Алгебра и анализ
Том. 15 (2003), вып. 3

St. Petersburg Math. J.
Vol. 15 (2004), No. 3, Pages 419–436
S 1061-0022(04)00815-5
Article electronically published on March 25, 2004

THE INCIDENCE OF THE PLANE WAVE ON AN ELASTIC WEDGE AT A CRITICAL ANGLE

V. V. KAMOTSKIĬ

Introduction

The problem of diffraction by an elastic wedge has a significant history. A rather detailed survey was presented in [1], so we shall restrict ourselves to a short commentary.

Up to the mid 1990s, most of the authors had been interested in the computational aspects of the problem, leaving the question of rigorous formulation aside (we mean the existence and uniqueness theorems). In the mid 1990s, a seminal approach, called the spectral function method, was worked out by G. Lebeau; this approach makes it possible to treat the diffraction problems in "wedge-like domains" and does not involve separation of variables. In particular, in [2] this method was applied to the problem of an elastic wedge immersed in a fluid: the solvability of the plane wave diffraction problem was proved, and its solution was represented in the form of a single layer potential. In [3], the same approach was applied to the elastic wedge problem. The existence and uniqueness of the solution was proved in the class of functions satisfying the radiation conditions formulated in [3]. The case of the critical incidence, i.e., the situation where a grazing plane wave traveling to the wedge tip appears, was not considered in [2] and [3]. Our aim in the present paper is to fill this gap; we prove the existence and uniqueness for the critical case as well.

§1. Notation and results

In this section we formulate the problem and results. Largely, we keep the notation introduced in [2] and [3].

1.1. Basic equations. Consider an elastic medium occupying a wedge

$$\Omega = \{(x, y) = (r\cos\theta, r\sin\theta) : 0 < \theta < \varphi\} \tag{1.1}$$

of opening $\varphi \in (0, 2\pi)$. We introduce two Cartesian systems (x_1, y_1) and (x_2, y_2) associated with the wedge faces (see Figure 1) and the corresponding polar angles θ_1 and θ_2 (measured in opposite directions from the x_1 and x_2 axes, respectively). The wedge faces are Γ_j, $j = 1, 2$:

$$\partial\Omega = \Gamma_1 \cup \Gamma_2, \tag{1.2}$$

$$\Gamma_j = \{y_j = 0, x_j \geq 0\} = \{\theta_j = 0\}, \quad j = 1, 2. \tag{1.3}$$

Below we agree to omit the index that indicates the edge if and only if it is equal to 1.

2000 *Mathematics Subject Classification.* Primary 35Q60.

Key words and phrases. Elastic wedge, diffraction, spectral functions, Green's tensor, existence of a solution.

Supported by RFBR (grant no. 01-01-00251).

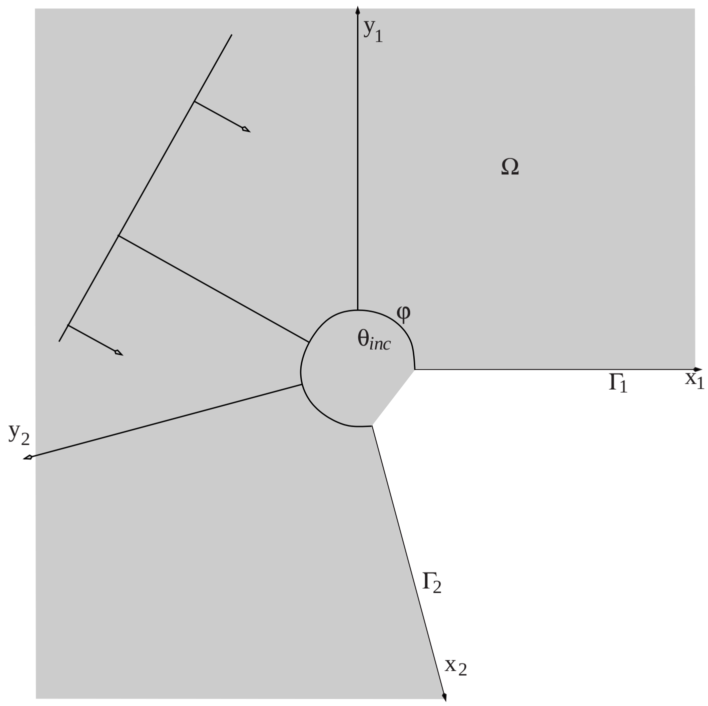

FIGURE 1. Geometry of the problem.

We consider the classical system of elasticity equations: it is assumed that the displacement field $u = (u^1, u^2)$ satisfies the dynamic Lamé system

$$\underline{\mu}\Delta u + (\underline{\lambda} + \underline{\mu})\nabla \operatorname{div} u + \rho\omega^2 u = 0 \quad \text{in } \Omega \tag{1.4}$$

and the zero normal traction condition on the faces of Ω:

$$(\underline{\lambda} \operatorname{div} u + 2\underline{\mu}\varepsilon(u))n = 0. \tag{1.5}$$

Here $\underline{\lambda}$ and $\underline{\mu}$ are the Lamé constants; ρ and ω denote the density and the frequency, respectively. In arbitrary Cartesian coordinates (z_1, z_2), the deformation tensor ε can be expressed as $\varepsilon_{i,j} = (u^i_{z_j} + u^j_{z_i})/2$.

We seek a solution of the diffraction problem, i.e., we assume that the total field u is the sum of an incident plane wave and the scattered field:

$$u = u^{\text{inc}} + u_0, \tag{1.6}$$

where u^{inc} is an incident longitudinal or transverse wave.

Now we pass to dimensionless coordinates: we introduce a new unknown function v by the formula

$$u_0(x, y) = v\Big(\frac{\omega}{c_L}x, \frac{\omega}{c_L}y\Big), \tag{1.7}$$

where $c_L = \sqrt{(\underline{\lambda} + 2\underline{\mu})/\rho}$ is the longitudinal wave speed. Thus, v satisfies the system

$$\begin{cases} (E+1)v = 0 & \text{in } \Omega, \\ Bv = -Bv^{\text{inc}}_* & \text{on } \Gamma; \end{cases} \tag{1.8}$$

here $E = \mu\Delta u + (\lambda+\mu)\nabla \operatorname{div} u$ is the dimensionless Lamé operator, and the dimensionless operator of normal traction is $Bv = (\lambda \operatorname{div} v + 2\mu\varepsilon(v))n$, where n is the inward normal.

The cases where $* = L, T$ correspond to the type of the incident plane wave: the incident longitudinal wave is given by

$$v_L^{\text{inc}} = \begin{pmatrix} \cos\theta_{\text{inc}} \\ \sin\theta_{\text{inc}} \end{pmatrix} e^{ir\cos(\theta-\theta_{\text{inc}})}, \tag{1.9}$$

and the transverse wave by

$$v_T^{\text{inc}} = \begin{pmatrix} -\sin\theta_{\text{inc}} \\ \cos\theta_{\text{inc}} \end{pmatrix} e^{i\nu_T r\cos(\theta-\theta_{\text{inc}})}. \tag{1.10}$$

We have the following relations between the true and the dimensionless quantities:

$$\mu = \frac{\underline{\mu}}{\rho c_L^2}, \quad \lambda = \frac{\underline{\lambda}}{\rho c_L^2}, \tag{1.11}$$

$$\nu_L = 1, \quad \nu_T = \frac{c_L}{c_T}, \quad \nu_R = \frac{c_L}{c_R}, \tag{1.12}$$

where c_R is the Rayleigh wave speed. Since $c_T = \sqrt{\underline{\mu}/\rho}$, we also have $\mu = \nu_T^{-2}$, $\lambda + 2\mu = 1$.

