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# ON GLOBAL SOLUTIONS AND ASYMPTOTIC BEHAVIOR OF PLANAR MAGNETOHYDRODYNAMICS WITH LARGE DATA

By

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**Abstract.** In this paper we consider an initial boundary value problem for planar magnetohydrodynamic compressible flows. By assuming that the adiabatic constant $\gamma$ is sufficiently close to 1, we prove the existence and uniqueness of global strong solutions with large initial data when all the viscosity, heat conductivity, and diffusivity coefficients are constant. Moreover, the asymptotic behavior of solutions is also investigated.

**1. Introduction.** Magnetohydrodynamics (MHD) concerns the motion of conducting fluids in an electromagnetic field. It covers a wide range of physical objects from liquid metals to cosmic plasmas; see [2], [12], [18], [19], [24]. The governing equations of a planar magnetohydrodynamic compressible flow in Lagrange coordinates take the form

$$\begin{cases} v_t - u_y = 0, \\ u_t + (p + \frac{1}{2}|\mathbf{b}|^2)_y = (\frac{\lambda u_y}{v})_y, \\ \mathbf{w}_t - \mathbf{b}_y = (\frac{\mu \mathbf{w}_y}{v})_y, \\ (v\mathbf{b})_t - \mathbf{w}_y = (\frac{\nu \mathbf{b}_y}{v})_y, \\ e_t + p u_y = (\frac{\kappa \theta_y}{v})_y + \frac{\lambda}{v} u_y^2 + \frac{\mu}{v}|\mathbf{w}_y|^2 + \frac{\nu}{v}|\mathbf{b}_y|^2, \end{cases} \tag{1.1}$$

where $v$ denotes the specific volume, $u \in \mathbb{R}$ the longitudinal velocity, $\mathbf{w} \in \mathbb{R}^2$ the transverse velocity, $\mathbf{b} \in \mathbb{R}^2$ the transverse magnetic field and $\theta$ the temperature. $p = p(v, \theta)$, $e = e(v, \theta)$ denote the pressure and internal energy, respectively. $\lambda = \lambda(v, \theta)$ and $\mu = \mu(v, \theta)$ are the viscosity coefficients of the flow, $\nu = \nu(v, \theta)$ is the magnetic diffusivity coefficient of the magnetic field, $\kappa = \kappa(v, \theta)$ is the heat conductivity coefficient. The system (1.1) is a three-dimensional magnetohydrodynamic flow which is uniform in the transverse directions; see [3].

There have been a number of studies on MHD by physicists and mathematicians because of its physical importance, complexity, rich phenomena, and mathematical challenges; see [1–5, 8, 15, 16, 21, 27–29] and the reference cited therein. In this paper, we

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consider a perfect gas for magnetohydrodynamic flow, that is,

$$p = R\frac{\theta}{v}, \quad e = C_v\theta = \frac{R\theta}{\gamma - 1}, \tag{1.2}$$

where $R$ is the gas constant, $C_v$ is the heat capacity of the gas at constant volume and $\gamma > 1$ is the adiabatic constant.

The system (1.1) is supplemented with the following initial and boundary conditions:

$$(v, u, \mathbf{w}, \mathbf{b}, \theta)(y, 0) = (v_0, u_0, \mathbf{w}_0, \mathbf{b}_0, \theta_0), \quad \forall y \in \Omega, \tag{1.3}$$

$$(u, \mathbf{w}, \mathbf{b}, \theta_x)|_{\partial\Omega} = 0, \tag{1.4}$$

where $\Omega = (0, 1)$ and the initial data (1.3) should be compatible with the boundary conditions (1.4).

Now, we recall some well-posedness results for the problem (1.1)-(1.4). The existence and uniqueness of local smooth solutions were proved firstly in [26], while the existence of global smooth solutions with small smooth initial data was shown in [15]. The large-time behavior of solutions was studied in [21], where the exponential stability of solutions was established. The global existence of weak solutions was also studied in [9]. Under the technical condition that $\kappa(v, \theta)$ satisfies

$$C^{-1}(1 + \theta^q) \le \kappa(v, \theta) \equiv \kappa(\theta) \le C(1 + \theta^q) \tag{1.5}$$

for some $q \ge 2$, Chen and Wang in [4] obtain the existence, uniqueness and Lipschitz continuous dependence of global strong solutions to (1.1)-(1.4) with large initial data. A similar result is obtained in [3] for real gases. Fan, Jiang and Nakamura [7] obtained global weak solutions to system (1.1)-(1.4) with large initial data and heat conductivity satisfying (1.5) with $q \ge 1$. Very recently, Hu and Ju [11] extended the above results to the case $q > 0$; also see [6] for solutions with vaccum. However, as mentioned in [10], the global existence of classical solution to the full perfect MHD equations with large data remains unsolved when all the viscosity, heat conductivity, and diffusivity coefficients are constant.

In the present paper, we assume that all the coefficients $\lambda, \mu, \nu, \kappa, R$ are positive constants. For simplicity, let $\lambda = \mu = \nu = \kappa = R = 1$. We shall show that under the condition that $\gamma - 1$ is sufficiently small, the global strong solution exists and neither shock waves nor vacuum and concentration appear in finite time with large initial data. Moreover, the asymptotic behavior of solutions is studied, and the exponential stability is established for large time $t$.

We mention that there exist some results concerning the smallness of $\gamma - 1$ in the one-dimensional Navier-Stokes equation and the Navier-Stokes-Possion equation; see [23], [14], [20], [25]. Our results can be regarded as an extension to the magnetohydrodynamic flow case.

Our main results are

THEOREM 1.1. Suppose that

- the initial data $(v_0, u_0, \mathbf{w}_0, \mathbf{b}_0, \theta_0)$ satisfying
$$\| \left( v_0, u_0, \mathbf{w}_0, \mathbf{b}_0, \frac{\theta_0 - 1}{\sqrt{\gamma - 1}} \right) \|_{H^1} \le E_0, \tag{1.6}$$
where $E_0$ is a positive constant independent of $\gamma - 1$;
- there exist $\gamma - 1$-independent positive constants $\underline{V}$, $\overline{V}$, $\underline{\Theta} < 1$, $\overline{\Theta} > 1$ such that
$$\underline{V} \le v_0(y) \le \overline{V}, \qquad \underline{\Theta} \le \theta_0(y) \le \overline{\Theta}; \tag{1.7}$$
- $\gamma - 1$ is sufficiently small.

