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AN ALTERNATIVE APPROACH TO ELASTODYNAMIC CRACK PROBLEMS IN AN ORTHOTROPIC MEDIUM*

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Abstract. A similarity transformation is used to reduce the system of second-order equations, governing elastodynamic plane problems in an orthotropic medium, to a first-order elliptic system of the Cauchy-Riemann type. A complex variable notation is then introduced to derive in a straightforward way the solution of two noticeable elastodynamic crack problems.

1. Introduction. The problem of determining the stress field induced by a steadily propagating crack in a two-dimensional elastic medium is of primary interest in fracture mechanics. A great many results have been obtained for isotropic materials, and dutiful mention is made to the more significant analytical studies.

Yoffè [1] discussed the plane problem of a crack of constant length moving with constant speed in an isotropic medium stressed at infinity. The same problem was solved later by Radok [2] who used a complex variable method.

Craggs [3] considered the shape of a semi-infinite crack loaded over a segment of its edges. In solving the related boundary value problem, he used Cauchy's integral representation which, however, does not permit one to determine analytically the singular terms of the solution.

The antiplane problem for various crack configurations has been solved by McClintock and Sukhatme [4]. Sih [5] revisited, from a mathematical point of view, the problems studied in [1–4], giving a general treatment founded on a complex variable formulation.

Problems in steady-state crack propagation in an isotropic strip of finite width have been examined by Sih and Chen [6, 7], Nilsson [8], Tait and Moodie [9, 10], and Singh et al. [11]. Recently, Georgiadis and Theocaris [12] gave a review on steady-state elastodynamic crack problems where the complex variables approach is emphasized.

The counterpart of solved problems for anisotropic materials is somewhat poor, which may be due to mathematical complexity of such problems. Atkinson [13] applied the method used in [3] to study the steady-state propagation of a semi-infinite crack in an

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aelotropic medium. He found out, as in [3], the singular part of the solution through a conjecture concerning the behavior of the material at the crack tip. Kassir and Tse [14] used an integral transform method to solve the plane problem of a steadily moving Griffith crack in an orthotropic medium. Danyluk and Singh [15] applied the same technique to obtain closed-form solutions to antiplane problems of a crack moving in an orthotropic layer.

The author [16] has recently extended the analysis of [15] to a strip made up of an anisotropic material with one plane of symmetry. In this paper a similarity transformation is used to transform the system of equations of motion, governing elastodynamic plane problems in an orthotropic medium, to a first-order elliptic system of the Cauchy–Riemann class. The complex variable approach is then used to solve, in a straightforward manner, the boundary value problems related to steady-state elastodynamic crack problems.

2. Mathematical preliminaries. For an orthotropic medium under plane strain conditions, the equations of motion become [17]

$$C_{11}\frac{\partial^2 u}{\partial x^2} + C_{66}\frac{\partial^2 u}{\partial y^2} + (C_{12} + C_{66})\frac{\partial^2 v}{\partial x \partial y} = \rho\frac{\partial^2 u}{\partial t^2}$$

and (2.1)

$$C_{66}\frac{\partial^2 v}{\partial x^2} + C_{22}\frac{\partial^2 v}{\partial y^2} + (C_{12} + C_{66})\frac{\partial^2 u}{\partial x \partial y} = \rho\frac{\partial^2 v}{\partial t^2},$$

in which $u = u(x, y, t)$, $v = v(x, y, t)$ are the displacement components in x and y directions, t is time, ρ is the density of the medium, and C_{ij} are the elastic moduli.

The stress-strain relations in terms of the displacement components are as follows:

$$\begin{aligned}
\sigma_{xx} &= C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y},\\
\sigma_{yy} &= C_{12}\frac{\partial u}{\partial x} + C_{22}\frac{\partial v}{\partial y},\\
\sigma_{xy} &= C_{66}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right),
\end{aligned} \tag{2.2}$$

where σ_{xx}, σ_{yy}, and σ_{xy} are the Cartesian stress components.

