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OF  
APPLIED MATHEMATICS

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VOLUME LVII

SEPTEMBER • 1999

NUMBER 3

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# QUARTERLY OF APPLIED MATHEMATICS

The QUARTERLY prints original papers in applied mathematics which have an intimate connection with applications. It is expected that each paper will be of a high scientific standard; that the presentation will be of such character that the paper can be easily read by those to whom it would be of interest; and that the mathematical argument, judged by the standard of the field of application, will be of an advanced character.

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This journal is indexed in *Science Citation Index*<sup>®</sup>, *SciSearch*<sup>®</sup>, *Research Alert*<sup>®</sup>, *CompuMath Citation Index*<sup>®</sup>, *Current Contents*<sup>®</sup>/*Physical, Chemical & Earth Sciences*, *Current Contents*<sup>®</sup>/*Engineering Computing & Technology*. It is also indexed by *Applied Science & Technology Index* and abstracted by *Applied Science & Technology Abstracts*.

Periodicals postage paid at Providence, Rhode Island.

Publication number 808680 (ISSN 0033-569X).

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## SUGGESTIONS CONCERNING THE PREPARATION OF MANUSCRIPTS FOR THE QUARTERLY OF APPLIED MATHEMATICS

The editors will appreciate the authors' cooperation in taking note of the following directions for the preparation of manuscripts. These directions have been drawn up with a view toward eliminating unnecessary correspondence, avoiding the return of papers for changes, and reducing the charges made for "author's corrections."

**Manuscripts:** Manuscripts should be typewritten double-spaced on one side only. Marginal instructions to the typesetter should be written in pencil to distinguish them clearly from the body of the text. The author should keep a complete copy.

The papers should be submitted in final form. Only typographical errors should be corrected in proof; composition charges for any major deviations from the manuscript will be passed on to the author.

**Titles:** The title should be brief but express adequately the subject of the paper. The name and initials of the author should be written as he/she prefers; all titles and degrees or honors will be omitted. The name of the organization with which the author is associated should be given in a separate line following his/her name.

**Mathematical Work:** As far as possible, formulas should be typewritten; Greek letters and other symbols not available on the average typewriter should be inserted using either instant lettering or by careful insertion in ink. Manuscripts containing pencilled material other than marginal instructions to the typesetter will not be accepted.

The difference between capital and lower-case letters should be clearly shown; care should be taken to avoid confusion between zero (0) and the letter O, between the numeral one (1), the letter l and the prime ('), between alpha and a, kappa and k, mu and u, nu and v, eta and n.

The level of subscripts, exponents, subscripts to subscripts, and exponents to exponents should be clearly indicated.

Single embellishments over individual letters are allowed; the only embellishment allowed above groups of letters is the overbar.

Double embellishments are not allowed. These may be replaced by superscripts following the symbols.

Complicated exponents and subscripts should be avoided. Any complicated expression that recurs frequently should be represented by a special symbol.

For exponentials with lengthy or complicated exponents the symbol exp should be used, particularly if such exponentials appear in the body of the text. Thus,

$$\exp[(a^2 + b^2)^{1/2}] \text{ is preferable to } e^{[a^2 + b^2]^{1/2}}.$$

Fractions in the body of the text and fractions occurring in the numerators or denominators of fractions should be written with the solidus. Thus,

$$\frac{\cos(x/2b)}{\cos(a/2b)} \text{ is preferable to } \frac{\cos \frac{x}{2b}}{\cos \frac{a}{2b}}.$$

In many instances the use of negative exponents permits saving of space. Thus,

$$\int u^{-1} \sin u \, du \text{ is preferable to } \int \frac{\sin u}{u} \, du.$$

Whereas the intended grouping of symbols in handwritten formulas can be made clear by slight variations in spacing, this procedure is not acceptable in typeset formulas. To avoid misunderstanding, the order of symbols should therefore be carefully considered. Thus,

$$(a + bx) \cos t \text{ is preferable to } \cos t(a + bx).$$

**Figures:** Figures should be drawn in black ink with clean, unbroken lines; do not use ball point pen. The paper should be of a nonabsorbant quality so that the ink does not spread and produce fuzzy lines. If the figures are intended for reduction, they should be drawn with heavy enough lines so that they do not become flimsy at the desired reduction. The notation should be of professional quality and in proportion for the expected reduction size. Figures that are unsuitable for reproduction will be returned to the author for redrawing. Legends accompanying figures should be written on a separate sheet.

**Bibliography:** References should be grouped together in a Bibliography at the end of the manuscript. References in text to the Bibliography should be made by numerals between square brackets.