Then there exists a unique global strong solution $(v, u, \mathbf{w}, \mathbf{b}, \theta)$ on $[0, 1] \times [0, +\infty)$ satisfying
$$\|(v, u, \mathbf{w}, \mathbf{b}, \frac{\theta - 1}{\sqrt{\gamma - 1}})\|_{H^1} + \int_0^{+\infty} \left( \|(u_y, \mathbf{w}_y, \mathbf{b}_y, \theta_y)\|_{H^1}^2 + \|(v_y, u_t, \mathbf{w}_t, \mathbf{b}_t, \theta_t)\|^2 \right) \mathrm{d}\tau \le C \tag{1.8}$$
and for each $(y, t) \in [0, 1] \times [0, +\infty)$,
$$C^{-1} \le v(y, t) \le C, \quad \underline{\Theta} \le \theta(y, t) \le \overline{\Theta}, \tag{1.9}$$
where $C > 0$ is constant depending only on the initial data.

THEOREM 1.2. Let $(v, u, \mathbf{w}, \mathbf{b}, \theta)$ be the solution obtained in Theorem 1.1. Then
$$\|(v - v^*, u, \mathbf{w}, \mathbf{b}, \theta - \theta^*)\|_{H^1} \to 0 \quad \text{as } t \to \infty, \tag{1.10}$$
where the constants $v^*, \theta^*$ are determined by
$$v^* = \int_0^1 v_0 \mathrm{d}y, \ \frac{\theta^*}{\gamma - 1} = \int_0^1 \left( \frac{\theta_0}{\gamma - 1} + \frac{1}{2}(u_0^2 + \mathbf{w}_0^2 + v_0 \mathbf{b}_0^2) \right) \mathrm{d}y. \tag{1.11}$$
Moreover, there are constants $\alpha, T_0, C > 0$, independent of $t$, such that
$$\|v - v^*\|_{H^1} + \|u\|_{H^1} + \|\mathbf{w}\|_{H^1} + \|\mathbf{b}\|_{H^1} + \|\theta - \theta^*\|_{H^1} \le Ce^{-\alpha t}, \quad t \ge T_0. \tag{1.12}$$

The existence and uniqueness of local solutions can be obtained by the standard Banach fixed point theorem due to the contraction of the operators defined by the linearized problem (cf. [22]). The global existence of solutions will be proved by the method of extending the local solutions with respect to time based on a priori global estimates.

Now, we present the outline of the proof of the main results. To prove Theorem 1.1, the main idea is to assume the a priori estimate on the absolute temperature $\theta$; see (2.1) below. With given estimate for $\theta$, the estimates for both magnetic field and hydrodynamic velocity (see (2.4)) can be obtained, which is crucial in getting uniform (with respect to time $t$) lower and upper bounds on $v$ by using the argument initiated in [17] and [13]. With uniform bounds for both $\theta$ and $v$ at hand, we can get the estimates for the higher order derivatives of solution $(v, u, \mathbf{w}, \mathbf{b}, \theta)$ by some delicate energy-type estimates. At the end, in order to extend the solution globally in time, we only need to close the a priori assumption (2.1) where we need the smallness of $\gamma - 1$. To prove Theorem 1.2, the idea is to use the established global a priori estimates which ensure that the solution $(v, u, \mathbf{w}, \mathbf{b}, \theta)$ decay to a constant state as time $t$ goes to infinity. The exponential decay of solution for large time $t$ can be obtained by using the argument initiated in [21].

**2. A priori estimates.** In this section, we suppose that the local solution $(v, u, \mathbf{w}, \mathbf{b}, \theta)$ has been extended to time $t = T$. We shall derive the necessary a priori estimates and apply the general continuation argument to get a global solution. First, we assume the following a priori estimates for the temperature $\theta$:

$$\frac{1}{2}\underline{\Theta} < \theta \leq 2\overline{\Theta}. \tag{2.1}$$

The smallness of $\gamma - 1$ is needed to close this a priori estimate.

In the following, $C$ denotes the generic constants which may depend on initial data but not depend on $T$ and $\gamma - 1$, not necessarily the same one at two different spaces.

We first have the following basic energy estimate.

LEMMA 2.1. Under the conditions in Theorem 1.1, we have for $0 \leq t \leq T$ that

$$\begin{aligned} &\int_0^1 \frac{\theta - 1 - \ln\theta}{\gamma - 1} + (v - \ln v - 1) + \frac{1}{2}(u^2 + |\mathbf{w}|^2 + v|\mathbf{b}|^2)\mathrm{d}y \\ &+ \int_0^t \int_0^1 \left( \frac{u_y^2}{v\theta} + \frac{|\mathbf{w}_y|^2}{v\theta} + \frac{|\mathbf{b}_y|^2}{v\theta} + \frac{\theta_y^2}{v\theta^2} \right) \mathrm{d}y\mathrm{d}\tau \leq C. \end{aligned} \tag{2.2}$$

*Proof.* By using system (1.1), we have the equality

$$\begin{aligned} &\left( \frac{\theta - \ln\theta - 1}{\gamma - 1} + (v - \ln v - 1) + \frac{1}{2}(u^2 + |\mathbf{w}|^2 + v|\mathbf{b}|^2) \right)_t + (pu - \mathbf{w}\mathbf{b})_y \\ &+ \frac{u_y^2 + |\mathbf{w}_y|^2 + |\mathbf{b}_y|^2}{v\theta} + \frac{\theta_y^2}{v\theta^2} = \left( \frac{\theta_y}{\theta v} + \frac{uu_y + \mathbf{w}\mathbf{w}_y + \mathbf{b}\mathbf{b}_y + \theta_y}{v} \right)_y. \end{aligned} \tag{2.3}$$

In view of (2.1), we notice that

$$\theta_0 - \ln\theta_0 - 1 \leq \frac{1}{\underline{\Theta}}(\theta_0 - 1)^2.$$

Thus, by initial assumption (1.6), we have

$$\int_0^1 \frac{\theta_0 - \ln\theta_0 - 1}{\gamma - 1}\mathrm{d}y \leq \frac{1}{\underline{\Theta}} \int_0^1 \frac{(\theta_0 - 1)^2}{\gamma - 1}\mathrm{d}y \leq C.$$

Now, integrating (2.3) over $[0,1] \times [0,t]$ and using boundary condition (1.4), we get (2.2) immediately. □

The following lemma gives the uniform bounds for both magnetic field and hydrodynamic velocity.