Introducing the transformation $X = x - ct$, $Y = y$, $t = t$, where c is a constant speed, and assuming $u = u(X, Y)$, $v = v(X, Y)$ allows the following simplification into Eq. (2.1) to be made:

$$\begin{aligned}
\frac{\partial^2 u}{\partial X^2} + 2\beta\frac{\partial^2 v}{\partial X \partial Y} + \alpha\frac{\partial^2 u}{\partial Y^2} &= 0,\\
\frac{\partial^2 v}{\partial X^2} + 2\beta_1\frac{\partial^2 u}{\partial X \partial Y} + \alpha_1\frac{\partial^2 v}{\partial Y^2} &= 0,
\end{aligned} \tag{2.3}$$

with

$$2\beta = \frac{C_{12}+C_{66}}{C_{11}(1-M_1^2)}, \qquad 2\beta_1 = \frac{C_{12}+C_{66}}{C_{66}(1-M_2^2)},$$
$$\alpha = \frac{C_{66}}{C_{11}(1-M_1^2)}, \qquad \alpha_1 = \frac{C_{22}}{C_{66}(1-M_2^2)}. \tag{2.4}$$

The quantities $M_j = c/v_j$ ($j = 1,2$), where $v_1 = (C_{11}/\rho)^{1/2}$ and $v_2 = (C_{66}/\rho)^{1/2}$, are the Mach numbers which, in what follows, will be assumed less than one (subsonic condition).

The system (2.3) may be rewritten as

$$I\frac{\partial\Phi}{\partial X} + A\frac{\partial\Phi}{\partial Y} = 0, \tag{2.5}$$

where I is the 4×4 identity matrix and

$$A = \begin{pmatrix} 0 & \alpha & 2\beta & 0 \\ -1 & 0 & 0 & 0 \\ 2\beta_1 & 0 & 0 & \alpha_1 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \qquad \Phi(X,Y) = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \end{pmatrix} \equiv \begin{pmatrix} u,_X \\ u,_Y \\ v,_X \\ v,_Y \end{pmatrix}, \tag{2.6}$$

in which (,) denotes differentiation.

It is assumed that A has no real eigenvalues so that the system (2.5) is elliptic. Hence, the characteristic equation for the matrix A

$$\lambda^4 + 2a_1\lambda^2 + a_2 = 0,$$

where

$$2a_1 = \alpha + \alpha_1 - 4\beta\beta_1, \qquad a_2 = \alpha\alpha_1,$$

provides the conjugate pairs of eigenvalues $\lambda_{1,2} = \pm ip$ and $\lambda_{3,4} = \pm iq$, with

$$p = \left(a_1 - \left(a_1^2 - a_2\right)^{1/2}\right)^{1/2}, \qquad q = \left(a_1 + \left(a_1^2 - a_2\right)^{1/2}\right)^{1/2}$$

positive constants.[1] The corresponding eigenvectors may be chosen to be

$$h^{(1)} = \frac{1}{\alpha - p^2}\begin{pmatrix} 2\beta p^2 \\ i2\beta p \\ -ip(\alpha - p^2) \\ \alpha - p^2 \end{pmatrix}, \quad \overline{h^{(1)}}, \quad h^{(2)} = \frac{1}{\alpha - q^2}\begin{pmatrix} 2\beta q^2 \\ i2\beta q \\ -iq(\alpha - q^2) \\ \alpha - q^2 \end{pmatrix}, \quad \overline{h^{(2)}}. \tag{2.7}$$

In the new basis

$$(\operatorname{Im} h^{(1)}, \operatorname{Re} h^{(1)}, \operatorname{Im} h^{(2)}, \operatorname{Re} h^{(2)}), \tag{2.8}$$

[1] Note added in proof. This is the occurrence in many cases of practical interest. The case of complex eigenvalues may be also considered in the context of this approach.

the matrix A is represented by the matrix $B = T^{-1}AT$ with

$$T = \begin{pmatrix} 0 & \dfrac{2\beta p^2}{\alpha - p^2} & 0 & \dfrac{2\beta q^2}{\alpha - q^2} \\ \dfrac{2\beta p}{\alpha - p^2} & 0 & \dfrac{2\beta q}{\alpha - q^2} & 0 \\ -p & 0 & -q & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}, \qquad (\det T \neq 0). \tag{2.9}$$

Moreover, the following invertible transformation,

$$\Phi(X, Y) = T\Psi(X, Y), \tag{2.10}$$

holds where $\Psi(X, Y)$ is a 4×1 matrix-valued function with real entries of the independent variables X and Y. Hence, in the basis (2.8) the system (2.5) may be rewritten as

$$I\frac{\partial \Psi}{\partial X} + B\frac{\partial \Psi}{\partial Y} = 0, \tag{2.11}$$

where it may be shown that

$$B = \begin{pmatrix} 0 & -p & 0 & 0 \\ p & 0 & 0 & 0 \\ 0 & 0 & 0 & -q \\ 0 & 0 & q & 0 \end{pmatrix}. \tag{2.12}$$

Consequently, combining (2.12) with (2.11) yields

$$\begin{aligned} \frac{\partial \Psi_1}{\partial X} &= \frac{\partial \Psi_2}{\partial Y_1}, & \frac{\partial \Psi_1}{\partial Y_1} &= -\frac{\partial \Psi_2}{\partial X}, \\ \frac{\partial \Psi_3}{\partial X} &= \frac{\partial \Psi_4}{\partial Y_1}, & \frac{\partial \Psi_3}{\partial Y_2} &= -\frac{\partial \Psi_4}{\partial X}, \end{aligned} \tag{2.13}$$

with $Y_1 = Y/p$ and $Y_2 = Y/q$.

