

QUARTERLY
OF
APPLIED MATHEMATICS

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In accordance with their general policy, the Editors welcome particularly contributions which will be of interest both to mathematicians and to scientists or engineers. Authors will receive galley proof only. The author's institution will be requested to pay a publication charge of \$30 per page to support production costs which, if honored, entitles the author to 100 free offprints. Detailed instructions will be sent with galley proofs.

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Design and Analysis of Clinical Trials—Concepts and Methodologies. By Shein-Chung Chow and Jen-Pei Liu, Wiley-Interscience, 2004, xiii+729 pp., \$130.00

This is the second edition of a volume in the Wiley Series in Probability and Statistics, first published in 1997. For this edition, sections have been updated and revised to reflect good clinical practice in regulatory review/approval processes, and recent developments in the field. Three new chapters (6, 14, and 15) have been added and there are many new topics to mirror recent developments. Chapter headings: 1. Introduction; 2. Basic statistical concepts; 3. Basic design considerations; 4. Randomization and blinding; 5. Designs for clinical trials; 6. Designs for cancer clinical trials; 7. Classification of clinical trials; 8. Analysis of continuous trials; 9. Analysis of categorical data; 10. Censored data and interim analysis; 11. Sample size determination; 12. Issues in efficacy evaluation; 13. Safety assessment; 14. Preparation and implementation of a clinical protocol; 15. Clinical data management. There is a bibliography of about 800 items, of which 280 are new for this edition.

Exploratory Data Mining and Data Cleansing. By Tamraparni Dasu and Theodore Johnson, Wiley-Interscience, 2003, xii+203 pp.

This is a volume in the Wiley Series in Probability and Statistics. Its focus is to develop a systematic process of data exploration and data quality management, that is, to present an integrated approach to data mining and data quality. Chapter headings: 1. Exploratory data mining and data cleaning: an overview; 2. Exploratory data mining; 3. Partitions and piecewise models; 4. Data quality; 5. Data quality: techniques and algorithms. There is a bibliography of 130 items.

The Theory of Measures and Integration. By Eric M. Vestrup, Wiley-Interscience, 2003, xv+594 pp., \$94.95

This is a volume in the Wiley Series in Probability and Statistics. It is its goal to present the major results of classical measure and integration theory in a clear, rigorous, and detailed fashion. It assumes a knowledge of advanced calculus and elementary analysis. In addition to the standard subjects, it also includes discussions of extensions, the structure of Borel and Lebesgue sets, set-theoretic considerations, the Riesz representation theorem, and the Hardy-Littlewood theorem. There are exercises of various degrees of difficulty at the end of every section, with hints or outlines of solutions provided for many. Chapter headings: 1. Set systems; 2. Measures; 3. Extensions of measures; 4. Lebesgue measure; 5. Measurable functions; 6. The Lebesgue integral; 7. Integrals relative to Lebesgue measure; 8. The L^p spaces; 9. The Radon-Nikodym theorem; 10. Products of two measure spaces; 11. Arbitrary products of measure spaces. There is a useful annotated bibliography of eight standard treatises that have influenced the presentation in this text.

Exact Solutions to Einstein's Field Equations. By Hans Stephani, Dietrich Kramer, Malcolm MacCallum, Cornelius Hoenselaers, and Eduard Herlt, Cambridge University Press, 2003, xxix + 701 pp., \$120.00

First published in 1980, this is the second edition of a volume in the Cambridge Monographs in Mathematical Physics series. It surveys the known solutions of Einstein's field equations for vacuum, Einstein-Maxwell, pure radiation, and perfect fluid sources. It starts by introducing the foundations of differential geometry and Riemannian geometry and the methods used to characterize, find, or construct solutions. The solutions are then considered, ordered by their symmetry group, their algebraic structure (Petrov type), or other invariant properties such as special subspaces or tensor fields and embedding properties. This edition has been expanded and updated, and contains five completely new chapters, covering topics such as generation methods and their application, colliding waves, classification of metrics by invariants, and homogeneous cosmologies. In preparing this new edition, the authors looked at over 4,000 new papers (the cut-off date being the end of 1999); the bibliography at the end of the work contains over 2,000 items. The 38 chapters are divided into five parts after the introduction: I. General methods (9 chapters); II. Solutions with groups of motion (15 chapters); III. Algebraically special solutions (8 chapters); IV. Spectral methods (4 chapters); V. Tables (chapter 38, The interconnections between the main classification schemes).