LEMMA 2.2.

$$\int_0^t \max_{y\in[0,1]} (u^2 + \mathbf{w}^2 + |\mathbf{b}|^2)\mathrm{d}\tau \leq C. \tag{2.4}$$

*Proof.* By using (1.4), (2.1) and Hölder inequality, we have

$$\begin{aligned} u^2+\mathbf{w}^2+\mathbf{b}^2 &= \left(\int_0^y u_y \mathrm{d}y\right)^2+\left(\int_0^y \mathbf{w}_y \mathrm{d}y\right)+\left(\int_0^y \mathbf{b}_y \mathrm{d}y\right)^2 \\ &\le \left(\int_0^1 |u_y| \mathrm{d}y\right)^2+\left(\int_0^1 |\mathbf{w}_y| \mathrm{d}y\right)^2+\left(\int_0^1 |\mathbf{b}_y| \mathrm{d}y\right)^2 \\ &\le \left(\int_0^1 \left(\frac{u_y^2}{v\theta}+\frac{\mathbf{w}_y^2}{v\theta}+\frac{\mathbf{b}_y^2}{v\theta}\right)\mathrm{d}y\right)\left(\int_0^1 v\theta \mathrm{d}y\right) \\ &\le C\int_0^1 \left(\frac{u_y^2}{v\theta}+\frac{\mathbf{w}_y^2}{v\theta}+\frac{\mathbf{b}_y^2}{v\theta}\right)\mathrm{d}y. \end{aligned}$$

Thus, in view of the energy estimates (2.2), we get (2.4) immediately. □

The next lemma shows the uniform upper and lower bounds of $v$.

LEMMA 2.3.

$$C^{-1}\le v(y,t)\le C. \tag{2.5}$$

*Proof.* Let

$$\phi(y,t)=\int_0^t\left(\frac{u_y}{v}-p-\frac{1}{2}|\mathbf{b}|^2\right)(y,\tau)\mathrm{d}\tau+\int_0^y u_0(x)\mathrm{d}x.$$

Then

$$\phi_t=\frac{u_y}{v}-p-\frac{1}{2}|\mathbf{b}|^2,\ \ \phi_y=u.$$

Therefore, using equation $(1.1)_1$, we have

$$(v\phi)_t-(u\phi)_y=u_y-\theta-\frac{1}{2}v\mathbf{b}^2-u^2.$$

Integrating the above equation over $[0,1]\times[0,t]$ and using the boundary conditions (1.4), we have

$$\int_0^1 v\phi \mathrm{d}y=\int_0^1 v_0\phi_0\mathrm{d}y-\int_0^t\int_0^1(\theta+\frac{1}{2}v|\mathbf{b}|^2+u^2)\mathrm{d}y\mathrm{d}\tau.$$

On the other hand, by equation $(1.1)_1$ and the boundary conditions (1.4), we have

$$\int_0^1 v\mathrm{d}y=\int_0^1 v_0\mathrm{d}y=v^*.$$

By continuity of $\phi$, for any $t>0$, there exists $a(t)\in[0,1]$ such that

$$\phi(a(t),t)=\frac{1}{v^*}\int_0^1 v\phi(y,t)\mathrm{d}y.$$

By definition of $\phi$ and using equation $(1.1)_1$, we have

$$\begin{aligned} \phi(a(t),t) &= \int_0^t\left(\frac{u_y}{v}-p-\frac{1}{2}|\mathbf{b}|^2\right)(a(t),\tau)\mathrm{d}\tau+\int_0^{a(t)}u_0(x)\mathrm{d}x \\ &= \ln v(a(t),t)-\ln v_0(a(t))-\int_0^t(p+\frac{1}{2}|\mathbf{b}|^2)(a(t),\tau)\mathrm{d}\tau+\int_0^{a(t)}u_0(x)\mathrm{d}x. \end{aligned}$$

Thus, we get

$$\frac{v_0(a(t))}{v(a(t),t)}\exp\left\{\int_0^t(p+\frac{1}{2}|\mathbf{b}|^2)(a(t),\tau)\mathrm{d}\tau\right\} \tag{2.6}$$
$$=\exp\left\{\frac{1}{v^*}\int_0^t\int_0^1(\theta+\frac{1}{2}v|\mathbf{b}|^2+u^2)\mathrm{d}y\mathrm{d}\tau+\int_0^{a(t)}u_0(x)\mathrm{d}x-\frac{1}{v^*}\int_0^1 v_0\phi_0\mathrm{d}y\right\}.$$

Now we derive the explicit formula for $v$ by using a similar argument as in [17]. Integrating $(1.1)_2$ with respect to $t$ and using the equation $(1.1)_1$, we have

$$u(y,t)-u_0(y)+\int_0^t(p+\frac{1}{2}|\mathbf{b}|^2)_y(y,\tau)\mathrm{d}\tau=\frac{\mathrm{d}}{\mathrm{d}y}(\ln v(y,t)-\ln v_0(y)). \tag{2.7}$$