The following examples show the desired arrangements: (for books—S. Timoshenko, *Strength of Materials*, vol. 2, Macmillan and Co., London, 1931, p. 237; for periodicals—Lord Rayleigh, *On the flow of viscous liquids, especially in three dimensions*, Phil. Mag. (5) 36, 354-372 (1893)). Note that the number of the series is not separated by commas from the name of the periodical or the number of the volume.

Authors' initials should precede their names rather than follow them.

In quoted titles of books or papers, capital letters should be used only where the language requires this. Thus, *On the flow of viscous fluids* is preferable to *On the Flow of Viscous Fluids*, but the corresponding German title would have to be rendered as *Über die Strömung zäher Flüssigkeiten*.

Titles of books or papers should be quoted in the original language (with an English translation added in parentheses, if this seems desirable), but only English abbreviations should be used for bibliographical details such as ed., vol., no., chap., p.

**Footnotes:** As far as possible, footnotes should be avoided. Footnotes containing mathematical formulas are not acceptable.

**Abbreviations:** Much space can be saved by the use of standard abbreviations such as Eq., Eqs., Fig., Sec., Art., etc. These should be used, however, only if they are followed by a reference number. Thus, "Eq. (25)" is acceptable but not "the preceding Eq." Moreover, if any one of these terms occurs as the first word of a sentence, it should be spelled out.

Special abbreviations should be avoided. Thus "boundary conditions" should always be spelled out and not be abbreviated as "b.c." even if this special abbreviation is defined somewhere in the text.

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0033-569X(199909)57:3;1-Q

*Methods of Multivariate Analysis.* By Alvin C. Rencher, John Wiley and Sons, 1995, xvi+627 pp., \$39.95

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is the first volume of a two-volume set on multivariate analysis. The second volume, tentatively entitled *An Introduction to Multivariate Statistical Inference and Applications*, expected to appear in 1996, will include proofs of many of the techniques covered in the first volume, and also introduces additional topics. Since the present volume is essentially a methods book, the author is giving the reader a choice between two different approaches. The author's objective has been to make this volume accessible to readers who have taken as few as two statistical methods courses. Careful intuitive explanations of the concepts are provided. A brief overview of the subject is given in Chapter 1. Chapter 2 reviews the fundamentals of matrix algebra. Chapters 3 and 4 give an introduction to sampling from multivariate populations. Chapters 5, 6, 7, 10, and 11 extend univariate procedures with one dependent variable (such as  $t$ -tests, analysis of variance, tests on variances, multiple regression, and multiple correlation) to analogous multivariate techniques involving several dependent variables. A review of each univariate procedure is included. Chapters 8, 9, 12, and 13 describe multivariate techniques that are not related to univariate procedures. These include finding functions of the variables that discriminate among groups in the samples and finding functions of the variables that reveal the essential dimensionality and characteristic patterns of the system. To illustrate multivariate applications, the author has provided many examples and exercises based on real data sets from a wide variety of disciplines. The diskette included with the text contains (1) all the data sets and (2) SAS command files for all the examples in the text. The contents of the diskette are described in an appendix. Chapter headings: 1. Introduction, 2. Matrix algebra, 3. Characterizing and displaying multivariate data, 4. The multivariate normal distribution, 5. Tests on one or two mean vectors, 6. Multivariate analysis of variance, 7. Tests on covariate matrices, 8. Discriminant analysis, 9. Classification analysis: allocation of observations to groups, 10. Multivariate regression, 11. Canonical regression, 12. Principal component analysis, 13. Factor analysis.

*Fractals, Random Shapes and Point Fields—Methods of Geometrical Statistics.* By Dietrich Stoyan and Helga Stoya, John Wiley and Sons, 1994, xiv+389 pp., \$59.95

This is a volume in the Wiley Series in Probability and Mathematical Statistics. It is its aim to present some statistical methods for the analysis of large groups of geometrical objects and structures that have non-regular random shapes or are randomly scattered in space, in a way that may also be understood by non-mathematicians, in particular by materials scientists, geologists, environmental researchers and biologists. The authors assume that the reader has a basic knowledge of mathematics and statistics and their aim was to write a clear and popular text that is nevertheless mathematically correct without containing proofs. They treat three different subjects: fractals, random shapes, and point fields (processes), restricting attention to planar structures. The chapter headings indicate the scope of the book: Part I: Fractals and methods for the determination of fractal dimension; 1. Introduction, 2. Hausdorff measure and dimension, 3. Deterministic fractals, 4. Random fractals, 5. Methods for the empirical determination of fractal dimensions; Part II: The statistics of shapes and forms; 6. Fundamental concepts, 7. Representation of contours, 8. Set theoretic analysis, 9. Point description of figures, 10. Examples; Part III: Point field statistics; 11. Fundamentals, 12. Finite point fields, 13. Poisson point fields, 14. Fundamentals of the theory of point fields, 15. Statistics for homogeneous point fields, 16. Point fields models.