1.2. Results. In general position, i.e., except for finitely many incidence angles (the wedge angle is fixed), the solvability of (1.8) was established in [3]. To be more specific, from the total field we extract the sum $\tilde{u}^p$ of all plane body waves occurring in the problem: the incident wave and all multiply reflected waves generated by it. This part of the field has the form

$$\widetilde{u}^p = \sum_{*=L,T} \sum_{k=1}^{N^*} c_{*,k} e^{i\nu_* r\cos(\theta-\theta_{*,k})} \chi_{*,k}, \tag{1.13}$$

where the $c_{*,k}$ are constant vectors, and the $\chi_{*,k}$ are the characteristic functions of the physical domain of the existence of the wave. All $c_{*,k}$ and $\theta_{*,k}$ can be calculated successively, in accordance with the laws of geometrical optics; the incident wave is reflected by a face and gives rise to the first generation of reflected waves, which, if reaching the other face of the wedge, gives rise to the second generation of reflected waves, or, without reaching, travels away to infinity, and so on. In [3], the existence of a solution was established under the assumption (hypothesis (H) in [2], [3]) that

$$\theta_{*,k} \neq 0 \quad \text{and} \quad \theta_{*,k} \neq \varphi \tag{1.14}$$

for all $(*, k)$. Of course, these relations should be viewed modulo 2π. Geometrically this means that neither the incident wave, nor any generated plane wave is a grazing one that travels along Γ_1 or Γ_2 to the wedge tip. In the present paper we prove the existence of a solution for these types of degeneration, with only one exception that corresponds to the case where the incident wave is a grazing wave with respect to a face (this situation needs special treatment even if we study the acoustic diffraction problem for the wedge with the Dirichlet boundary conditions).

The approach to be used differs from that employed in [2] and [3]: in those papers the boundary value problem was formulated for the total field (the reflected waves have not been extracted from the solution), and the solvability of the problem was proved by using spectral functions, which led naturally to the above constraints on the angle of incidence. The problem is that the grazing waves cannot be represented in the form of a single layer potential, i.e., via spectral functions.

In this paper we follow a different pattern: first, we extract all the reflected waves, and multiply them by a cut-off function equal to 1 outside the ball centered at the wedge tip and of radius 2, and equal to 0 in the unit ball. The resulting problem is split into two; the first — the "problem at infinity" — is solved with the help of the spectral function method, and for solving the other problem — the “local” one — we apply the Green tensor.

The remaining part of the paper is organized as follows: first we briefly recall the method of spectral functions in application to the elastic wedge problem (see [2] for the original treatment of the case of an immersed wedge, and [3] for the case under consideration) and next we apply the results of [3] to the critical cases.

§2. Spectral function method

2.1. Outgoing solutions. Consider the general boundary value problem, namely, the problem for the Lamé system with “general” boundary data:

$$\begin{cases} (E+1)v = 0 \quad \text{in } \Omega, \\ Bv|_{\Gamma_1} = f_1, \\ Bv|_{\Gamma_2} = f_2. \end{cases} \tag{2.1}$$

We shall show that if $f = (f_1, f_2)$ satisfies certain analytical constraints, then (2.1) admits a solution that satisfies the limiting absorption principle. More specifically, as in [2], we seek v in the form of the sum of two “single layer” potentials corresponding to the two wedge faces; i.e., we write

$$v = v_1 + v_2, \tag{2.2}$$

where v_1 and v_2 are the potentials defined *in the entire space* $\mathbb{R}^2$ and satisfying the following system:

$$(E+1)v_j = -\left[\begin{pmatrix}\alpha_j \\ \beta_j\end{pmatrix} \otimes \delta_j\right]. \tag{2.3}$$

Here δ_j is the integration measure on Γ_j; for an arbitrary $\psi \in C_0^\infty(\mathbb{R}^2)$ the action of δ_j on ψ is defined by the identity

$$\langle \delta_j, \psi \rangle = \int_0^\infty \psi(x_j, 0)\, dx_j. \tag{2.4}$$

The functions α_j and β_j are unknown densities that belong to a special function class $\mathcal{A}$ (see below). As has already been mentioned, the unknown displacements v_j must satisfy the limiting absorption principle. Thus, for $\varepsilon \in (0, \pi)$ the v_j^ε are viewed as well-defined *in the entire* $\mathbb{R}^2$ by the formulas

$$v_j^\varepsilon = -(E + e^{-2i\varepsilon})^{-1}\left[\begin{pmatrix}\alpha_j \\ \beta_j\end{pmatrix} \otimes \delta_j\right]. \tag{2.5}$$

So, we are ready for the following definition.

Definition 2.1. We say that g belongs to the class $\mathcal{A}$ if $g \in \mathcal{S}'(\mathbb{R}), \operatorname{supp} g \subset \mathbb{R}^+$, and the Fourier transform

$$\widehat{g}(\xi) = \int_{\mathbb{R}} e^{-ix\xi} g(x)\, dx \tag{2.6}$$

satisfies the requirements

$$\exists C_0 > 0 \text{ such that } \sup_{-\pi<\theta<0} \int_{C_0}^\infty |\widehat{g}(\rho e^{i\theta})|^2\, d\rho < \infty; \tag{2.7}$$

(2.8) $\widehat{g}$ is holomorphic in the vicinity of the points ν_T, ν_L, and ν_R.

In what follows we shall also use the notation $\widehat{\mathcal{A}}$ for the class of Fourier transforms of the distributions in $\mathcal{A}$.

For the potentials with densities in $\mathcal{A}$ we have the following regularity result [2, Lemma 2.2].

Lemma 2.2. *Suppose $\alpha_j, \beta_j \in \mathcal{A}$, $j = 1, 2$. Then formula* (2.5) *gives rise to well-defined tempered distributions v_j^ε that converge in $\mathcal{S}'(\mathbb{R}^2)$ as $\varepsilon \to 0$, and their limits v_j^0 satisfy the equation*

$$(E+1)v_j^0 = -\begin{pmatrix}\alpha_j\\ \beta_j\end{pmatrix} \otimes \delta_j. \tag{2.9}$$

Moreover, the v_j^ε possess the following regularity for all $\varepsilon \in [0,\pi)$:

(i) *v_j^ε is continuous in the entire space $\mathbb{R}^2$, and the trace $v_j^\varepsilon|_\Gamma$ belongs to $H^1_{\text{loc}}(\Gamma)$, $j = 1, 2$;*
(ii) *the one-sided normal derivatives are well defined: the functions $\partial_n^\pm v_j^\varepsilon$ are of class $L^2_{\text{loc}}(\Gamma)$ and are tempered distributions in the vicinity of ∞.*

This lemma allows us to give the following definition.

Definition 2.3. By an *outgoing solution* of the problem (2.10) we mean a solution representable in the form of an outgoing single layer potential, i.e., in the form

$$v = v_1 + v_2, \tag{2.10}$$

where

$$v_j = -\lim_{\varepsilon\to 0}(E + e^{-2i\varepsilon})^{-1}\left[\begin{pmatrix}\alpha_j\\ \beta_j\end{pmatrix}\otimes\delta_j\right] \tag{2.11}$$

with $\alpha_j, \beta_j \in \mathcal{A}$, $j = 1, 2$.