Now, doing integration with respect to $y$ at fixed $t$ from the point $a(t)$ to an arbitrary $y\in[0,1]$, we obtain

$$\int_{a(t)}^y(u(x,t)-u_0(x))\mathrm{d}x+\int_0^t(p+\frac{1}{2}|\mathbf{b}|^2)(y,\tau)\mathrm{d}\tau-\int_0^t(p+\frac{1}{2}|\mathbf{b}|^2)(a(t),\tau)\mathrm{d}\tau$$
$$=\ln v(y,t)-\ln v(a(t),t)-\ln v_0(y)+\ln v_0(a(t)). \tag{2.8}$$

Using (2.6), we rewrite the above equation as

$$\frac{1}{v(y,t)}\exp\left\{\int_0^t p(y,\tau)\mathrm{d}\tau\right\}=\frac{1}{v_0(y)}Y(t)B(y,t)D(y,t), \tag{2.9}$$

where

$$Y(t)=\exp\left\{\frac{1}{v^*}\int_0^t\int_0^1(\theta+\frac{1}{2}v|\mathbf{b}|^2+u^2)\mathrm{d}y\mathrm{d}\tau\right\},$$
$$B(y,t)=\exp\left\{\int_{a(t)}^y[u_0(x)-u(t,x)]\mathrm{d}x+\int_0^{a(t)}u_0(x)\mathrm{d}x-\frac{1}{v^*}\int_0^1 v_0\phi_0\mathrm{d}y\right\},$$
$$D(y,t)=\exp\left\{-\int_0^t\frac{1}{2}|\mathbf{b}|^2(y,\tau)\mathrm{d}\tau\right\}.$$

Multiplying (2.9) by $\theta$, integrating over $[0,t]$, and recalling that $p=\frac{\theta}{v}$, we have that

$$\exp\left\{\int_0^t p(y,\tau)\mathrm{d}\tau\right\}=1+\frac{1}{v_0(y)}\int_0^t Y(\tau)B(y,\tau)D(y,\tau)\theta(y,\tau)\mathrm{d}\tau. \tag{2.10}$$

Putting (2.10) into (2.9), we finally get that

$$v(y,t)=v_0(y)Y(t)^{-1}B^{-1}(y,t)D^{-1}(y,t)\left(1+\frac{1}{v_0}\int_0^t Y(\tau)B(y,\tau)D(y,\tau)\theta(y,\tau)\mathrm{d}\tau\right). \tag{2.11}$$

From Lemma 2.1 and Lemma 2.2, we easily get that

$$C^{-1}\le B(y,t)\le C,\quad C<D(y,t)<1. \tag{2.12}$$

Thus, by using (2.1), (2.11) and (2.12), we have

$$v(y,t) \le C + C\int_0^t \exp\left\{-\frac{1}{v^*}\int_\tau^t\int_0^1 (\theta + \frac{1}{2}v|\mathbf{b}|^2 + u^2)\mathrm{d}y\mathrm{d}\tau\right\} \tag{2.13}$$

$$\le C + C\int_0^t \exp\{-C(t-\tau)\}\mathrm{d}\tau \le C. \tag{2.14}$$

On the other hand, by using (1.7), (2.1), (2.11), (2.12), and Lemma 2.1, we have that for any $t \ge 1$,

$$v(y,t) \ge C\int_0^t \exp\left\{-\frac{1}{v^*}\int_\tau^t\int_0^1 (\theta + \frac{1}{2}v|\mathbf{b}|^2 + u^2)\mathrm{d}y\mathrm{d}\tau\right\}$$

$$\ge C\int_0^t \exp\{-C(t-\tau)\}\mathrm{d}\tau \ge C(1-\exp\{-Ct\}) \ge C \tag{2.15}$$

and for $t < 1$,

$$v(y,t) \ge C\exp\{-\frac{1}{v^*}\int_0^t\int_0^1 \theta + \frac{1}{2}v|\mathbf{b}|^2 + u^2\mathrm{d}y\mathrm{d}\tau\} \ge C. \tag{2.16}$$

Therefore, the proof of this lemma is finished. □

Using (2.1), Lemma 2.1 and Lemma 2.3, we have the following estimates.

LEMMA 2.4.

$$\int_0^t\int_0^1 (u_y^2 + |\mathbf{w}_y|^2 + |\mathbf{b}_y|^2 + \theta_y^2)\mathrm{d}y\mathrm{d}\tau \le C. \tag{2.17}$$

The following lemma gives the estimates of derivatives of $v$.

LEMMA 2.5.

$$\int_0^1 v_y^2\mathrm{d}y + \int_0^t\int_0^1 v_y^2\mathrm{d}y\mathrm{d}\tau \le C. \tag{2.18}$$

*Proof.* We rewrite the momentum equation $(1.1)_2$ as

$$(u - \frac{v_y}{v})_t = -(p + \frac{1}{2}|\mathbf{b}|^2)_y. \tag{2.19}$$

Multiplying the above equation by $u - \frac{v_y}{v}$ and integrating the result, we have

$$\begin{aligned}
&\frac{1}{2}\int_0^1 (u - \frac{v_y}{v})^2(y,t)\mathrm{d}y \\
= \ &\frac{1}{2}\int_0^1 (u - \frac{v_y}{v})^2(y,0)\mathrm{d}y - \int_0^t\int_0^1 (-\frac{\theta v_y}{v^2} + \frac{\theta_y}{v} + \mathbf{b}\mathbf{b}_y)(u - \frac{v_y}{v})\mathrm{d}y\mathrm{d}\tau \\
\le \ &C - \int_0^t\int_0^1 \frac{\theta v_y^2}{v^3}\mathrm{d}y\mathrm{d}\tau + \int_0^t\int_0^1 \frac{\theta v_y u}{v^2}\mathrm{d}y\mathrm{d}\tau - \int_0^t\int_0^1 (\frac{\theta_y}{v} + \mathbf{b}\mathbf{b}_y)(u - \frac{v_y}{v})\mathrm{d}y\mathrm{d}\tau.
\end{aligned}$$