Assuming continuity of the partial derivatives up to the second order, the Cauchy-Riemann equations (2.13) grant that Ψ_j ($j = 1, 2$) and Ψ_k ($k = 3, 4$) are pairs of conjugate harmonic functions in the $z_1 = X + iY_1$ and $z_2 = X + iY_2$ plane, respectively.

Combining (2.10), (2.9), and (2.6) and substituting into (2.2) leads to

$$\begin{aligned} \sigma_{XX} = \frac{C_{66}}{\alpha}\Bigg\{&\left[\frac{2\beta p^2}{(\alpha - p^2)(1 - M_1^2)} + (2\beta - \alpha)\right]\Psi_2 \\ &+ \left[\frac{2\beta q^2}{(\alpha - q^2)(1 - M_1^2)} + (2\beta - \alpha)\right]\Psi_4\Bigg\}, \end{aligned} \tag{2.14}$$

$$\sigma_{YY} = C_{66}\left[p^2 k_1 \Psi_2 + q^2 k_2 \Psi_4\right], \tag{2.15}$$

$$\sigma_{XY} = C_{66}\left[pk_3\Psi_1 + qk_4\Psi_3\right], \tag{2.16}$$

in which

$$k_1 = \frac{1}{\alpha - p^2}\left[(1 - M_2^2)(\alpha - p^2) - 2\beta\right], \qquad k_2 = \frac{1}{\alpha - q^2}\left[(1 - M_2^2)(\alpha - q^2) - 2\beta\right],$$

$$k_3 = -(k_1 + M_2^2), \qquad k_4 = -(k_2 + M_2^2).$$

Introducing complex notation by setting

$$\Omega_1(z_1) = \Psi_1 + i\Psi_2, \qquad \Omega_2(z_2) = \Psi_3 + i\Psi_4, \tag{2.17}$$

where $\Omega_1(z_1)$ and $\Omega_2(z_2)$ are holomorphic functions, the more relevant stress components (2.15) and (2.16) become

$$\sigma_{YY} = C_{66}\,\mathrm{Im}\left[p^2 k_1 \Omega_1(z_1) + q^2 k_2 \Omega_2(z_2)\right], \tag{2.18}$$

$$\sigma_{XY} = C_{66}\,\mathrm{Re}\left[p k_3 \Omega_1(z_1) + q k_4 \Omega_2(z_2)\right]. \tag{2.19}$$

By means of the representation (2.18) and (2.19), closed-form solutions to boundary value problems, related to some elastodynamic plane problems of cracks in an orthotropic medium, may be obtained.

3. Two remarkable examples. The linear relationships (2.18) and (2.19) allow one to reduce some boundary value problems to appropriate Dirichlet problems for sectionally holomorphic functions. The first example is to find the solution of the plane problem referring to a semi-infinite crack, steadily moving at a constant rate c along the x-axis, when a uniform traction p_0 applied to a finite segment a of its edges and the medium undisturbed at infinity is assumed. Taking into account the symmetry and referring to the moving coordinate system (X, Y), the boundary conditions become

$$\begin{aligned} \sigma_{YY}(X,0) &= -p_0, && -a < X < 0,\\ \sigma_{YY}(X,0) &= 0, && -\infty < X < -a,\\ \sigma_{XY}(X,0) &= 0, && |X| < \infty, \end{aligned} \tag{3.1}$$

with all stress components vanishing at infinity.