2.2. Integral system, spectral functions. Passing to the Fourier transforms in (2.5), we easily obtain the following integral representation of potentials:

$$v_j^\varepsilon(x_j, y_j) = \frac{i}{4\pi}e^{2i\varepsilon}\int_{\Gamma_0} e^{ix_j\xi}\sum_{*=L,T} e^{i|y_j|\zeta_*^\varepsilon(\xi)}M_*^\varepsilon(\xi, \operatorname{sgn} y_j)\Sigma_j(\xi)\,d\xi \tag{2.12}$$

for all $\varepsilon \in [0,\pi)$, where Γ_0 is the contour depicted in Figure 2, $\zeta_*^\varepsilon(\xi)$ is the branch of $\sqrt{e^{-2i\varepsilon}\nu_*^2 - \xi^2}$ distinguished by the requirements that it is continuous along Γ_0 and its imaginary part is nonnegative on the real axis. The matrices M_*^ε have the following representation:

$$M_L^\varepsilon(\xi, t) = \begin{pmatrix}\dfrac{\xi^2}{\zeta_L^\varepsilon(\xi)} & t\xi\\ t\xi & \zeta_L^\varepsilon(\xi)\end{pmatrix}, \tag{2.13}$$

$$M_T^\varepsilon(\xi, t) = \begin{pmatrix}\zeta_T^\varepsilon(\xi) & -t\xi\\ -t\xi & \dfrac{\xi^2}{\zeta_T^\varepsilon(\xi)}\end{pmatrix}, \tag{2.14}$$

and the Σ_j are the Fourier transforms of (α_j, β_j):

$$\Sigma_j(\xi) = \int_0^\infty e^{-ix_j\xi}\begin{pmatrix}\alpha_j(x_j)\\ \beta_j(x_j)\end{pmatrix}dx_j. \tag{2.15}$$

Now Σ_1 and Σ_2 are the new unknowns for the problem; they will be referred to as *spectral functions.*

To $v^\varepsilon = v_1^\varepsilon + v_2^\varepsilon$ with v_j represented by (2.12), we apply the operator of normal traction,

$$Bu = (\lambda \operatorname{div} u + 2\mu\varepsilon(u))n, \tag{2.16}$$

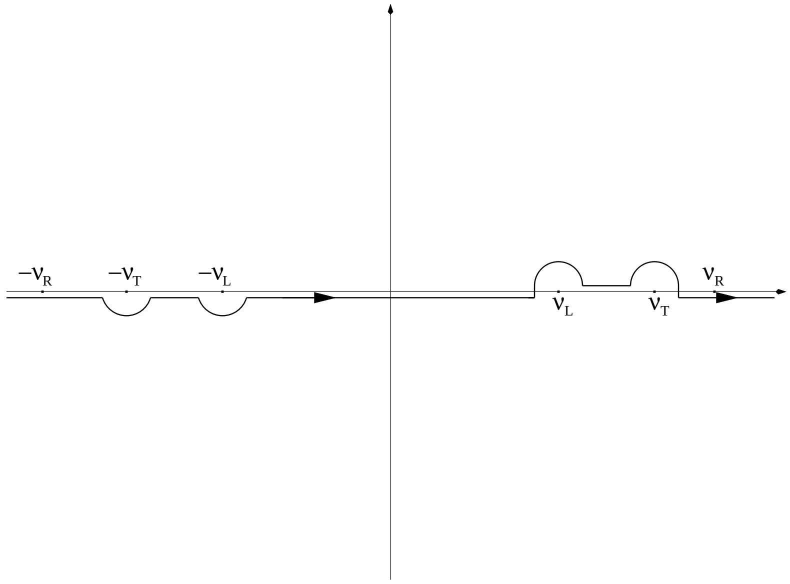

FIGURE 2. The contour Γ_0 in the ξ-plane.

where n is the inward normal; this results in the following lemma (see [2]).

Lemma 2.4. *The Fourier transform of the normal traction $B(v^\varepsilon)$ on the face Γ_1 is given by the following formula:*

$$\int_0^\infty e^{-ix\xi} B(v^\varepsilon)\,dx = \frac{1}{2}[DM^\varepsilon(\Sigma_1) + TM^\varepsilon(\Sigma_2)](\xi) \tag{2.17}$$

for all $\xi \in \mathbb{C}$ with $\operatorname{Im}\xi < 0$, where DM^ε and TM^ε are the integral operators defined for the $\widehat{\mathcal{A}}^2$-functions by the relations

$$DM^\varepsilon(g)(\xi) = \int_{\Gamma_0} DM^\varepsilon(\xi,\zeta) g(\zeta)\,d\zeta, \tag{2.18}$$

$$TM^\varepsilon(g)(\xi) = \int_{\Gamma_0} TM^\varepsilon(\xi,\zeta) g(\zeta)\,d\zeta. \tag{2.19}$$

The kernels of these operators look like this:

$$DM^\varepsilon(\xi,\zeta) = \frac{1}{2\pi i}\frac{1}{\xi-\zeta} dm(\zeta e^{i\varepsilon}), \tag{2.20}$$

where

$$dm(z) = \begin{pmatrix} -1 & A(z) \\ B(z) & -1 \end{pmatrix} \tag{2.21}$$

with

$$A(z) = \frac{z}{\zeta_T(z)}(1 - 2\mu Q(z)), \tag{2.22}$$

$$B(z) = -\frac{z}{\zeta_L(z)}(1 - 2\mu Q(z)), \tag{2.23}$$

$$Q(z) = \zeta_L(z)\zeta_T(z) + z^2, \tag{2.24}$$

and

$$TM^\varepsilon(\xi,\zeta) = \frac{1}{2\pi i} \sum_{*=L,T} D_*^\varepsilon(\xi,\zeta) tm_*(\zeta e^{i\varepsilon}, \operatorname{sgn}\sin\varphi), \tag{2.25}$$

where

$$D_*^\varepsilon(\xi,\zeta) = \frac{1}{\xi - (\zeta\cos\varphi + |\sin\varphi|\zeta_*^\varepsilon(\zeta))}, \tag{2.26}$$

and the rank 1 *matrices* tm_L *and* tm_T *are given by*

$$\begin{cases} tm_L(z,t) = \big(-\dfrac{\cos\chi}{\sin\chi}\overline{f}_L; t\overline{f}_L\big);\ \overline{f}_L = \begin{bmatrix} -\mu\sin\psi \\ \mu\cos\psi + \mu - 1 \end{bmatrix}, \\ z = \nu_L\cos\chi,\ \zeta_L = -\nu_L\sin\chi,\ \psi = 2(\varphi + t\chi), \end{cases} \tag{2.27}$$

$$\begin{cases} tm_T(z,t) = \big(-t\overline{f}_T; -\dfrac{\cos\chi}{\sin\chi}\overline{f}_T\big);\ \overline{f}_T = \begin{bmatrix} -\mu\cos\psi \\ \mu\sin\psi + \mu - 1 \end{bmatrix}, \\ z = \nu_T\cos\chi,\ \zeta_T = -\nu_T\sin\chi,\ \psi = 2(\varphi + t\chi). \end{cases} \tag{2.28}$$

For symmetry reasons, a similar result is valid for the other face of the wedge. Now we formulate a certain problem in terms of spectral functions; this problem is equivalent to the original boundary value problem.

Lemma 2.5. *A function* v *is a solution of* (2.1) *if and only if for all* ξ *in the lower half-plane the spectral function* $\Sigma = (\Sigma_1, \Sigma_2)$ *satisfies the following system*:

$$\begin{cases} DM^0\Sigma_1(\xi) + TM^0\Sigma_2(\xi) = W_1(\xi), \\ TM^0\Sigma_1(\xi) + DM^0\Sigma_2(\xi) = W_2(\xi), \end{cases} \tag{2.29}$$

where

$$W_1 = 2\widehat{f}_1(\xi), \tag{2.30}$$

$$W_2 = 2\widehat{f}_2(\xi). \tag{2.31}$$

The proof follows from the preceding lemma and the boundary conditions in (2.1).

To simplify our presentation, we introduce the following operator:

$$\mathcal{L}^\varepsilon = \begin{pmatrix} DM^\varepsilon & TM^\varepsilon \\ TM^\varepsilon & DM^\varepsilon \end{pmatrix}. \tag{2.32}$$

For $W^0(\xi) = (W_1, W_2)$, we rewrite (2.29) as

$$\mathcal{L}^0\Sigma = W^0. \tag{2.33}$$

So, we have obtained the following result: the problem we started with is reduced to an integral system[1], and the boundary data has contributed to the right-hand side of that system. Next, we shall obtain the solvability result for (2.33) in the class $\widehat{\mathcal{A}}^4$ under certain assumptions concerning the regularity of the function W^0 and its behavior at infinity. In what follows, it will be shown that, in essence, the original problem of the critical plane wave incidence reduces precisely to this situation.