Using Young inequality, (2.1), Lemma 2.2 and Lemma 2.3, we have

$$\begin{aligned}
\int_0^t\int_0^1 \frac{\theta v_y u}{v^2}\mathrm{d}y\mathrm{d}\tau \ &\le \ \frac{1}{2}\int_0^t\int_0^1 \frac{\theta v_y^2}{v^3}\mathrm{d}y\mathrm{d}\tau + C\int_0^t \max_{y\in[0,1]} u^2 \int_0^1 \frac{\theta}{v}\mathrm{d}y\mathrm{d}\tau \\
&\le \ \frac{1}{2}\int_0^t\int_0^1 \frac{\theta v_y^2}{v^3}\mathrm{d}y\mathrm{d}\tau + C.
\end{aligned}$$

On the other hand, using (2.1) and Lemmas 2.1-2.4, we have

$$
\begin{aligned}
&\int_0^t\int_0^1 \frac{\theta_y}{v}(u-\frac{v_y}{v})\mathrm{d}y\mathrm{d}\tau\\
&\le \int_0^t\int_0^1 \frac{\theta_y^2}{v\theta^2}\mathrm{d}y\mathrm{d}\tau+\int_0^t \max_{y\in[0,1]} u^2\int_0^1 \frac{\theta^2}{v}\mathrm{d}y\mathrm{d}\tau+\frac{1}{4}\int_0^t\int_0^1 \frac{\theta v_y^2}{v^3}\mathrm{d}y\mathrm{d}\tau+C\int_0^t\int_0^1 \frac{\theta_y^2}{\theta v}\mathrm{d}y\mathrm{d}\tau\\
&\le C+\frac{1}{4}\int_0^t\int_0^1 \frac{\theta v_y^2}{v^3}\mathrm{d}y\mathrm{d}\tau
\end{aligned}
$$

and

$$
\begin{aligned}
\int_0^t\int_0^1 \mathbf{b}\mathbf{b}_y(u-\frac{v_y}{v})\mathrm{d}y\mathrm{d}\tau &\le \int_0^t\int_0^1 |\mathbf{b}_y|^2\mathrm{d}y\mathrm{d}\tau+C\int_0^t \max_{y\in[0,1]}|\mathbf{b}|^2\int_{\mathbb{R}}(u-\frac{v_y}{v})^2\mathrm{d}y\mathrm{d}\tau\\
&\le C+C\int_0^t \max_{y\in[0,1]}|\mathbf{b}|^2\int_{\mathbb{R}}(u-\frac{v_y}{v})^2\mathrm{d}y\mathrm{d}\tau.
\end{aligned}
$$

So, summing up the above estimates, we get

$$
\int_0^1 (u-\frac{v_y}{v})^2\mathrm{d}y+\int_0^t\int_0^1 \frac{\theta v_y^2}{v^3}\mathrm{d}y\mathrm{d}\tau\le C+C\int_0^t \max_{y\in[0,1]}|\mathbf{b}|^2\int_{\mathbb{R}}(u-\frac{v_y}{v})^2\mathrm{d}y\mathrm{d}\tau. \tag{2.20}
$$

Using Lemma 2.2 and Gronwall's inequality, we get

$$
\int_0^1 (u-\frac{v_y}{v})^2\mathrm{d}y+\int_0^t\int_0^1 \frac{\theta v_y^2}{v^3}\mathrm{d}y\mathrm{d}\tau\le C. \tag{2.21}
$$

Therefore, in view of (2.1), Lemma 2.1 and Lemma 2.3, the estimate (2.18) follows immediately. □

Next, we give the estimates on derivatives of $\mathbf{w},\mathbf{b}$. For convenience, we present the following interpolation inequalities, which will be frequently used in the coming text, for any $\delta>0$:

$$
|f|^2\le(1+\delta^{-1})\int_0^1|f|^2\mathrm{d}y+\delta\int_0^1|f_y|^2\mathrm{d}y, \tag{2.22}
$$

$$
|f_y|^2\le(1+\delta^{-1})\int_0^1|f_y|^2\mathrm{d}y+\delta\int_0^1|f_{yy}|^2\mathrm{d}y. \tag{2.23}
$$

Lemma 2.6.

$$
\begin{aligned}
&\int_0^1(|\mathbf{w}_y|^2+|\mathbf{b}_y|^2)\mathrm{d}y \qquad (2.24)\\
&\quad+\int_0^t\int_0^1(|\mathbf{w}_{yy}|^2+|\mathbf{b}_{yy}|^2+|\mathbf{b}_y|^4+|\mathbf{w}_y|^4+|\mathbf{w}_t|^2+|\mathbf{b}_t|^2)\mathrm{d}y\mathrm{d}\tau\le C,
\end{aligned}
$$

$$
\max_{(y,\tau)\in[0,1]\times[0,t]}(|\mathbf{w}|+|\mathbf{b}|)\le C. \tag{2.25}
$$

*Proof.* Multiplying equation $(1.1)_3$ by $\mathbf{w}_{yy}$, integrating the result, using (2.17), (2.18) and (2.23), we have

$$\begin{aligned}
&\frac{1}{2}\int_0^1 \mathbf{w}_y^2(y,t)\mathrm{d}y\\
&=\frac{1}{2}\int_0^1 \mathbf{w}_y^2(y,0)\mathrm{d}y-\int_0^t\int_0^1\frac{\mathbf{w}_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+\int_0^t\int_0^1\frac{v_y\mathbf{w}_y\mathbf{w}_{yy}}{v^2}\mathrm{d}y\mathrm{d}\tau-\int_0^t\int_0^1 b_y\mathbf{w}_{yy}\mathrm{d}y\mathrm{d}\tau\\
&\le C-\int_0^t\int_0^1\frac{\mathbf{w}_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+\int_0^t\int_0^1(|\mathbf{b}_y|+|v_y||\mathbf{w}_y|)\mathbf{w}_{yy}\mathrm{d}y\mathrm{d}\tau\\
&\le C-\frac{3}{4}\int_0^t\int_0^1\frac{\mathbf{w}_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+C\int_0^t\int_0^1|\mathbf{b}_y|^2\mathrm{d}y\mathrm{d}\tau+C\int_0^t\max_{y\in[0,1]}|\mathbf{w}_y|^2\int_0^1 v_y^2\mathrm{d}y\mathrm{d}\tau\\
&\le C-\frac{1}{2}\int_0^t\int_0^1\mathbf{w}_{yy}^2\mathrm{d}y\mathrm{d}\tau+\int_0^t\int_0^1|\mathbf{w}_y|^2\mathrm{d}y\mathrm{d}\tau\le C-\frac{1}{2}\int_0^t\int_0^1\mathbf{w}_{yy}^2\mathrm{d}y\mathrm{d}\tau.
\end{aligned}$$