*Continuous Univariate Distributions, Volume 2.* By Norman L. Johnson, Samuel Kotz, and N. Balakrishnan, John Wiley and Sons, 1995, xix+719 pp., \$85.00

This is the second edition of a volume in the Wiley Series in Probability and Mathematical Statistics. For this edition, the length of each chapter has been, on the average, about doubled, and the number of references increased almost threefold. The chapter on quadratic forms has been postponed to a projected volume dealing with *Continuous Multivariate Distributions*. The chapter headings, continuing from volume I, are: 22. Extreme value distributions, 23. Logistic distributions, 24. Laplace (double exponential) distributions, 25. Beta distributions, 26. Uniform (rectangular) distributions, 27.  $F$ -distributions, 28.  $t$ -distributions, 29. Noncentral  $\chi^2$ -distributions, 30. Noncentral  $F$ -distributions, 31. Noncentral  $t$ -distributions, 32. Distributions of correlation coefficients, 33. Lifetime distributions and miscellaneous orderings.

*Geographical Data Analysis.* By Nigel Walford, John Wiley and Sons, 1995, viii+446 pp., \$59.95

This book has three specific objectives: to explain basic statistical techniques and demonstrate their application to quantitative geography; to equip students with the knowledge and skills necessary for carrying out research projects; and to make a link between statistical analysis and the substantive topics taught as part of systematic geography units. It follows the analysis progress of four exemplar projects, drawn from physical and human geography. Each project shows how an initial geographical question can lead to the formulation of tentative hypotheses, how data can be collected and input on computers, and then how basic statistical analysis can be undertaken.

*Quaternionic Quantum Mechanics and Quantum Fields.* By Stephen L. Adler, Oxford University Press, 1995, xii+586 pp., \$75.00

This is volume 88 in the International Series of Monographs on Physics. It aims to give a development and exposition of the quaternionic generalization of standard complex quantum mechanics. In the latter, the wave functions and probability amplitudes are represented by complex numbers. However, it has been known since a 1936 paper by Birkhoff and von Neumann that more general quantum mechanical systems, and in particular a quaternionic quantum mechanics, can in principle be constructed. The author is convinced that quaternionic quantum mechanics (q.q.m.) represents largely uncharted, and potentially very interesting, terrain in theoretical physics, giving both physical and mathematical motivations for the attempt, and hopes that this work will encourage its further exploration by others. The chapter headings are as follows. Part I (Introduction and general formalism): 1. Introduction, 2. General framework of q.q.m., 3. Further general results in q.q.m.; Part II (Nonrelativistic q.q.m.): 4. One particle quantum mechanics—general formalism, 5. Stationary state methods and phase methods, 6. Scattering theory and bound states, 7. Methods for time development, 8. Single-channel time-dependent formal scattering theory, 9. Multiparticle and multichannel methods, 10. Further multiparticle topics; Part III (Relativistic quaternionic quantum mechanics): 11. Relativistic single-particle wave equations: spin-0 and spin-1/2, 12. More on relativistic wave equations: the spin-1 gauge potential, Lagrangian formulations, and the Poincaré group, 13. Quaternionic quantum field theory, 14. Outlook.

*Probability, Stochastic Processes, and Queueing Theory—The Mathematics of Computer Performance Modeling.* By Randolph Nelson, Springer-Verlag, 1995, xxii+583 pp., \$44.00

The scope of this text is evident from the following table of contents. 1. Introduction, Part I (Probability): 2. Randomness and probability, 3. Combinatorics, 4. Random variables and distributions, 5. Expectation and fundamental theorems, Part II (Stochastic processes): 6. The Poisson process and renewal theory, 7. The M/G/1 queue, 8. Markov processes, 9. Matrix geometric solutions, 10. Queueing networks, 11. Epilogue and special topics. The text is thus an introduction to basic probability theory, and to some aspects of stochastic processes, in particular, Markov processes and queues, with emphasis on the latter. An unusual topic, not often treated in elementary texts, are matrix geometric solutions. The subjects stressed are those relevant to computer performance modeling, although there are no concrete applications to this subject given. The text is very readable and much care has been taken to illustrate all concepts introduced by intuitive and useful applications.