[1]The system under consideration is not an integral equation system in the usual sense: we consider equations (2.29) with $\xi \in \mathbb{C}_-$; the possibility of reduction to a system of integral equations on the contour Γ_0 is insignificant for our approach.

2.3. An isomorphism theorem. In order to describe the properties of the system (2.33), we introduce the following notation and definitions: the lower half-plane will be denoted by $\mathbb{C}_- = \{z : \operatorname{Im} z < 0\}$, and the Hilbert space of L^2-functions on the positive semiaxis will be called H^+.

Definition 2.6. The space H^+ consists of all functions h holomorphic in $\mathbb{C}_-$ and such that

$$\sup_{c>0} \int_{\mathbb{R}} |h(x - ic)|^2 \, dx < \infty. \tag{2.34}$$

The following result is crucial for the spectral function techniques. In the case of an elastic wedge, it was obtained in [3] with the help of the arguments used in [2].

Theorem 2.7. *For all $\varepsilon \in (0, \pi)$, the operator $\mathcal{L}^\varepsilon$ is an isomorphism of the space $(H^+)^2 \oplus (H^+)^2$ onto itself.*

Basically, this theorem is a consequence of two results. The first is the well-posedness of the Dirichlet and Neumann boundary value problems (i.e., the problems with prescribed displacements and normal tractions, respectively) for the operator $E + e^{2i\varepsilon}$ in the wedges, and the second is the possibility to represent a solution of the Neumann problem in terms of a "single layer" potential, i.e., via (2.12). The details of the proof can be found in [2], [3]; here we shall restrict ourselves to the idea of the proof. The injectivity of $\mathcal{L}^\varepsilon$ follows from the uniqueness result for the Neumann problem: assuming that there exists a nontrivial solution of the system

$$\mathcal{L}^\varepsilon X = 0, \tag{2.35}$$

we conclude that there exists a nontrivial single layer potential solution of the problem

$$\begin{cases} (E + e^{-2i\varepsilon})u = 0 \quad \text{in } \Omega, \\ Bu|_\Gamma = 0, \end{cases} \tag{2.36}$$

which is not true because the corresponding form is coercive. Thus, in Ω this potential is zero; since it is continuous and the Dirichlet problem for the complement of Ω is solvable uniquely, it is zero globally. Since X depends linearly on the jumps of the normal traction of the potential, X is also zero.

The surjectivity of $\mathcal{L}^\varepsilon$ is a consequence of the solvability of the Neumann problem in Ω and the Dirichlet problem in $\mathbb{R}^2 \setminus \Omega$. Indeed, the solution of the equation

$$\mathcal{L}^\varepsilon X = Y \tag{2.37}$$

with $Y = 2(\widehat{y}_1, \widehat{y}_2) \in H^+$ can be constructed as follows: let u_+ be defined as a solution of the Neumann problem in Ω,

$$\begin{cases} (E + e^{-2i\varepsilon})u_+ = 0 \quad \text{in } \Omega, \\ Bu_+|_{\Gamma_1} = y_1, \\ Bu_+|_{\Gamma_2} = y_2, \end{cases} \tag{2.38}$$

and let u_- be the solution to the Dirichlet Problem in $\mathbb{R}^2 \setminus \Omega$,

$$\begin{cases} (E + e^{-2i\varepsilon})u_- = 0 \quad \text{in } \mathbb{R}^2 \setminus \Omega, \\ u_-|_{\Gamma_1} = u_+|_{\Gamma_1}, \\ u_-|_{\Gamma_2} = u_+|_{\Gamma_2}. \end{cases} \tag{2.39}$$

Then

$$u(x) = \begin{cases} u_+(x) & \text{if } x \in \Omega, \\ u_-(x) & \text{if } x \in \mathbb{R}^2 \setminus \Omega \end{cases} \tag{2.40}$$

can be represented in the form of a single layer potential, and the Fourier transform of the normal traction jumps will be the solution X of (2.37).

The isomorphism theorem is a key point in the spectral function method: though involving a "nonphysical" $\varepsilon \neq 0$, it nevertheless enables us to deduce the following result (see [3]), due to a direct analytical link between the operators $\mathcal{L}^\varepsilon$ and $\mathcal{L}^0$.

Theorem 2.8. *Suppose W is a function satisfying the following conditions*:

(i) $W \in \widehat{\mathcal{A}}^4$, *and W admits analytic continuation to* $\mathbb{C}^\delta_- = \{z : \arg(z) \in (-\pi, \delta)\}$ *with an arbitrarily small* $\delta > 0$;

(ii) *for all* $\varepsilon \in (0, \delta)$ *we have* $W^\varepsilon \in H^+$, *where* $W^\varepsilon(z) = W(e^{i\varepsilon} z)$.

Then there exists $X \in \widehat{\mathcal{A}}^4$ *that satisfies the equation*

$$\mathcal{L}^0 X = W. \tag{2.41}$$

The idea of the proof is fairly transparent: we fix some $\varepsilon \in (0, \delta)$ and define X^ε as the solution of the equation

$$L^\varepsilon X^\varepsilon = W^\varepsilon. \tag{2.42}$$

By the isomorphism theorem, this solution exists and belongs to H^+; then $X(z) = X^\varepsilon(e^{-i\varepsilon} z)$ is a solution of (2.41). The verification of this statement reduces to a change of variables in the integrals involved in the definition of the operators DM and TM.

Since the link between the integral operators that correspond to the nonperturbed problem and to the problem with absorption is quite simple and transparent, application of the spectral function method in the solvability analysis for problems in wedge-like domains looks quite attractive. We finish this section with an obvious corollary to the results cited.

Theorem 2.9 (on the existence). *If* $f = (f_1, f_2) \in \mathcal{A}^4$ *and, moreover,* $W = 2\widehat{f}$ *satisfies the assumptions* (i) *and* (ii) *of Theorem* 2.8*, then problem* (2.1) *has an outgoing solution.*

2.4. A uniqueness theorem and the Green tensor. By using the techniques of spectral functions, in [3] the Green tensor for an elastic wedge was constructed, the radiation conditions were formulated, and the uniqueness theorem was proved.

To formulate the uniqueness theorem, we need the following definitions: the intersection of the circle of radius R and centered at the wedge tip with the wedge Ω will be denoted by $\partial_R\Omega$; fixing some small $\varepsilon > 0$, we introduce the following covering of $\partial_R\Omega$:

$$\partial_R\Omega = \partial_R^{(i)}\Omega \cup \partial_R^{(b,1)}\Omega \cup \partial_R^{(b,2)}\Omega, \tag{2.43}$$

where the first element is the inner part,

$$\partial_R^{(i)}\Omega = \{(r, \theta) : r^{-1+\epsilon} < \theta < \varphi - r^{-1+\epsilon}\}, \tag{2.44}$$

and the remaining two are arc intervals close to Γ_j:

$$\partial_R^{(b,1)}\Omega = \{(r, \theta) : \theta < 2r^{-1+\epsilon}\}, \tag{2.45}$$

$$\partial_R^{(b,2)}\Omega = \{(r, \theta) : \varphi - \theta < 2r^{-1+\epsilon}\}. \tag{2.46}$$

In the sequel we shall need the following uniqueness theorem proved in [3].