Thus, we have

$$\int_0^1|\mathbf{w}_y|^2\mathrm{d}y+\int_0^t\int_0^1\mathbf{w}_{yy}^2\mathrm{d}y\mathrm{d}\tau\le C. \tag{2.26}$$

Rewrite $(1.1)_4$ as

$$\mathbf{b}_t=-\frac{u_y}{v}\mathbf{b}+\frac{1}{v}(\mathbf{w}+\frac{\mathbf{b}_y}{v})_y. \tag{2.27}$$

Multiplying the above equation by $\mathbf{b}_{yy}$, integrating the result, using Lemma 2.1, (2.17), (2.22) and (2.23), we have

$$\begin{aligned}
&\frac{1}{2}\int_0^1|\mathbf{b}_y|^2(y,t)\mathrm{d}y=\frac{1}{2}\int_0^1|\mathbf{b}_y|^2(y,0)\mathrm{d}y\\
&-\int_0^t\int_0^1\frac{\mathbf{b}_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+\int_0^t\int_0^1\frac{u_y\mathbf{b}-\mathbf{w}_y}{v}\mathbf{b}_{yy}\mathrm{d}y\mathrm{d}\tau+\int_0^t\int_0^1\frac{v_y\mathbf{b}_y\mathbf{b}_{yy}}{v^3}\mathrm{d}y\mathrm{d}\tau\\
&\le C-\frac{3}{4}\int_0^t\int_0^1\frac{\mathbf{b}_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau\\
&\qquad+C\max_{(y,\tau)\in[0,1]\times[0,t]}|\mathbf{b}|^2\int_0^t\int_0^1 u_y^2\mathrm{d}y\mathrm{d}\tau+C\int_0^t\max_{y\in[0,1]}|\mathbf{b}_y|^2\int_0^1 v_y^2\mathrm{d}y\mathrm{d}\tau\\
&\le C-\frac{1}{2}\int_0^t\int_0^1\frac{\mathbf{b}_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+C\int_0^1|\mathbf{b}|^2\mathrm{d}y+\frac{1}{4}\int_0^1|\mathbf{b}_y|^2\mathrm{d}y+C\int_0^t\int_0^1|\mathbf{b}_y|^2\mathrm{d}y\mathrm{d}\tau\\
&\le C-\frac{1}{2}\int_0^t\int_0^1\frac{\mathbf{b}_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+\frac{1}{4}\int_0^1|\mathbf{b}_y|^2\mathrm{d}y.
\end{aligned}$$

Thus

$$\int_0^1|\mathbf{b}_y|^2\mathrm{d}y+\int_0^t\int_0^1|\mathbf{b}_{yy}|^2\mathrm{d}y\mathrm{d}\tau\le C. \tag{2.28}$$

As a consequence of (2.27) and (2.28), we have

$$\begin{aligned}\int_0^t\int_0^1(|\mathbf{w}_y|^4+|\mathbf{b}_y|^4)\mathrm{d}y\mathrm{d}\tau &\le \int_0^t(\max_{y\in[0,1]}|\mathbf{w}_y|^2+\max_{y\in[0,1]}|\mathbf{b}_y|^2)\int_0^1(|\mathbf{w}_y|^2+|\mathbf{b}_y|^2)\mathrm{d}y\mathrm{d}\tau\\ &\le C\int_0^t\int_0^1(|\mathbf{w}_y|^2+|\mathbf{b}_y|^2+|\mathbf{w}_{yy}|^2+|\mathbf{b}_{yy}|^2)\mathrm{d}y\mathrm{d}\tau\le C.\end{aligned}$$

Moreover, by equations $(1.1)_3$ and (2.27), using (2.18), (2.26) and (2.28), we have

$$\begin{aligned}\int_0^t\int_0^1\mathbf{w}_t^2\mathrm{d}y\mathrm{d}\tau &\le 2\int_0^t\int_0^1\mathbf{b}_y^2\mathrm{d}y\mathrm{d}\tau+2\int_0^t\int_0^1\Big(\frac{\mathbf{w}_{yy}}{v}-\frac{v_y\mathbf{w}_y}{v^2}\Big)^2\mathrm{d}y\mathrm{d}\tau\\ &\le C+\int_0^t\max_{y\in[0,1]}|\mathbf{w}_y|^2\int_0^1 v_y^2\mathrm{d}y\mathrm{d}\tau\\ &\le C+C\int_0^t\int_0^1\mathbf{w}_y^2\mathrm{d}y\mathrm{d}\tau+C\int_0^t\int_0^1\mathbf{w}_{yy}^2\mathrm{d}y\mathrm{d}\tau\le C\end{aligned}$$

and

$$\begin{aligned}\int_0^t\int_0^1\mathbf{b}_t^2\mathrm{d}y\mathrm{d}\tau &\le 2\int_0^t\int_0^1\frac{u_y^2\mathbf{b}^2}{v}\mathrm{d}y\mathrm{d}\tau+2\int_0^t\int_0^1\frac{1}{v^2}\left(\mathbf{w}+\frac{\mathbf{b}_y}{v}\right)_y^2\mathrm{d}y\mathrm{d}\tau\\ &\le C+C\int_0^t\max_{y\in[0,1]}|\mathbf{b}_y^2|\int_0^1 v_y^2\mathrm{d}y\mathrm{d}\tau\le C.\end{aligned}$$