Using (2.18) and (2.19), the stress boundary conditions (3.1) yield the following Dirichlet problems (see Appendix):

$$\mathrm{Re}\,\Lambda_j(X) = \begin{cases} p_0/C_{66}, & -a < X < 0,\\ 0, & -\infty < X < -a, \end{cases} \tag{3.2}$$

where

$$\begin{aligned} \Lambda_1(z_1) &= \frac{ip(pk_1k_4 - qk_2k_3)}{k_4}\Omega_1(z_1),\\ \Lambda_2(z_2) &= -\frac{iq(pk_1k_4 - qk_2k_3)}{k_3}\Omega_2(z_2) \end{aligned} \tag{3.3}$$

are sectionally analytic functions, behaving respectively as $O(1/z_i)$ at infinity. The solutions to (3.2) are given by [18]:

$$\Lambda_j(z_j) = \frac{p_0}{i\pi C_{66}\sqrt{z_j}} \int_{-a}^{0} \frac{\sqrt{X}\,dX}{X - z_j}. \tag{3.4}$$

Evaluating the Cauchy-type integral (3.4), combining with (3.3), and substituting into (2.18) and (2.19), yields:

$$\sigma_{YY} = \frac{p_0}{\pi(pk_1k_4 - qk_2k_3)} \operatorname{Im}\left\{ pk_1k_4\left[2i\sqrt{\frac{a}{z_1}} - \log\left(\frac{\sqrt{z_1}+i\sqrt{a}}{\sqrt{z_1}-i\sqrt{a}}\right)\right] - qk_2k_3\left[2i\sqrt{\frac{a}{z_2}} - \log\left(\frac{\sqrt{z_2}+i\sqrt{a}}{\sqrt{z_2}-i\sqrt{a}}\right)\right]\right\}, \tag{3.5}$$

$$\sigma_{XY} = \frac{p_0k_3k_4}{\pi(pk_1k_4 - qk_2k_3)} \operatorname{Re}\left\{2i\left(\sqrt{\frac{a}{z_1}} - \sqrt{\frac{a}{z_2}}\right) + \log\left[\frac{\left(\sqrt{z_2}+i\sqrt{a}\right)\left(\sqrt{z_1}-i\sqrt{a}\right)}{\left(\sqrt{z_2}-i\sqrt{a}\right)\left(\sqrt{z_1}+i\sqrt{a}\right)}\right]\right\}. \tag{3.6}$$

Introducing polar coordinates (r_j, θ_j), measured from the tip of the moving crack, with

$$r_1 = (r/p)\left(p^2\cos^2\theta + \sin^2\theta\right)^{1/2}, \qquad r_2 = (r/q)\left(q^2\cos^2\theta + \sin^2\theta\right)^{1/2},$$
$$\tan\theta_1 = (\tan\theta)/p, \qquad \tan\theta_2 = (\tan\theta)/q,$$

being (r, θ) polar coordinates in the physical plane, permits us to represent the closed-form expressions of the relevant stress components as

$$\sigma_{YY} = \frac{p_0}{\pi(pk_1k_4 - qk_2k_3)}\left\{ pk_1k_4\left[2\sqrt{\frac{a}{r_1}}\cos\frac{\theta_1}{2} + \tan^{-1}\Theta_1(r_1,\theta_1)\right] - qk_2k_3\left[2\sqrt{\frac{a}{r_2}}\cos\frac{\theta_2}{2} + \tan^{-1}\Theta_2(r_2,\theta_2)\right]\right\} \tag{3.7}$$

$$\sigma_{XY} = \frac{p_0k_3k_4}{\pi(pk_1k_4 - qk_2k_3)}\left\{2\left[\sqrt{\frac{a}{r_1}}\sin\frac{\theta_1}{2} - \sqrt{\frac{a}{r_2}}\sin\frac{\theta_2}{2}\right] + \frac{1}{2}\log\left|\frac{f_2(r_2,\theta_2)}{f_1(r_1,\theta_1)}\right|\right\}, \tag{3.8}$$

where

$$\Theta_j(r_j,\theta_j) = \frac{2\sqrt{ar_j}\cos\theta_{j/2}}{a - r_j}, \qquad f_j(r_j,\theta_j) = \frac{(r_j + a) + 2\sqrt{ar_j}\sin\theta_j/2}{(r_j + a) - 2\sqrt{ar_j}\sin\theta_j/2}.$$

When an isotropic medium under plane strain is considered for which $C_{11} = C_{22} = \lambda + 2\mu$, $C_{12} = \lambda$, $C_{66} = \mu$, where λ and μ are Lamé's elastic constants, the above results are in agreement with those reported in [3, 5]. In particular, the stress component (3.7) coincides with that given in [12] where the above problem has been recently revisited.

The second example finds the solution of the plane problem referring to a straight crack of constant length $2a$, with uniform tractions p_0 applied to its edges, propagating along the x-axis with constant speed c, in an orthotropic medium undisturbed at infinity.