Bifurcation Theory—An Introduction with Applications to Partial Differential Equations. By Hansjörg Kielhöfer, Springer, 2004, vii + 346 pp.

This is volume 156 in the Applied Mathematical Sciences series. It gives a unified presentation in an abstract setting of the main theorems in bifurcation theory, as well as more recent and lesser-known results. It covers both the local and global theories of one-parameter bifurcations for operators acting in infinite-dimensional Banach spaces and shows how to apply the theory to problems involving partial differential equations. In addition to existence, qualitative properties such as stability and nodal structure of bifurcating solutions are treated in depth. The material is organized in three chapters: Local Theory, Global Theory, and Applications. There is a bibliography of 153 items.

Differential Equations—Linear, Nonlinear, Ordinary, Partial. By A. C. King, J. Billingham, and S. R. Otto, Cambridge University Press, 2003, xi + 541 pp., \$100.00 (hardcover), \$40.00 (paperback)

The reader of this text is assumed to have some basic knowledge of linear, constant coefficient ordinary differential equations, real analysis, and linear algebra. The fifteen chapters are divided into two parts: I. Linear equations (chapters 1–7) and II. Nonlinear equations and advanced techniques (chapters 8–15). Chapter headings: 1. Variable coefficient, second order, linear, ordinary differential equations; 2. Legendre functions; 3. Bessel functions; 4. Boundary value problems, Green's functions and Sturm-Liouville theory; 5. Fourier series and the Fourier transform; 6. Laplace transforms; 7. Classification, properties and complex variable methods for second order partial differential equations; 8. Existence, uniqueness, continuity and comparison of solutions of ordinary differential equations; 9. Nonlinear ordinary differential equations: phase plane methods; 10. Group theoretical methods; 11. Asymptotic methods: basic ideas; 12. Asymptotic methods: differential equations; 13. Stability, instability and bifurcations; 14. Time-optimal control in the phase plane; 15. An introduction to chaotic systems. There are seven appendices on background material, including a short introduction to MATLAB (which is used extensively in the book to illustrate the material and which is involved for several lengthier projects among the exercises).

Modeling Neural Development. Edited by Arjen van Ooyen. MIT Press, 2003. xiv + 322 pp., \$60.00

This is a volume in the Developmental Cognitive Neuroscience series. It is intended primarily for computational and experimental neuroscientists. Its 14 chapters by various authors bring together examples from different levels of organization (from molecule to system) and from different phases of development (from neurulation to cognition) that demonstrate the power of modeling for investigating the development of the nervous system. In most cases, each chapter contains an overview of the biology of the topic in question, a brief review of the modeling efforts in the field, a discussion in more detail of some of the models, and some perspectives on future theoretical and experimental work. Chapters 1 and 2 discuss the very early development of the nervous system; chapters 3 through 5 cover neuronal morphogenesis and neurite development; chapters 6 through 8 focus on different aspects of the self-organization of neurons into networks; and chapters 9 through 12 discuss the refinement of connectivity and the development of specific connectivity patterns, involving neuronal death and remodeling of axonal arborizations and changes in synaptic strength. Chapter 13 focuses on the hypothesis that the development of connections and learning in the mature brain depend critically on structural plasticity at the axodendritic interface, assuming an important role for individual dendrites in computation. Chapter 14 discusses the link between developmental processes at the cellular level and those at the systems level and explores how structural brain development relates to cognitive development. There are extensive bibliographies after each chapter.