Thus the proof of this lemma is finished. □

Now, multiplying $(1.1)_2$ by $u_{yy}$ and integrating the result, and using (2.1), (2.17), (2.18), (2.24), Lemma 2.1 and Lemma 2.3, we get

$$\begin{aligned}&\frac12\int_0^1 u_y^2\mathrm{d}y+\int_0^t\int_0^1\frac{u_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau\\ &=\int_0^t\int_0^1\left(-(p+\frac12|\mathbf{b}|^2)_y u_{yy}-\frac{v_yu_yu_{yy}}{v^2}\right)\mathrm{d}y\mathrm{d}\tau+\frac12\int_0^1u_y^2(y,0)\mathrm{d}y\\ &\le C+\frac14\int_0^t\int_0^1\frac{u_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+C\int_0^t\int_0^1\frac{\theta_y^2}{v\theta^2}\mathrm{d}y\mathrm{d}\tau+\int_0^t\int_0^1\frac{v_y^2\theta^2}{v^3}\mathrm{d}y\mathrm{d}\tau+\int_0^t\max_{y\in[0,1]}|u_y|^2\mathrm{d}\tau\\ &\le\frac12\int_0^t\int_0^1\frac{u_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+C.\end{aligned}$$

Thus we get

$$\int_0^1 u_y^2\mathrm{d}y+\int_0^t\int_0^1u_{yy}^2\mathrm{d}y\mathrm{d}\tau\le C \tag{2.29}$$

and

$$\int_0^t\int_0^1 u_y^4\mathrm{d}y\mathrm{d}\tau\le\int_0^t\max_{y\in[0,1]}|u_y|^2\int_0^1u_y^2\mathrm{d}y\mathrm{d}\tau\le\int_0^t\int_0^1u_{yy}^2\mathrm{d}y\mathrm{d}\tau\le C. \tag{2.30}$$

Using the Sobolev imbedding theorem, we have

$$\max_{(y,\tau)\in[0,1]\times[0,t]}u\le C. \tag{2.31}$$

Moreover, using equations $(1.1)_2$ and (2.1), Lemma 2.3, Lemma 2.5, and Lemma 2.6, we have

$$\int_0^t\int_0^1 u_t^2\mathrm{d}y\mathrm{d}\tau\leq C. \tag{2.32}$$

Combining (2.29), (2.30), (2.31) and (2.32) together, we obtain the following lemma.

LEMMA 2.7.

$$\max_{(y,\tau)\in[0,1]\times[0,t]} u+\int_0^1 u_y^2\mathrm{d}y+\int_0^t\int_0^1(u_y^4+u_{yy}^2+u_t^2)\mathrm{d}y\mathrm{d}\tau\leq C. \tag{2.33}$$

The estimates for the derivatives of $\theta$ are given in the following lemma.

LEMMA 2.8.

$$\int_0^1\frac{\theta_y^2}{\gamma-1}+\int_0^t\int_0^1(\theta_{yy}^2+\frac{\theta_t^2}{(\gamma-1)^2})\mathrm{d}y\mathrm{d}\tau\leq C. \tag{2.34}$$

*Proof.* Taking derivatives with respect to $y$ in temperature equation $(1.1)_5$, we have

$$\frac{\theta_{yt}}{\gamma-1}+(pu_y)_y=\left(\frac{\theta_y}{v}\right)_{yy}+\left(\frac{u_y^2+|\mathbf{w}_y|^2+|\mathbf{b}_y|^2}{v}\right)_y. \tag{2.35}$$

Multiplying the above equation by $\theta_y$, integrating the results, using (2.1), (2.18), (2.24), (2.33) and Lemma 2.3, we have

$$\begin{aligned}
\int_0^1\frac{\theta_y^2}{\gamma-1}\mathrm{d}y&=\int_0^1\frac{\theta_{0y}^2}{\gamma-1}\mathrm{d}y-\int_0^t\int_0^1\left(\left(\frac{\theta_y}{v}\right)_y+(pu_y)-\frac{u_y^2+|\mathbf{w}_y|^2+|\mathbf{b}_y|^2}{v}\right)\theta_{yy}\mathrm{d}y\mathrm{d}\tau\\
&\leq C-\frac12\int_0^t\int_0^1\frac{\theta_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+\int_0^t\int_0^1\theta_y^2v_y^2\mathrm{d}y\mathrm{d}\tau+\int_0^t\int_0^1(u_y^2+u_y^4+|\mathbf{w}_y|^4+|\mathbf{b}_y|^4)\mathrm{d}y\mathrm{d}\tau\\
&\leq C-\frac12\int_0^t\int_0^1\frac{\theta_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau+\int_0^t\max_{y\in[0,1]}\theta_y^2\mathrm{d}\tau\\
&\leq C-\frac14\int_0^t\int_0^1\frac{\theta_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau.
\end{aligned}$$

Thus, we get

$$\int_0^1\frac{\theta_y^2}{\gamma-1}\mathrm{d}y+\int_0^t\int_0^1\frac{\theta_{yy}^2}{v}\mathrm{d}y\mathrm{d}\tau\leq C. \tag{2.36}$$

By using equations $(1.1)_5$ and (2.1), (2.36), Lemma 2.3, Lemma 2.5, Lemma 2.6 and Lemma 2.7, we get

$$\int_0^t\int_0^1\frac{\theta_t^2}{(\gamma-1)^2}\leq C. \tag{2.37}$$

So, combining (2.36) and (2.37), we get (2.34), and the proof of this lemma is finished. □

**3. Proof of Theorem 1.1 and Theorem 1.2.** In view of the a priori estimates established in section 2, we only need to close the a priori estimate (2.1) to complete the proof of Theorem 1.1. We need the smallness of $\gamma-1$ in order to do this. Firstly, simple calculations imply the following inequality:

$$\frac{1}{4\overline{\Theta}}(\theta-1)^2\leq\theta-\ln\theta-1. \tag{3.1}$$

The above inequality together with the basic energy estimate (2.2) yields

$$\|\theta-1\|_{L^2}\le C(\gamma-1)^{\frac{1}{2}}. \tag{3.2}$$

So, by the Sobolev imbedding theorem, we have

$$\|\theta-1\|_{L^\infty}\le\int_0^1|\theta-1|\mathrm{d}y+\int_0^1|\theta_y|\mathrm{d}y\le\|\theta-1\|_{L^2}+\|\theta_y\|_{L^2}\le C(\gamma-1)^{\frac{1}{2}}. \tag{3.3}$$

Therefore, choose $\gamma-1$ sufficiently small such that

$$1+C(\gamma-1)^{\frac{1}{2}}\le\overline{\Theta},\qquad 1-C(\gamma-1)^{\frac{1}{2}}\ge\underline{\Theta}. \tag{3.4}$$

Combining (3.3) and (3.4), we get

$$\underline{\Theta}\le\theta(y,t)\le\overline{\Theta}. \tag{3.5}$$

By the continuity argument, we see that the estimate (2.1) will always hold for $0\le t\le T$, and the proof of Theorem 1.1 is completed.