Taking the symmetry into account and referring to the moving coordinate sysem (X, Y), attached to the midpoint of the crack, the boundary conditions may be written as

$$\begin{aligned} \sigma_{YY}(X,0) &= -p_0, \qquad |X| < a, \\ \sigma_{XY}(X,0) &= 0, \qquad |X| < \infty, \end{aligned} \tag{3.9}$$

with all stress components vanishing at infinity. Starting again with (2.18) and (2.19), the sectionally analytic functions (3.3) are determined from the boundary value problem (see [18], §46.4, p. 457):

$$\operatorname{Re} \Lambda_j(X) = p_0/C_{66}, \qquad |X| < a. \tag{3.10}$$

The solution to (3.10), unbounded (Hölderian) at the ends $x = \pm a$, is (cf. [18])

$$\Lambda_j(z_j) = \frac{p_0}{C_{66}}\left(\frac{z_j}{F(z_j)} - 1\right), \tag{3.11}$$

where $F(z_j) = (z_j^2 - a^2)^{1/2}$ is the Plemelj function.

Combining (3.11) with (3.3) and substituting into (2.14), (2.15), and (2.16) yields the following expressions for the stress components:

$$\begin{aligned} \sigma_{XX} = \frac{p_0}{\alpha(pk_1k_4 - qk_2k_3)}\Bigg\{ &\frac{k_4}{p}\left[\frac{2\beta p^2}{(\alpha - p^2)(1 - M_1^2)} + (2\beta - \alpha)\right]\operatorname{Re}\left[1 - \frac{z_1}{F(z_1)}\right] \\ &- \frac{k_3}{q}\left[\frac{2\beta q^2}{(\alpha - q^2)(1 - M_1^2)} + (2\beta - \alpha)\right]\operatorname{Re}\left[1 - \frac{z_2}{F(z_2)}\right]\Bigg\}, \end{aligned} \tag{3.12}$$

$$\sigma_{YY} = \frac{p_0}{(pk_1k_4 - qk_2k_3)}\left\{pk_1k_4 \operatorname{Re}\left[1 - \frac{z_1}{F(z_1)}\right] - qk_2k_3 \operatorname{Re}\left[1 - \frac{z_2}{F(z_2)}\right]\right\}, \tag{3.13}$$

$$\sigma_{XY} = \frac{-p_0k_3k_4}{(pk_1k_4 - qk_2k_3)}\left\{\operatorname{Im}\left[1 - \frac{z_1}{F(z_1)}\right] - \operatorname{Im}\left[1 - \frac{z_2}{F(z_2)}\right]\right\}. \tag{3.14}$$

It may be shown, with a lot of algebra, that the above results coincide with those presented in [14] where an integral transform method has been employed to solve the problem.

The isotropic case is recovered according to [5].

Appendix. By (2.19), the boundary condition (3.1) has the form

$$pk_3\left[\Omega_1^+(X) + \overline{\Omega_1^+(X)}\right] + qk_4\left[\Omega_2^+(X) + \overline{\Omega_2^+(X)}\right] = 0, \qquad |X| < \infty, \tag{A1}$$

where a bar denotes complex conjugation and $\Omega_i^+(X)$, $i = 1, 2$, are the limiting values of $\Omega_i(z_i)$ as $z_i \to X$ from the upper half-plane, $Y > 0$. By defining two functions

$$f_1(z_1) = \begin{cases} pk_3\Omega_1(z_1), & Y > 0, \\ -pk_3\bar{\Omega}_1(z_1), & Y < 0 \end{cases}, \qquad f_2(z_2) = \begin{cases} qk_4\Omega_2(z_2), & Y > 0 \\ -qk_4\bar{\Omega}_2(z_2), & Y < 0 \end{cases},$$

where $\bar{\Omega}_i(z_i) = \overline{\Omega_i(\bar{z}_i)}$, the relation (A1) becomes

$$[f_1(X) + f_2(X)]^+ = [f_1(X) + f_2(X)]^-, \qquad |X| < \infty. \tag{A2}$$

Recalling that the stress components must vanish at infinity, the Liouville theorem will ensure that $f_1(z_1) + f_2(z_2) = 0$ when $z_1 = z_2$. Consequently,

$$pk_3\Omega_1(z_1) + qk_4\Omega_2(z_2) = 0, \; z_1 = z_2. \tag{A3}$$

By (A3) and (3.3) the stress component (2.18) becomes

$$\sigma_{YY} = -C_{66}\,\mathrm{Re}\,\Lambda_1(z_1) = -C_{66}\,\mathrm{Re}\,\Lambda_2(z_2), \; z_1 = z_2, \tag{A4}$$

whence boundary value problem (3.1) yields problem (3.2).

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