Theorem 2.10 (on uniqueness). *Suppose that* $u \in H^1_{\mathrm{loc}}(\Omega)$ *satisfies the Lamé system in the wedge* Ω *and the zero normal traction condition on* Γ. *Let u satisfy the following growth conditions for some* $\epsilon > 0$ *as* $R \to \infty$:

$$\|u; L^2(\partial_R^{(i)}\Omega)\| = o(R^{1/2}), \qquad \|Bu; L^2(\partial_R^{(i)}\Omega)\| = o(R), \tag{2.47}$$

$$\|u; L^1(\partial_R^{(b,j)}\Omega)\| = o(R^{1/2}), \qquad \|Bu; L^1(\partial_R^{(b,j)}\Omega)\| = o(R^{1/2}), \tag{2.48}$$

together with the inner radiation conditions

$$\int_{\partial_R^{(i)}\Omega} \left| iu^r + \frac{\partial u^r}{\partial r} \right|^2 \to 0, \quad \int_{\partial_R^{(i)}\Omega} \left| i\nu_T u^\theta + \frac{\partial u^\theta}{\partial r} \right|^2 R^2 \to 0 \tag{2.49}$$

and the surface radiation conditions

$$\int_{\delta_R^{(b,j)}\Omega} \left| i\nu_R u + \frac{\partial u}{\partial x_j} \right|^2 \to 0, \quad j = 1, 2. \tag{2.50}$$

Then $u = 0$.

Also, we shall use a result on the existence of the Green tensor in the wedge with stress-free boundary.

Theorem 2.11 (on the existence of the Green tensor)**.** *There exists a unique tensor* $G(x, y, x_0, y_0)$, $(x, y, x_0, y_0) \in \Omega^4$, *that solves the problem*

$$\begin{cases} (E_{x,y} + 1)G = -\delta_{(x^0,y^0)}\mathbb{I}, \\ B_{x,y}G|_\Gamma = 0, \end{cases} \tag{2.51}$$

and satisfies (2.47) *and* (2.48) *as well as the radiation conditions* (2.49), (2.50).

§3. Plane wave diffraction problem

When considering the problem of the plane wave diffraction, in the papers [2], [3], a direct approach was used, i.e., problem (2.1) was formulated for the case where f is the trace of the incident plane wave, and only after that, at the level of integral equations, the analytic properties of the operators DM and TM were invoked to reduce the problem to Theorem 2.8. Precisely this approach led to the technical constraints (1.14) on the incidence angle. Here we proceed by a different method: we distinguish the problem at infinity and the local problems. To solve the former we use spectral functions, and the latter is solved with the help of the Green tensor. This algorithm of constructing (or proving the existence of) solutions was elaborated for the analysis of the critical angle incidence case, but, surely, it works in the general nondegenerate case as well.

First we extract the plane waves arising in the problem; this component will be denoted by u^p. We seek the solution of the problem in the form

$$u = (1 - \psi)u^p + u^\infty + u^l, \tag{3.1}$$

where ψ is a smooth cut-off function supported on the unit ball with center at the wedge tip and equal to 1 in the ball of radius $\frac{1}{2}$.

The problem "at infinity" is formulated for u^∞; it eliminates the jumps of $(1 - \psi)u^p$ *outside* the unit ball and the nonhomogeneities of Bu^p in the boundary condition:

$$\begin{cases} (E + 1)u^\infty = -[(E + 1)((1 - \psi)u^p)]|_{r>2} \quad \text{in } \Omega, \\ Bu^\infty|_\Gamma = -Bu^p|_\Gamma. \end{cases} \tag{3.2}$$

We solve this problem in two steps: first, we find a precise solution $u^{\infty,1}$ of (3.2) in the entire space, and the remaining problem for $u^{\infty,2} = u^\infty - u^{\infty,1}$ with nonhomogeneities only in the boundary conditions will be solved with the help of the spectral function method.

The local problem is formulated for u^l:

$$\begin{cases} (E + 1)u^l = -[(E + 1)((1 - \psi)u^p)]|_{r<2} \quad \text{in } \Omega, \\ Bu^l|_\Gamma = (-B((1 - \psi)u^p)| + Bu^p)_\Gamma. \end{cases} \tag{3.3}$$

Indeed, it is easily seen that all the nonuniformities of this problem are compactly supported.

The remaining part of the paper is devoted to the proof of the following result.

Theorem 3.1. *For any incident plane wave except for the grazing ones that travel along one of the faces of the wedge in the direction of the wedge tip, there exists a unique solution of the problem*

$$\begin{cases} (E+1)u = 0, \\ Bu|_\Gamma = 0, \end{cases} \tag{3.4}$$

with the representation

$$u = (1-\psi)u^p + u^{\infty,1} + u^{\infty,2} + u^l, \tag{3.5}$$

in which the first two terms can be found precisely, and the last two satisfy the growth conditions (2.47)*,* (2.47) *and the radiation conditions* (2.49)*,* (2.50)*.*

3.1. Reflected waves. Thus, to the maximal possible extent, we extract the plane waves arising in the problem: the incident wave and the generated waves (these are body waves) as well as the exponentially decaying surface waves arising when the transverse wave is incident at an angle greater than the critical angle.[2] In this calculation we shall consider the physical domain of the existence of the waves, and the characteristic functions of the "enlightened domains" will be denoted by $\chi_{*,k}$. We obtain the following representation for this "plane" component of the field:

$$u^p = \sum_{*=L,T} \sum_{k=1}^{N^*} c_{*,k} e^{i\nu_* r\cos(\theta-\theta_{*,k})} \chi_{*,k}. \tag{3.6}$$

All terms in this expression can be found successively in accordance with the laws of geometrical optics; for their evaluation we adopt the following rules:

(a) If at some step a reflected (or incident) wave meets the face Γ_j at an angle greater than the critical angle and generates a longitudinal wave decaying exponentially (with the distance from the face), then the domain of existence of this longitudinal wave will be $\Pi_j = \{x_j > 0, y_j > 0\} \cap \Omega$ (Figure 3). This longitudinal wave is not involved in the subsequent iterations.

(b) If the transverse wave is incident under the critical angle, then we distinguish two different cases, depending on the wave direction:

- (b.1) if the incident wave generates a longitudinal wave grazing *towards the tip*, then for this (longitudinal) wave we assume that its domain of existence is $\{x_j > 0, y\} \cap \Omega$;
- (b.2) otherwise, i.e., if the incident wave is already outgoing with respect to the face, we assume that it does not generate a longitudinal wave.

We note that using this algorithm we construct u^p with the following property: depending on the geometry of the problem, the normal tractions $Bu^p|_{\Gamma_1}$ and $Bu^p|_{\Gamma_2}$ are linear combinations of functions of either of two types:

(I) exponentially decaying at infinity;

(II) the trace of an outgoing grazing plane body wave, transverse or longitudinal.

This is easily seen, because nonuniformity in the boundary condition arises only when the situation described in (a) realizes for the wedge angle $\varphi \le \pi/2$, either in the case (b.2), or if one of the reflected waves is a grazing wave with respect to one of the faces

[2]The Rayleigh waves cannot be extracted at this step, because their amplitudes are unknown functions of all problem parameters.

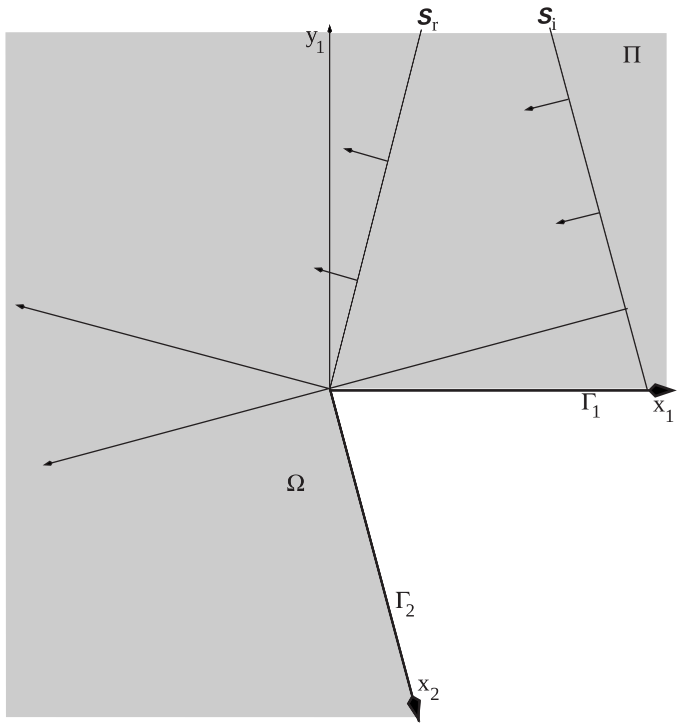

FIGURE 3. When the transverse wave S_i is incident at an angle greater than the critical angle, it gives rise to the reflected transverse wave S_r and an exponentially decaying longitudinal wave with domain of existence Π_j.