Now, we show the convergence of $(v,u,\mathbf{w},\mathbf{b},\theta)$ to the constant state $(v^*,0,0,0,\theta^*)$ in $H^1(0,1)$ as $t$ goes to infinity. To this end, we observe that by $(1.1)_1$, Lemmas 2.5-2.8, and the equality

$$\int_0^1(u_xu_{xt}+\mathbf{w}_x\mathbf{w}_{xt}+\mathbf{b}_x\mathbf{b}_{xt}+\theta_x\theta_{xt})\mathrm{d}y=-\int_0^1(u_{xx}u_t+\mathbf{w}_{xx}\mathbf{w}_t+\mathbf{b}_{xx}\mathbf{b}_t+\theta_{xx}\theta_t)\mathrm{d}y, \tag{3.6}$$

we have

$$\int_0^\infty\left\{\frac{\mathrm{d}}{\mathrm{d}t}\left(\|v_x\|^2+\|u_x\|^2+\|\mathbf{w}_x\|^2+\|\mathbf{b}_x\|^2+\|\theta_x\|^2\right)\right\}\mathrm{d}t\le C, \tag{3.7}$$

which combined with Lemmas 2.4-2.5 yields

$$\|v_x\|^2+\|u_x\|^2+\|\mathbf{w}_x\|^2+\|\mathbf{b}_x\|^2+\|\theta_x\|^2\to0,\quad\text{as }\ t\to\infty. \tag{3.8}$$

Using Poincaré's inequality, we have

$$\|v-v^*\|_{H^1}+\|u\|_{H^1}+\|\mathbf{w}\|_{H^1}+\|\mathbf{b}\|_{H^1}\to0,\quad\text{as }\ t\to\infty. \tag{3.9}$$

So, in view of (3.8), in order to complete the proof of the convergence of solutions, it remains to show that

$$\|\theta-\theta^*\|\to0,\quad\text{as}\quad t\to\infty. \tag{3.10}$$

By using equations $(1.1)_1-(1.1)_4$, we can rewrite equation $(1.1)_5$ as

$$E_t+\left[(p+\frac{1}{2}\mathbf{b}^2)u-\mathbf{w}\mathbf{b}\right]_y=\left(\frac{uu_y+\mathbf{w}\mathbf{w}_y+\mathbf{b}\mathbf{b}_y+\theta\theta_y}{v}\right)_y, \tag{3.11}$$

where $E(t)=\frac{\theta}{\gamma-1}+\frac{1}{2}(u^2+\mathbf{w}^2+v\mathbf{b}^2)$. Recalling the definition of $\theta^*$, integrating the above equation over $[0,1]\times[0,t]$, we have

$$\int_0^1\left(\frac{\theta}{\gamma-1}+\frac{1}{2}(u^2+\mathbf{w}^2+v\mathbf{b}^2)\right)\mathrm{d}y=\frac{\theta^*}{\gamma-1}, \tag{3.12}$$

which together with Poincaré's inequality, Lemma 2.3, Lemma 2.6, Lemma 2.7 and (3.7), gives

$$\begin{aligned}
&\|\frac{\theta}{\gamma-1}-\frac{\theta^*}{\gamma-1}+\frac{1}{2}(u^2+\mathbf{w}^2+v\mathbf{b}^2)\|^2\\
&\le C\|\theta_x+uu_x+\mathbf{w}\mathbf{w}_x+v\mathbf{b}\mathbf{b}_x+\frac{1}{2}\mathbf{b}^2v_x\|^2\\
&\le C\|\theta_x\|^2+\max_{(y,t)\in[0,1]\times[0,\infty)}(u^2+\mathbf{w}^2+v^2\mathbf{b}^2+\mathbf{b}^4)(\|u_x\|^2+\|\mathbf{w}_x\|^2+\|\mathbf{b}_x\|^2+\|v_x\|^2)\\
&\le C(\|v_x\|^2+\|u_x\|^2+\|\mathbf{w}_x\|^2+\|\mathbf{b}_x\|^2+\|\theta_x\|^2). \qquad (3.13)
\end{aligned}$$

It follows by the triangle inequality that

$$\begin{aligned}
\|\frac{\theta-\theta^*}{\gamma-1}\|^2 &\le \|\frac{\theta}{\gamma-1}-\frac{\theta^*}{\gamma-1}+\frac{1}{2}(u^2+\mathbf{w}^2+v\mathbf{b}^2)\|^2+\|\frac{1}{2}(u^2+\mathbf{w}^2+v\mathbf{b}^2)\|^2\\
&\le C(\|v_x\|^2+\|u_x\|^2+\|\mathbf{w}_x\|^2+\|\mathbf{b}_x\|^2+\|\theta_x\|^2)\to 0, \quad \text{as} \quad t\to\infty. \qquad (3.14)
\end{aligned}$$

We know that $v-v^*, u, \mathbf{w}, \mathbf{b}, \theta-\theta^*$ become small in $H^1$-norm for large $t$; thus we can apply an argument similar to those used in [21] to obtain (1.12) in Theorem 1.2 (the exponential convergence of $(v, u, \mathbf{w}, \mathbf{b}, \theta)$ to the constant state as $t$ goes to infinity). Thus the proof of Theorem 1.2 is completed.

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