warm-hearted, kind and distinguished colleague, he unfortunately passed away in 1974 when he was hardly 37 years old" (from the preface). This collection of his publications is intended as a tribute to his memory from his friends and admirers.

Structural control. Proceedings of the International IUTAM Symposium. Edited by H. H. E. Leipholz. North-Holland Publishing Co., Amsterdam and New York, 1980. xv + 810 pp. \$87.75.

These are the Proceedings of the International IUTAM Symposium held at the University of Waterloo in June, 1979. The aim of the symposium was to bring three groups of people in contact with each other: (a) researchers concerned with the fundamentals of control and optimization theory; (b) researchers and engineers involved in the application of control and optimization theory to industrial processes and to vehicles as well as structures in the area of aerospace mechanics; (c) researchers and engineers interested and active in the application of passive and active control to civil engineering structures. The editor summarizes the main results of the symposium as follows: it came to light that there are immediate, useful, and relevant applications of optimal control to civil engineering structures in the following cases: (i) control of bridges against excessive deformations and accelerations due to moving loads and wind forces; (ii) control of tall buildings, masts, and towers against wind forces; (iii) control of buildings against earthquake effects; (iv) control of building foundations against changing soil conditions. There are 43 papers in the proceedings.

Methods and applications of interval analysis. By Ramon E. Moore. SIAM, Philadelphia, 1979. xi + 190 pp. \$16.00.

This monograph is the second in the series SIAM Studies in Applied Mathematics. It is designed for a one-semester senior-graduate course and is accessible to a person with a standard undergraduate training in the mathematical sciences with some knowledge of computer programming. It provides an account of some of the principal methods and applications of *interval analysis*. The basis for the new methods is an extension of the concept of a "real" number and it is shown in chapter 2 that an *interval* of "real" numbers can be thought of as a *new kind of number*, represented by a pair of "real" numbers, namely its endpoints. By properly rounding the endpoints of machine-computed results of interval arithmetic, intervals containing results of infinite precision can be computed; arbitrarily narrow intervals containing exact real arithmetic results can be computed by carrying enough digits. Based on these ideas, the author develops the theory of interval arithmetic and reports several applications, as the table of contents demonstrates: 1. Introduction; 2. Finite representations; 3. Finite evaluation; 4. Finite convergence; 5. Computable sufficient conditions for existence and convergence; 6. Safe starting regions for iterative methods; 7. Applications to mathematical programming (optimization); 8. Applications to operator equations; 9. An application in finance: internal rates-of-return.

Lectures on the logic of computer programming. By Zohar Manna. SIAM, Philadelphia, Pa., 1980. iv + 49 pp. \$7.50.

This is volume 31 of the CBMS-NSF regional conference series in applied mathematics. The lecture notes deal with six aspects of the application of mathematical logic to the computer programming process: 1. Partial correctness of programs: proving that a given program produces the intended results whenever it halts; 2. Termination of programs: proving that a given program will eventually halt; 3. Total correctness of programs: proving both that a given program is partially correct and that it terminates; 4. Systematic program annotation: describing the intermediate behavior of a given program; 5. Synthesis of programs: constructing a program to meet given specifications; 6. Termination of production systems: proving that a system of rewriting rules always halts.

Integer programming: facets, subadditivity and duality for group and semi-group problems.

By Ellis L. Johnson. SIAM, Philadelphia, Pa., 1980. vii + 68 pp. \$9.00.

This is volume 32 of the CBMS-NSF regional conference series in applied mathematics. These notes deal with the author's approach to and view of integer programming, drawing on the work of Gomory, Fulkerson and Araoz. The chapter headings are: 1. Integer programming; 2. Cuts, knapsacks, and a cyclic group problem; 3. Finite abelian groups; 4. Gomory's corner polyhedra; 5. Blocking polyhedra and master group problems; 6. Araoz's semigroup; 7. Blockers and polars for master semigroup problems; 8. Subadditive and minimal valid inequalities; 9. Subadditive characterizations; 10. Duality.

Arithmetic complexity of computations. By Shmuel Winograd. SIAM, Philadelphia, Pa., 1980. iii + 93 pp. \$10.50.

This is volume 33 of the CBMS-NF regional conference series in applied mathematics. It presents the author's pathbreaking contributions to the subject, and its two major problem areas are: 1. What is the minimum number of arithmetic operations needed to perform a computation? 2. How can we obtain a better algorithm when improvement is possible? In the presentation, there is balance between the mathematical aspects of the theory and the applicability of the results, with emphasis on applications to signal processing. Chapter headings: 1. Introduction; 2. Three examples; 3. General background; 4. Product of polynomials; 5. FIR filters; 6. Product of polynomials modulo a polynomial; 7. Cyclic convolution and discrete Fourier transform.

Fundamentals of Maxwell's kinetic theory of a simple monatomic gas. By C. Truesdell and R. G. Muncaster. Academic Press, Inc., New York, 1980. xxvii + 593 pp. \$54.00.

There are twenty-six chapters in this book, grouped in seven parts: A. Continuum thermomechanics; B. Basic structures of the kinetic theory; C. The Maxwell-Boltzmann equation and its elementary consequences; D. Particular molecular models and exact solutions for moments; E. The system of equations for the moments; F. Existence, uniqueness, and qualitative behavior; G. Grossly and momentally determined solutions and the iterative procedures of the kinetic theory.

The main purpose of this book is to examine Maxwell's second kinetic theory of a moderately rarefied, simple, monatomic gas rigorously, to uncover gaps in past analyses and to illuminate them as challenges to future research by mathematicians. Quoting from the prologue: "We have attempted to apply to Maxwell's theory the standards of conceptual analysis, logical hygiene, and mathematical rigor to which the rational thermomechanics of continua developed in the past quarter century has accustomed us."

Measurement theory with applications to decisionmaking, utility, and the social sciences. By

Fred S. Roberts. Addison-Wesley Publishing Co., Reading, Mass., 1979. xxii + 420 pp. \$24.50.

This is volume 7 of the *Encyclopedia of Mathematics and its Applications*, edited by Gian-Carlo Rota. It is a volume in the section Mathematics and the Social Sciences (of which the section editor is the author of this volume). An introduction to measurement theory for nonspecialists, this book puts measurement in the social and behavioral sciences on a firm foundation using a mathematical formulation. Results are applied to such topics as measurements of utility, psychophysical scaling, and decision-making about pollution, energy, transportation, and health. The book ties together a large body of mathematical research work from philosophy of science, mathematical psychology, economics, and elsewhere. The mathematical results and questions presented here should be of interest to both practicing mathematicians and students of mathematics, since the author sets forth an area of mathematics which is not well known to most mathematicians, but which has many potentially significant applications. The tools and insights of measurement theory introduced here should be of interest to both theoretical and practical social scientists and to policy advisers, managers, and others concerned with making complex decisions. Chapter headings: 1. Relations; 2. Fundamental measurement, derived measurement, and the uniqueness problem; 3. Three representation problems: ordinal, extensive, and difference measurement; 4. Applications to psychophysical scaling; 5. Product structures; 6. Nontransitive indifference, probabilistic consistency, and measurement without numbers; 7. Decision making under risk or uncertainty; 8. Subjective probability.