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NOTE ON THE FUNCTIONAL DIFFERENTIAL EQUATION $\frac{d}{dx}\{\phi(x)\} - a\phi(x^\lambda) = 0$*

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The functional differential equation

$$\frac{d}{dx}\{\phi(x)\} - a\phi(x^\lambda) = 0, \quad x > 0 \tag{1}$$

does not appear to have received any attention in books upon functional differential equations (1). The purpose of this note is to point out that, by very simple processes, it may be transformed into an equation about which much is known.

Let

$$\phi(x) = x^{1/(1-\lambda)}\psi(x). \tag{2}$$

The equation (1) becomes

$$x^{1/(1-\lambda)}\frac{d}{dx}\{\psi(x)\} + \frac{1}{1-\lambda}x^{\lambda/(1-\lambda)}\psi(x) - ax^{\lambda/(1-\lambda)}\psi(x^\lambda) = 0$$

which can be rewritten as

$$x\frac{d}{dx}\{\psi(x)\} - a\psi(x^\lambda) - b\psi(x) = 0 \tag{3}$$

where $b = 1/(\lambda - 1)$.

Let

$$x = e^t, \tag{3a}$$

$$\psi(x) = y(t). \tag{3b}$$

Equation (3) can easily be seen to transform into

$$\frac{d}{dt}\{y(t)\} - ay(\lambda t) - by(t) = 0. \tag{4}$$

This is an equation about which much is known ([2], [3], [4]).

* Received August 26, 1986.

Clearly, were the interval $x < 0$ of interest, all that would be necessary would be to use the transformation

$$x = -e^t.$$

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