*Time Dependent Problems and Difference Methods.* By Bertil Gustafsson, Heinz-Otto Kreiss, and Joseph Olinger, John Wiley and Sons, 1995, xi+642 pp., \$79.95

This is a volume in Pure and Applied Mathematics, A Wiley-Interscience Series of Texts, Monographs, and Tracts. The authors' primary goal is to discuss material relevant to the derivation and analysis of numerical methods for computing approximate solutions to partial differential equations for time-dependent problems arising in the sciences and engineering. The material for differential equations and numerical methods is carried out in parallel. The book is organized in two parts: Part I (Chapters 1–8) discusses problems with periodic solutions and Part II (Chapters 9–13) discusses initial-boundary-value problems. The chapter headings are: 1. Fourier series and trigonometric interpolation; 2. Model equations; 3. Higher order accuracy; 4. Well-posed problems; 5. Stability and convergence for numerical approximations of linear and nonlinear problems; 6. Hyperbolic equations and numerical methods; 7. Parabolic equations; 8. Problems with discontinuous solutions; 9. The energy method for initial-boundary-value problems; 10. The Laplace Transform method for initial-boundary-value problems; 11. The energy method for difference approximations; 12. The Laplace Transform method for difference approximations; 13. The Laplace Transform method for fully discrete approximations: normal mode analysis.

*Saddlepoint Approximations.* By Jens Ledet Jensen, Oxford University Press, 1995, xii+332 pp., \$80.00

This is volume 16 in the Oxford Statistical Science series. It aims to give a self-contained and detailed account of saddlepoint approximations, a technique that provides good approximations to very small tail probabilities or to the density in the tails of the distributions, and to cover most aspects of the field. Saddlepoint approximations. There are ten chapters: 1. Basic notions; 2. Saddlepoint approximations for i.i.d. sums; 3. Asymptotic expansions of integrals; 4. Transformations of the basic approximation; 5. Tests in exponential families; 6. Uniform saddlepoint approximations; 7. Compound sums; 8. Alternative approximations; 9. Markov chains; 10. Nonstandard situations.

*Limit Theorems of Probability Theory—Sequences of Independent Random Variables.* By Valentin V. Petrov, Oxford University Press, 1995, ix+292 pp., \$90.00

This is volume 4 in the Oxford Studies in Probability Series. It can be used as a basis of a course in probability for graduate or advanced undergraduate students, since only some fundamentals of probability theory (summarized in the first chapter) are assumed. The book can also serve as a reference book for specialists in probability and statistics, since every chapter has addenda with precise formulations of many results, mostly recent and inaccessible in book form. Much attention is paid to the Russian probabilistic school headed by A. N. Kolmogorov and Yu. V. Linnik. The chapter headings are: 1. Some basic concepts and theorems of probability theory; 2. Probability inequalities for sums of independent random variables; 3. Weak limit theorems: convergence to infinitely divisible distributions; 4. Weak limit theorems: the central limit theorem and the weak law of large numbers; 5. Rates of convergence in the central limit theorem; 6. Strong limit theorems: the strong law of large numbers; 7. Strong limit theorems: the law of the iterated logarithm.

*Modelling Frequency and Count Data.* By J. K. Lindsey, Oxford University Press, 1995, ix+291 pp., \$55.00

This is volume 15 in the Oxford Statistical Science Series. It originated from teaching a third year categorical data analysis course to non-mathematical majors. It assumes an introductory statistics course and some previous knowledge of log linear/logistic and of generalized linear models. Chapters 6, 7, and 10 contain more complex material and may be difficult for non-mathematics students. It is assumed that the reader has access to software permitting the fitting of generalized linear models; the examples, including the graphics, were produced by GLIM4 and replication of the examples will be easiest with GENSTAT, GLIM, and S-Plus. The book is divided into two parts: I. Frequency data (Chapters 1–6) and Count data (Chapters 7–10), and the chapter headings are: 1. One-way frequency tables; 2. Larger tables; 3. Regression models; 4. Ordinal variables; 5. Zero frequencies; 6. Fitting distributions; 7. Counting processes; 8. Markov chains; 9. Structured transition matrices; 10. Overdispersion and cluster models. There is an appendix on GLIM macros, and a bibliography with 156 items.

*Degree Theory in Analysis and Applications.* By Irene Fonseca and Wilfrid Gangbo, Oxford University Press, 1995, vii+211 pp., \$55.00

This is volume 2 in the Oxford Lecture Series in Mathematics and its Applications. It focuses on the recent developments of degree theory for Sobolev functions, which cannot be found in the existing literature dealing with the notion of topological degree for continuous functions. In recent years, the need to extend the notion of degree to nonsmooth functions was motivated in part by applications to nonlinear analysis and, specifically, to nonlinear elasticity. Chapter headings: 1. Degree theory for continuous functions; 2. Degree theory in finite-dimensional spaces; 3. Some applications of the degree theory to topology; 4. Measure theory and Sobolev spaces; 5. Properties of the degree for Sobolev functions; 6. Local invertibility of Sobolev functions and applications; 7. Degree in infinite-dimensional spaces.