Introduction to Classical Integrable Systems. By Olivier Babelon, Denis Bernard, and Michel Talon, Cambridge University Press, 2004, xi + 601 pp., \$120.00

This is a volume in the Cambridge Monographs on Mathematical Physics series. The authors develop the thesis that the subject of classical integrable systems has a profound unity and that the various approaches are simply changes of points of view on the same underlying reality. All ideas presented in the book find their roots in the notion of Lax pairs. After an introductory chapter, chapter 2 (Integrable Dynamical Systems) introduces the classical definition of integrable systems through the Liouville theorem and presents the Lax pair formulation. Chapter 3 (Synopsis of Integrable Systems) explains how to construct Lax pairs with spectral parameter for finite and infinite-dimensional systems. Chapter 4 (Algebraic Methods) discusses the abstract group theoretical formulation of the theory. The analytical aspects of the theory are discussed in chapter 5. The general constructions are illustrated by the examples of the closed Toda chain in chapter 6 and the Calogero-Moser model in chapter 7. Chapters 8 and 9 describe, respectively, the isomonodromic deformation problem and the infinite Grassmannian and integrable hierarchies. Chapters 10 and 11 are devoted to the classical study of the Kadomtsev-Petviashvili (KP) and Korteweg-de Vries (KdV) hierarchies, respectively. Chapter 12 studies the class of Toda and sine-Gordon field theories, exhibiting the relations between their conformal and integrable properties. The sine-Gordon model is presented in the framework of the Classical Inverse Scattering Method in chapter 13. The last three chapters may be viewed as mathematical appendices, provided to help the reader. Chapter 14 presents the basic facts about symplectic geometry, the natural language for classical mechanics and integrable systems. Because mathematical tools from Riemannian geometry and Lie groups are used throughout the book, they are presented concisely in chapters 15 (Riemann Surfaces) and 16 (Lie Algebras). Since a most active field of recent research is concerned with quantum integrable systems and exactly soluble models in statistical mechanics, the authors have always had quantum theory in mind when writing the book and have therefore introduced all classical objects which have a well-known quantum counterpart, which explains their emphasis on Hamiltonian methods. Poisson brackets, classical r matrices, Lie-Poisson properties of dressing transformations, and the method of separation of variables. A short list of references is given at the end of each chapter.

The Universality of the Radon Transform. By Leon Ehrenpreis, Oxford University Press, 2003, xii + 727 pp., \$149.00

This is a volume in the Cambridge Monographs on Mathematical Physics series. One of its principal aims is to show how the Radon transform impinges on such varied branches of pure mathematics as integral geometry, partial differential equations, Lie groups, holomorphic functions of several variables, asymptotic analysis, and number theory. Chapter 1 of the book gives a heuristic treatment of much of the material presented in the later chapters: 2. The nonparametric Radon transform; 3. Harmonic functions in R^n ; 4. Harmonic functions and Radon transform on algebraic varieties; 5. The nonlinear and Fourier transforms; 6. The parametric Radon transform; 7. Radon transform on groups; 8. Radon transform as the interrelation of geometry and analysis; 9. Extension of solutions of differential equations; 10. Periods of Eisenstein and Poincaré series. In an Appendix, Peter Kuchment and Eric Todd Quinto coordinate the abstract nature of the book with more concrete and practical aspects of the Radon transform, such as X-ray computed tomography (CT). Attenuated and exponential Radon transforms are described in the context of single photon emission computed tomography (SPECT) and a section of the Appendix discusses hyperbolic integral geometry and electric impedance tomography (EIT). The main part of the book and the Appendix each have a bibliography, with a total of 334 items.

Electromagnetism of Continuous Media—Mathematical Modelling and Applications. By Maurice Fabrizio and Angelo Morro, Oxford University Press, 2003, xvii + 668 pp., \$124.50

This is a volume in the Oxford Science Publications series. The twelve chapters are divided into three parts. Part I presents the basic notions of electromagnetism. In chapter 1, the electromagnetic system is viewed as a set of fields subject to balance laws that result in Maxwell's equations and boundary conditions, and includes an elementary introduction to constitutive equations. Chapter 2 deals with Green's functions and retarded potentials. Chapter 3 investigates various topics of electromagnetism in the specific case when time dependence is harmonic, with a fixed frequency. Chapter 4 is devoted to materials with memory, including the simple case of waves in water. Part II (chapters 5–9) develops the thermodynamics of the electromagnetic field and shows the effects of thermodynamic restrictions on existence and uniqueness problems, wave propagation, and extremum principles. Chapters 5 and 6 are devoted to thermodynamics of electromagnetic systems, chapter 5 involving isothermal systems and chapter 6 systems where the dependence on the temperature and on the temperature gradient are considered explicitly given through their histories. Chapter 7 investigates various boundary initial-value problems and chapter 8 outlines classical results about wave propagation in linear materials. Chapter 9 provides a review of variational formulations for various problems in electromagnetism. Part III exhibits topics which are based on more involved models of electromagnetic media. Chapter 10 deals with nonlinear media and chapter 11 provides a general, thermodynamic, scheme of nonlocal electromagnetism. Chapter 12 is devoted to magnetic hysteresis where nonlinearity and memory effects are strictly interrelated. Appendices provide short introductions to Bessel functions, Sobolev spaces, compact operators and eigenfunctions, differential operators in curvilinear coordinate systems, and a finite formulation of electromagnetism. There is a bibliography of 239 items.