(i.e., if the shadow-light boundary coincides with one of the faces). In the first case the contribution to nonuniformity is of type (I), and in the other two cases this contribution is of type (II).

3.2. Problem at infinity. Our next step is the construction of u^∞. First we get rid of nonhomogeneity in (3.2).

We shall construct an outgoing solution $u^{\infty,1}$ of the equation *in the entire space*:

$$(E+1)u^{\infty,1} = h, \tag{3.7}$$

where h denotes the right-hand side in (3.2),

$$h = -\{(E+1)(1-\psi)u_p\}|_{\{r>2\}\cap\Omega}. \tag{3.8}$$

We explain that this notation makes sense. First, we note that $(1-\psi)u_p$ is regarded as a distribution *in the entire space* $\mathbb{R}^2$; application of the dynamic Lamé operator leads, in general, to a singular distribution, but a detailed investigation shows that, outside the unit ball, $(E+1)(1-\psi)u_p$ is a first order distribution and is equal to zero everywhere except for finitely many rays. These rays are the wedge faces and the shadow-light boundaries of the outgoing plane waves. The restriction to Ω in (3.8) means that the singularities generated by the wedge faces are not involved in h. On each of the rays corresponding to the shadow-light boundaries, $(E+1)(1-\psi)u_p$ is a linear combination of the integration measure on the ray multiplied by a smooth (exponential) function and the normal derivative of this measure multiplied by a smooth function. Thus, the restriction in (3.8) is understood as the well-defined product

$$(E+1)(1-\psi)u_p|_{\{r>2\}\cap\Omega} := \{(E+1)(1-\psi)u_p\}H(r-2)\chi_{\Omega_\varepsilon}, \tag{3.9}$$

where H is the Heaviside function, and $\Omega_\varepsilon \subsetneqq \Omega$ is also a wedge containing all the shadow-light boundaries.

Obviously, if a term in (3.6) produces a nonzero contribution to h, then $\chi_k^* \neq \chi_\Omega$, which corresponds to a plane wave that exists only in a part of the wedge, and thus has a shadow-light boundary. Without loss of generality we may assume that h is determined by only one term of this type. We introduce the Cartesian coordinates (z_1, z_2) with z_1 coinciding with the wave direction:

$$z_1 = \cos\theta_0^* x_1 + \sin\theta_0^* y_1, \tag{3.10}$$

$$z_2 = \sin\theta_0^* x_1 - \cos\theta_0^* y_1. \tag{3.11}$$

Suppose that the outgoing wave in question is a longitudinal wave, existing for positive z_1 in the domain $z_2 > 0$, i.e., we suppose that

$$u_p(z_1, z_2) = \begin{pmatrix}1\\0\end{pmatrix} e^{-iz_1}\chi_{\Omega_+}, \tag{3.12}$$

where $\Omega_+ = \{\theta \in (0, \theta_0)\}$. Then the corresponding h can be represented in the form

$$h = -\mu\frac{\partial}{\partial z_2}\delta^{(1)}e^{-iz_1}\begin{pmatrix}1\\0\end{pmatrix} + i(\lambda+\mu)\delta^{(1)}e^{-iz_1}\begin{pmatrix}0\\1\end{pmatrix}, \tag{3.13}$$

where $(\delta^{(1)}, f) = \int_2^\infty f(z_1)\,dz_1$ for all $f \in C_0^\infty(\mathbb{R}^2)$.

In contrast with the approach adopted in the preceding section, we define $u^{\infty,1}$ as the sum of a single layer potential and a double layer potential:

$$u^{\infty,1} = u^{\infty,1}_{(1)} + u^{\infty,1}_{(2)}, \tag{3.14}$$

where

$$u^{\infty,1}_{(1)} = \frac{i}{4\pi}\int_{\Gamma_0} e^{iz_1\xi}\sum_{*=L,T} e^{i|z_2|\zeta_*(\xi)}M_*(\xi, \operatorname{sgn} z_2)\Sigma_L^{(1)}(\xi)\,d\xi, \tag{3.15}$$

$$u^{\infty,1}_{(2)} = \frac{i}{4\pi}\int_{\Gamma_0} e^{iz_1\xi}\sum_{*=L,T} e^{i|z_2|\zeta_*(\xi)}M_*(\xi, \operatorname{sgn} z_2)\operatorname{sgn} z_2\zeta_*(\xi)\Sigma_L^{(2)}(\xi)\,d\xi; \tag{3.16}$$

here the spectral functions of these potentials look like this:

$$\Sigma_L^{(1)}(\xi) = -\frac{\lambda+\mu}{\xi+1-i0}\begin{pmatrix}0\\1\end{pmatrix}e^{-2i(\xi+1)}, \tag{3.17}$$

$$\Sigma_L^{(2)}(\xi) = \frac{\mu}{\xi+1-i0}\begin{pmatrix}1\\0\end{pmatrix}e^{-2i(\xi+1)}. \tag{3.18}$$

For the transverse wave, the corresponding spectral functions have the form

$$\Sigma_T^{(1)}(\xi) = \frac{\lambda+\mu}{\xi+\nu_T-i0}\begin{pmatrix}1\\0\end{pmatrix}e^{-2i(\xi+\nu_T)}, \tag{3.19}$$

$$\Sigma_T^{(2)}(\xi) = \frac{1}{\xi+\nu_T-i0}\begin{pmatrix}0\\1\end{pmatrix}e^{-2i(\xi+\nu_T)}. \tag{3.20}$$

If $\varphi > \pi/2$ and we have a wave decaying exponentially (with the distance from the wedge face), then we choose the z_1-axis in the direction of the wave fastest decay, i.e., we assume that $\theta_0 = \pi/2$ and consider the wave

$$u_p(z_1, z_2) = \begin{pmatrix}\alpha\\i\beta\end{pmatrix}e^{i\alpha z_2-\beta z_1}\chi_{\Pi_1}, \tag{3.21}$$

where $\alpha > 1$ and $\beta = \sqrt{\alpha^2 - 1}$. In this case we obtain the spectral functions

$$\Sigma^{(1)}(\xi) = \frac{-1}{\xi - i\beta}\begin{pmatrix} \beta^2\lambda + 2\mu\alpha\beta i \\ \alpha^2\lambda + \beta^2 + \mu\alpha\beta i \end{pmatrix} e^{-2i(\xi - i\beta)}, \tag{3.22}$$

$$\Sigma^{(2)}(\xi) = \frac{1}{\xi - i\beta}\begin{pmatrix} -\alpha \\ \beta i \end{pmatrix} e^{-2i(\xi - i\beta)}. \tag{3.23}$$

The next step is elimination of the nonhomogeneities in the boundary condition. We show that there exists an outgoing solution $u^{\infty,2}$ of the problem

$$\begin{cases} (E+1)u^{\infty,2} = 0 \quad \text{in } \Omega, \\ Bu^{\infty,2}|_\Gamma = -(Bu^{\infty,1} + Bu^p)|_\Gamma. \end{cases} \tag{3.24}$$

Then $u^\infty = u^{\infty,1} + u^{\infty,2}$ is a solution of (3.2).

It is easy to show that all cases of a nonzero traction of $Bu^p|_\Gamma$ studied in the preceding section can be eliminated by the use of the spectral function techniques. Indeed, we show that for the traces of type (II) there exists an outgoing solution. Suppose that in the case corresponding to (b.2) a transverse wave meets the face Γ_1 at the critical angle:

$$u_i(x_1, y_1) = \begin{pmatrix} \sin\theta_{\mathrm{cr}} \\ \cos\theta_{\mathrm{cr}} \end{pmatrix} e^{-i\nu_T(x\cos\theta_{\mathrm{cr}} - y\sin\theta_{\mathrm{cr}})} \tag{3.25}$$

(here $\cos\theta_{\mathrm{cr}} = \nu_T^{-1}$). As we have agreed, this wave only generates *a transverse* reflected wave

$$u_r(x_1, y_1) = \begin{pmatrix} -\sin\theta_{\mathrm{cr}} \\ \cos\theta_{\mathrm{cr}} \end{pmatrix} e^{-i\nu_T(x\cos\theta_{\mathrm{cr}} + y\sin\theta_{\mathrm{cr}})}. \tag{3.26}$$

Then, obviously,

$$B(u_i + u_r)|_{\Gamma_1}(x_1) = \begin{pmatrix} 0 \\ 1 \end{pmatrix} 2i\nu_T^{-1}\tan 2\theta_{\mathrm{cr}} e^{-ix_1} \tag{3.27}$$

satisfies the hypothesis of Theorem 2.9. The cases where one of the reflected waves (longitudinal or transverse) is a grazing one with respect to the opposite face can be analyzed in the same way.

In the situation where $\varphi \le \pi/2$ and case (I) occurs, i.e., on one of the faces we have the trace of an exponentially decaying wave, the arguments are also not complicated. Suppose, for instance, that

$$Bu_p|_{\Gamma_1}(x_1) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{-\beta\sin\varphi x_1 + i\alpha\cos\varphi x_1}. \tag{3.28}$$

This function also satisfies the hypothesis of Theorem 2.9. Thus, the aforesaid implies that there exists an outgoing solution $u_p^{\infty,2}$ of the problem

$$\begin{cases} (E+1)u_p^{\infty,2} = 0 \quad \text{in } \Omega, \\ Bu_p^{\infty,2}|_\Gamma = -Bu^p|_\Gamma. \end{cases} \tag{3.29}$$

The nonhomogeneities coming from $u^{\infty,1}$ are also in good agreement with Theorem 2.9. Indeed, the single and double layer potentials, as well as their derivatives, can be written as

$$v(z_1, z_2) = \int_{\Gamma_0} \sum_{* = L,T} e^{iz_1\xi + i|z_2|\zeta_*(\xi)} N_*(\xi, \zeta_*(\xi))\Sigma(\xi)\, d\xi, \tag{3.30}$$

where N_* is the product of M_* and a polynomial in ξ and ζ. The degree of that polynomial is zero for the single layer potential and is one for the double layer potential; each differentiation increases the degree by one. Thus, the normal traction operation on Γ_j can also be represented as in (3.30). Next, we note that for $z_2 \neq 0$ or $z_1 < 0$ (i.e., off

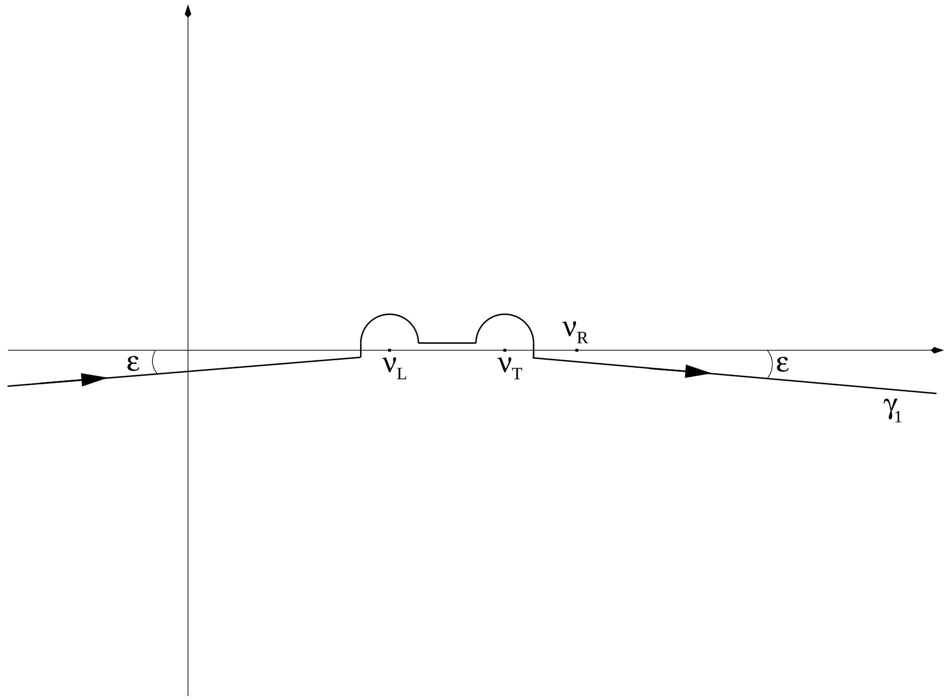

FIGURE 4. The contour γ_1 of integration.

the shadow-light boundary) we can transform the contour of integration in (3.30) into γ_1 (see Figure 4). Therefore, for all $\zeta \in \mathbb{C}_-$ the Fourier transform of $v|_{\Gamma_j}$ can be written as follows:

$$(v|_{\Gamma_j})\widehat{\ }(\zeta) = -i \int_{\gamma_1} \sum_{*=L,T} \frac{1}{\zeta - T_*(\xi)} N_*(\xi, \zeta_*(\xi)) \Sigma(\xi)\, d\xi, \tag{3.31}$$

where $T_*(\xi) = \xi \cos\psi + \sin\psi\zeta_*(\xi)$ is the "shift operator" and $\psi \in (0, \pi]$ is the angle between $\{z_1 > 0, z_2 = 0\}$ and Γ_j. The exponential terms in expressions for the spectral functions and the properties of T_* make it possible to estimate N_* roughly, showing that $(v|_{\Gamma_j})\widehat{\ }$ satisfies the hypothesis of Theorem 2.8. Let us check this statement.

In order to describe the properties of T_*, we introduce the change of variable $\xi = \nu_* \cos\theta$. In this representation, T_* adds ψ to θ:

$$T_*(\nu_* \cos\theta) = \nu_* \cos(\theta + \psi). \tag{3.32}$$

Thus, the branch of T_* continuous on γ_1, in which we are interested, acts in the following way: a point of the complex plane determined by its elliptic coordinates (α, β),