Theory and Computation in Hydrodynamic Stability. By W. O. Criminale, T. L. Jackson, and R. D. Joslin. Cambridge University Press, 2003. xxii + 441 pp., \$90.00

This is a volume in the Cambridge Monographs on Mechanics series, of which the founding editor was George Batchelor. It covers both classical and modern aspects of the subject, developing it from the simplest physical problems, progressing to the most complex, considering linear and nonlinear situations, and analysing temporal and spatial stability. All relevant fluid flows are treated, including those where the fluid may be compressible, geophysical fluid flow, and those requiring salient geometries for description. This new treatise on the stability of fluid motion represents a complete updating of all aspects of the field, providing both analytical and numerical means for solving any problem. The scope of the treatment covers the full range of the dynamics, ranging from the transient to asymptotic behaviour as well as linear and nonlinear formulations. The subject of direct numerical simulations on computers is also included. Chapter headings: 1. Introduction and problem formulation; 2. Temporal stability of inviscid incompressible flows; 3. Temporal stability of viscous incompressible flows; 4. Spatial stability of incompressible flows; 5. Stability of compressible flows; 6. Centrifugal stability; 7. Geophysical flow; 8. Transient dynamics; 9. Nonlinear stability; 10. Transition and receptivity; 11. Direct numerical simulation; 12. Flow control and optimization; 13. Investigating hydrodynamic instabilities with experiments. There is a bibliography with 486 items.

The Theory of Angular Momentum -Gribov Lectures on Theoretical Physics. By V. N. Gribov. Cambridge University Press, 2003. xii + 297 pp., \$90.00

This is a volume in the Cambridge Monographs in Mathematical Physics series. It provides a unique and rigorous introduction to the theory of complex angular momenta, based on the methods of field theory. It commences with a Foreword by Yuri Dokshitzer, which traces the origin of the book to the lecture course given in 1969 by Vladimir N. Gribov (who died in 1997) at the Ioffe Physico-technical Institute in Leningrad, at a time when the physics of high energy hadron interactions was being created. The basic physical results and the approaches Gribov developed are now being rediscovered in a new context: in the microscopic theory of hadrons provided by quantum chromodynamics. The lecture notes were taken down by a group of 12 students and colleagues and were edited by L. Frankfurt; they were published in the form of two preprints (the "Kharkov preprint") and became a rarity. More than 25 years later, an English version was prepared by E. Kuraev, B. Shaikhatdenov, and V. Bytev, which was then edited by G. Korchemsky and L. Lipatov. The final English translation was carried out by V. von Schlippe. This Foreword is followed by an Introduction by Yuri Dokshitzer and Leonid Frankfurt, who share responsibility for the final scientific editing. The Introduction places the work in its historical context, including references to relevant papers. Chapter headings: 1. High energy hadron scattering; 2. Physics of the t -channel and complex angular momenta; 3. Singularities of partial waves and unitarity; 4. Properties of Regge poles; 5. Regge poles in high energy scattering; 6. Scattering of particles with spin; 7. Fermion Regge problems; 8. Regge poles in perturbation theory; 9. Reggeization of an electron; 10. Vector field theory; 11. Inconsistency of the Regge pole picture; 12. Two-reggeon exchange and branch point singularities in the l -plane; 13. Properties of Mandelstam branch singularities; 14. Reggeon diagrams; 15. Interacting reggeons; 16. Reggeon field theory; 17. The structure of weak and strong coupling solutions. There are three appendices: space-time description of the hadron interactions at high frequencies, character of inclusive spectra and fluctuations produced in inelastic processes by multipomeron exchange, and theory of the heavy pomeron.