$$x = \nu_* \cos\alpha \cosh\beta,$$
$$y = -\nu_* \sin\alpha \sinh\beta,$$

i.e., the intersection of the ellipse

$$(x/\cosh\beta)^2 + (y/\sinh\beta)^2 = \nu_*^2 \tag{3.33}$$

with foci $\pm\nu_*$ and the hyperbola

$$(x/\cos\alpha)^2 - (y/\sin\alpha)^2 = \nu_*^2 \tag{3.34}$$

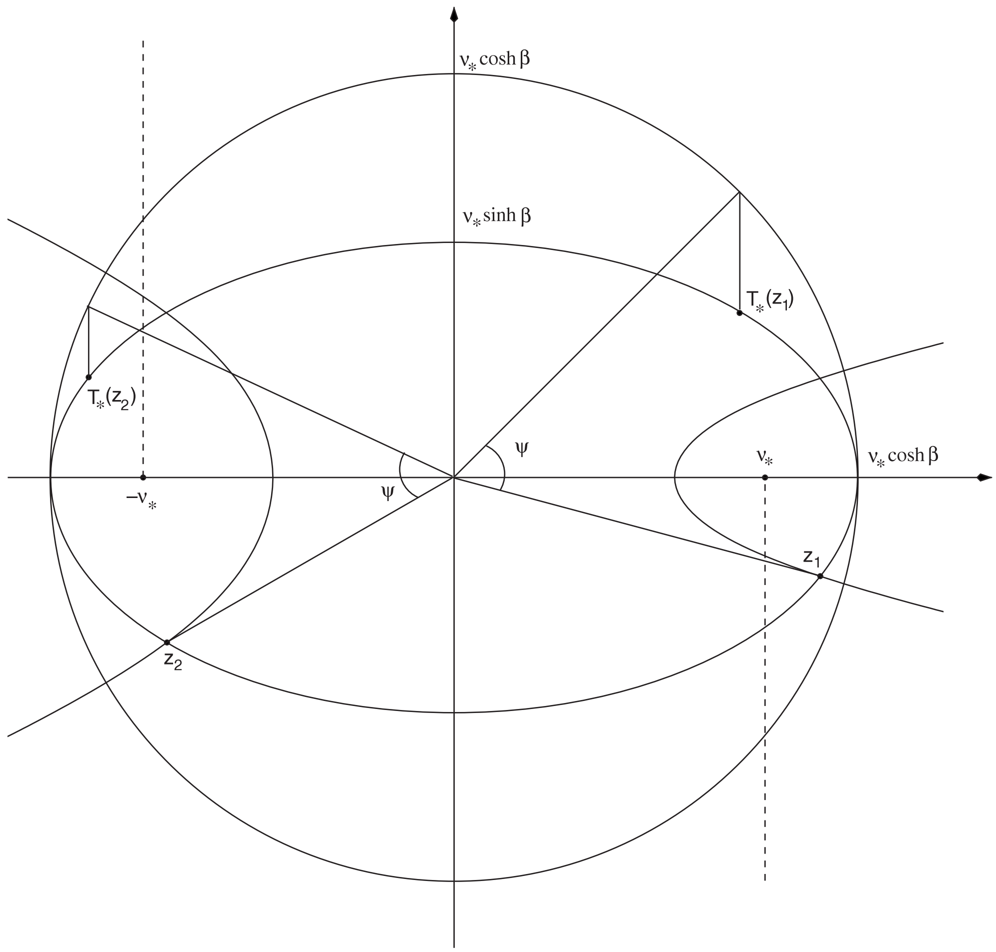

FIGURE 5. The action of T_*; the dashed lines are the branch cuts that distinguish the proper branch.

with the same foci is mapped to a point on the same ellipse; this point is determined by the simple rule illustrated in Figure 5.

We see that (3.31) determines an analytic function in $\mathbb{C} \setminus (-\infty, 1]$ with decay rate suitable for Theorem 2.8, at least "lower than" $T_*(\gamma_1)$. Indeed, to check that for the spectral functions (3.17–(3.20), (3.22), and (3.23) the corresponding $\widehat{v}^\delta(z) = \widehat{v}(e^{i\delta}z)$ belongs to H^+ for $\delta < \psi$, it suffices to investigate the image of the contour γ_1 under the action of T_* (see Figure 6): due to exponential terms in the expressions for the spectral functions, the numerator of the integrand in (3.31) decays exponentially, and for $\zeta \in \mathbb{C}_-^{\psi-\varepsilon} = \{z : \arg z \in (-\pi, \psi - \varepsilon)\}$ the denominator is of the order of $|\zeta|$. We arrive at the estimate

$$(v|_{\Gamma_j})\hat{\ }(\zeta) = O(|\zeta|^{-1}) \tag{3.35}$$

for $\zeta \in \mathbb{C}_-^{\psi-\varepsilon}$. This ensures that $(v|_{\Gamma_j})\hat{\ }(e^{i\delta}\cdot) \in H^+$, so that the hypotheses of Theorem 2.8 are satisfied. The remaining part can be treated in a similar way.

So, we have demonstrated that the traces of the potentials in question satisfy the requirements of Theorem 2.8; this proves the existence of $u^{\infty,2}$, and consequently, the existence of u^∞.

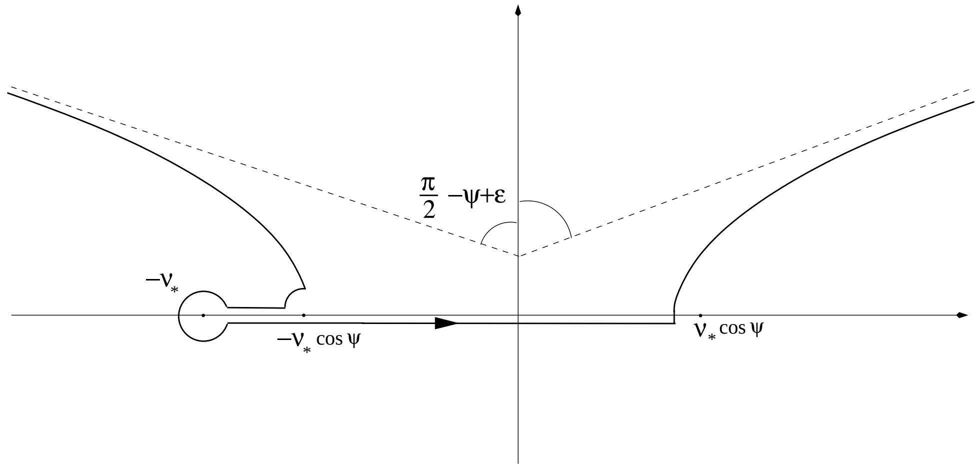

FIGURE 6. The image of γ_1 under T_*.

3.3. The local problem. The investigation of problem (3.3) will be simple, because we can apply the Green tensor. Formally, the solution of (3.3) can be written as

$$u^l(x) = -\int_{1/2<r<2} G(x,y)f(y)\,dy - \int_{\Gamma\cap\{r<1\}} G(x,y)g(y)\,dy, \tag{3.36}$$

where f denotes the nonhomogeneity in (3.3), and g is the boundary data. The "volume integral" in (3.36) can be reshaped to a sum of integrals in the usual sense: integrals along the shadow-light boundaries, and the volume integral coming from the cut-off function. The first-mentioned integrals are of the form

$$\int_{I_k} \left(G(x,y)[B(1-\psi)u_p](y) - B_y G(x,y)[(1-\psi)u_p(y)]\right)\,dy. \tag{3.37}$$

Here I_k is the interval lying on the shadow-light boundary and contained in the annulus $\{1/2 < r < 2\}$, and $[(1-\psi)u_p(y)]$ and $[B(1-\psi)u_p(y)]$ are (respectively) the jumps of the function itself and of its normal traction on that interval.

Observe that the first problem could also be solved with the help of the Green tensor, except for the degenerate cases where an oscillating function nonvanishing at infinity arises in the boundary condition, because the Green tensor contains the Rayleigh waves traveling along the wedge faces, which are nonvanishing at infinity.

So, we have established the existence of a solution of the form (3.1). To finish the proof of Theorem 3.1, it remains to discuss uniqueness. It is easily seen that all the components constructed implicitly satisfy the required growth restrictions as well as the radiation conditions. This follows from their asymptotics at infinity, which can be obtained by application of the steepest descent method for the evaluation of integral (2.12) (examples of such calculations can be found in [2] and [3]).

REFERENCES

[1] V. V. Kamotski, V. M. Babich, V. A. Borovikov, and L. Ju. Fradkin, *On Budaev and Bogy's approach to diffraction by a 2D traction-free elastic wedge*: *Theoretical aspects*, Proc. Roy. Soc. London Ser. A (to appear).

[2] J.-P. Croisille and G. Lebeau, *Diffraction by an immersed elastic wedge*, Lecture Notes in Math., vol. 1723, Springer-Verlag, Berlin, 1999. MR **2001e:**74049
[3] V. Kamotski and G. Lebeau, *Diffraction by an elastic wedge with stress-free boundary*: *existence and uniqueness*, Proc. Roy. Soc. London Ser. A (to appear).

St. Petersburg Branch, Steklov Mathematical Institute, Russian Academy of Sciences, Fontanka 27, St. Petersburg 191023, Russia
E-mail address: vladimir@pdmi.ras.ru

Received 18/FEB/2003
Translated by THE AUTHOR